On Negatively Interdependent Preferences in Rent-Seeking Contests

Inauguraldissertation
zur Erlangung des akademischen Grades
Doktor rerum politicarum
der Technischen Universität Dortmund

Sina Risse

Dortmund, May 2010

Preface

This dissertation draws on research I undertook during the time in which I held a scholarship of the Ruhr Graduate School in Economics (RGS Econ) and later while I was a teaching and research assistant at the chair of microeconomics at the Technische Universität Dortmund. The present thesis has strongly been influenced by and profited from discussions with professors and fellow students of the RGS Econ, as well as presentations at various national and international conferences. I am very grateful to all who supported my work in that way. In particular, I would like to thank Wolfgang Leininger, who supervised my dissertation. I am also very thankful to my coauthor Tobias Wenzel. Moreover, I would like to thank Burkhard Hehenkamp, Jörg Franke, Annika Herr and Tobias Guse, who all helped me to improve this thesis at various stages of its development. Financial support by the RGS Econ and Alfried Krupp von Bohlen und Halbach-Stiftung is gratefully acknowledged.

Dortmund, May 2010

Sina Risse

Contents

List of Symbols

i, j, l	index for a player
X, Y	group of players
n	number of potential players; number of players in group X
m	number of players in group Y
k	number of active players; highest index of the set $\mathcal{K}$
$\mathcal{N}$	set of potential players
$\mathcal{M}$	set of a subgroup of all players
$\mathcal{K}$	set of active players
ξ	permutation of $\mathcal{N}$
V	prize, rent
$V_n, V_m, \tilde{V}$	reduced rents
x_i	effort of player i
x_{-i}	effort of opponents of player i
$\mathbf{x}$	vector of efforts
x_{1i}, x_{2i}	effort of player i of group X in contest stage 1 resp. stage 2
y_{1j}, y_{2j}	effort of player j of group Y in contest stage 1 resp. stage 2
x^*	equilibrium effort
x^{abs}, x^{rel}	equilibrium effort of absolute resp. relative payoff maximizers of group X
y^{abs}, y^{rel}	equilibrium effort of absolute resp. relative payoff maximizers of group Y
RD	rent dissipation
R	aggregated efforts in a contest

p_i	winning probability of player i
p_i^m	winning probability of player i in the subgroup $\mathcal{M}$
p_X, p_Y	winning probability of group X resp. group Y
p_{xi}, p_{yj}	winning probability of player i of group X resp. player j of group Y
π_x, π_y	probabilities that group X resp. Y reaches the second stage
ω	constant factor for adjusting winning probabilities
r	technology parameter
$r_{max}(k)$	upper bound of r for participation of k players
δ	difference in the upper bound of the technology parameter due to heterogeneity
C_i	cost function
Π_i	payoff function
Π_{-i}	payoffs of the opponents of player i
$\bar{\Pi}$	average payoff
F_i	utility function
F_i^{abs}, F_i^{rel}	utility of absolute resp. relative payoff maximizers
α_i	preference parameter of player i
λ_i	transformed preference parameter of player i
$\bar{\lambda}$	average preferences parameter
Γ_k	harmonic mean of the first k values of the sequence $(\frac{1}{\lambda_i})$
β_i	heterogeneity parameter of player i
$\bar{\beta}_n$	average heterogeneity parameter of n players
$\tilde{\beta}_i$	difference of the weakest active player from the average heterogeneity
μ	maximum of all heterogeneity parameter β_i
$O(\mu)$	accuracy of an approximation
$\bar{x}_n$	average effort level of n players
e_i	unknown difference in the effort levels of player i to the average effort level
E	sum of all differences e_i

ε_{ij}	(cross) elasticities of the effort with respect to the preferences parameter
d	heterogeneity measure
$G(q)$	exponential distribution of the noise
a	real parameter in the exponential distribution
$g(q)$	probability density function of the noise
p	market price in duopoly
q_i	firm i's output
c	unit cost for producing the output
S_i	sales function
S	total market volume
$\tilde{g}_i$	incentive contract: linear combination of sales and profits
g_i	relative performance incentive contract
w_i	weight on payoff in incentive contract
$\bar{w}$	equilibrium weight with simultaneous contracting
w_i^*	equilibrium weight with two players sequential contracting
$\hat{w}_i$	equilibrium weight with three players sequential contracting
$\bar{\Pi}_i$	equilibrium payoff with simultaneous contracting
Π_i^*	equilibrium payoff with two players sequential contracting
$\hat{\Pi}_i$	equilibrium payoff with three players sequential contracting
a_i, b_i	weights in the manager's total compensation package

Chapter 1

Introduction

1.1 The Tullock Contest

1.1.1 Economic Relevance and Characterization of Contests

Competition in which exogenously given assets are allocated as a function of the various efforts expended by players in trying to win these assets is a very common phenomenon. These interaction for goods or rents in which players expend effort in trying to get ahead of their rivals is embraced by the term *contest*. Effort in contests can be expended in monetary form or in the form of other valuable resources depending on the external circumstances. Some areas of application of the contest theory are promotional competition (Friedman (1958), Mills (1961) and Schmalensee (1976)), litigation (Farmer and Pecorino (1999), Wärneryd (2000), Baye, Kovenock, and de Vries (2005) and Robson and Skaperdas (2008)), internal labor market tournaments (Lazear and Rosen (1981) and Rosen (1986)), R&D contests (Loury (1979) and Nalebuff and Stiglitz (1983)) and sports (surveyed by Szymanski (2003)).

A contest in this work can be characterized by the following elements:

- A (finite) set of contestants, also called players or agents, denoted by $\mathcal{N} = \{1, 2, \ldots, n\}$.
- An exogenously given prize with the valuation $V \in \mathbb{Q}_+$, which will be allocated among the players.

- A set of possible actions, yielding a vector $\mathbf{x} = (x_1, \ldots, x_n)$, $x_i \in \mathbb{R}_0^+$ $\forall i \in \{1, \ldots, n\}$ of efforts. These efforts determine the probability of obtaining the whole prize or the fraction of the prize obtained by each player. In the work at hand we focus on the 'winner takes all' interpretation which means that one player earns the whole prize.

- Function $p_i(x_1, \ldots, x_n)$ $\forall i \in \{1, \ldots, n\}$ that maps the vector of efforts into winning probabilities. Usually, this function is called *contest success function (CSF)*. For a given vector of efforts the winning probabilities p_i are between zero and one and sum up to 1.

- Function $C_i(x_i)$ $\forall i \in \{1, \ldots, n\}$ that states the cost of providing a given level of effort. We will assume in this work that $C_i(x_i) = x_i \quad \forall i \in \mathcal{N}$.

Assuming that players are risk-neutral with payoffs linear on the expected prize and costs, the payoff function of player i depends on the effort choices and is given as

$$\Pi_i(x_1, \ldots, x_n) = p_i(x_1, \ldots, x_n)V - x_i.$$

1.1.2 Rent-Seeking

The literature of contest theory has developed from the seminal contributions by Tullock (1967, 1980) and Krüger (1974) who studied a specific contest, rent-seeking. Rent-seeking is a contest in which the expended efforts are presumed to be wasted from a welfare perspective. Tullock (1967) explored this feature of rent-seeking without naming it with this specific term. He sought to explain the low profit of a firm that has monopoly power due to an entry barrier by reasoning that such a firm may have had to invest in achieving that barrier and in keeping it high. The part in his article that was to be most important in the development of the concept of rent-seeking was the investment in the activity of securing protection from the government (see Tullock (1967), p.231).[1] The term *rent-seeking* had not been introduced until Krüger (1974). Since then a whole school of researchers analyzed the implications of rent-seeking in many specific contexts such as competition for temporal and non-temporal monopoly rents, the choice between lobbying

[1] See Tullock (2003) for his account of the development of the concept rent-seeking.

and litigation (Rubin, Curran, and Curran (2001)), election campaigns, legal conflicts such as lawsuit, contested tenders and public projects, bribery, rivalry between tribes and military combats, R&D and patent races (Loury (1979), Beath, Katsoulacos, and Ulph (1989), Nti (1997)), conflict and appropriation (Garfinkel and Skaperdas (1996)), non-price competition (Huck, Konrad, and Müller (2002)) and contests between cities and countries to host prestige events such as the Olympic Games (Corchon (2000)). For an early survey of the rent-seeking literature see Nitzan (1994), for a more recent collection of papers see Lockard and Tullock (2001) and Congleton, Hillman, and Konrad (2008). The rent-seeking literature is concerned with the existence and characterization of Nash equilibria and, in particular, with the relationship between total rent-seeking outlays in equilibrium and the value of the contested rent. The ratio $RD = \sum_{i=1}^{n} x_i/V$ between these two values is called the rent dissipation. The analysis of the basic rent-seeking game depends on the assumptions made regarding the payoff function, the contest success function and the number and characteristics of the players. Players' preferences differ concerning their attitude towards risk, their valuations of the rent and, in general, their utility, which in all determines their strategy sets and their payoffs. In this work we concentrate on the varieties generated through different utility functions of the players.

1.1.3 The Tullock Contest Success Function

Perhaps the most popular contest success function, which is typically attributed to Tullock (1980), is used in several areas of economics. Tullock was the first who used it to study the problem of rent-seekers who expend resources to influence the policy outcome in their favor. Therefore, this famous contest success function is labeled after him and we call a contest with such a logit form CSF and a cost function $C_i(x_i) = x_i$ a *Tullock contest.* This CSF is a simple and tractable function, which assumes that a contestant i's probability of winning the contest depends on this contestant's own effort and the sum of all efforts

$$p_i(x_1, \ldots, x_n) = \begin{cases} \frac{x_i^r}{\sum_{j=1}^{n} x_j^r} & \text{if} \quad \max\{x_1, \ldots, x_n\} > 0 \\ 1/n & \text{otherwise} \end{cases} .$$

The probability for individual i to win the contest, p_i, is increasing in i's own effort and decreasing in rivals' effort. The technology parameter $r \in$

$\mathbb{R}_0^+ \cup \{\infty\}$ represents the efficiency of the given contest technology. r is also called the discriminatory power of the contest, because it determines how much impact a player's own effort has on his winning probability. For $r = 1$, the winning probability is equal to the share of expenditure of a player in the total expenditure. It is like a lottery in which one monetary unit buys one lottery ticket, and the winner is drawn from the set of all tickets with each ticket winning with the same probability. Therefore, a contest with $r = 1$ is called a *lottery contest*. This case of constant marginal efficiency is particularly popular because of its analytical tractability. Whereas, the parameter $r \neq 1$ is important for the marginal impact of an increase in a contestant's effort. For small values of the technology parameter, $0 < r < 1$, the marginal effectiveness decreases in effort but the reaction function is always positive as long as we have symmetric players and no entry fee. A strictly positive reaction function means that always all symmetric players participate in the contest. In the limit case $r = 0$ each player regardless of the effort has a winning probability of $p_i = \frac{1}{n}$. Therefore, none of the players would spent positive effort in the equilibrium. In contrast, for higher values, $r > 1$, the marginal effectiveness increases in effort and it is possible that not all players participate actively with a positive effort choice in the contest. As $r \to \infty$ the contest success function converges towards a function with no noise, the *all-pay-auction*, in which no pure strategy equilibrium exists.[2] In the limit case, $r = \infty$, the player with the highest effort wins the prize with certainty.

The Tullock contest success function is the underlying CSF for the whole work. In Chapter 3 we analyze the influence of different technology parameters r on the contest equilibria where we explicitly allow for $r > 1$. Afterwards, in the first two sections of Chapter 4 we assume $r < 1$ and finally in Chapter 2 and the last section of Chapter 4 we use in view of tractability the most restrictive assumption of $r = 1$.

Why is this Tullock contest so popular? Beside the tractability there are two different types of justification.

Axiomatic Foundation

Bell, Keeney, and Little (1975), Skaperdas (1996), Clark and Riis (1998)

[2] See e.g. Baye, Kovenock, and de Vries (1996).

and Kooreman and Schoonbeek (1997) give systems of axioms about how conflict is decided as a function of the players' efforts so that these sets of axioms imply that the contest success function are logit form variants of the famous Tullock contest success function. Bell, Keeney, and Little (1975) were probably the first to address this problem in the framework of promotional competition for market shares. Their pioneering work in this field, the axiomatic foundation, can be also translated into the contest framework. Skaperdas (1996) derives the Tullock contest function for a lottery contest ($r = 1$) from several intuitive axioms. Skaperdas' axioms are:

1. $\sum_{i \in \mathcal{N}} p_i(\mathbf{x}) = 1$ and $p_i(\mathbf{x}) \geq 0 \ \forall i \in \mathcal{N}$ and $x_i > 0 \Rightarrow p_i(x_i) > 0$.
2. $\forall i \in \mathcal{N} : p_i(\mathbf{x})$ is increasing in x_i, and decreasing in $x_j \ \forall j \neq i$.
3. For any permutation ξ of $\mathcal{N}$ we have $p_{\xi(i)}(\mathbf{x}) = p(x_{\xi_1}, \ldots, x_{\xi_n}) \forall i \in \mathcal{N}$.
4. $p_i^m(\mathbf{x}) = \frac{p_i(\mathbf{x})}{\sum_{j \in \mathcal{M}} p_j(\mathbf{x})} \ \forall i \in \mathcal{M}$ and $\forall \mathcal{M} \subseteq \mathcal{N}$ with $|\mathcal{M}| \geq 2$.
5. $p_i^m(\mathbf{x})$ is independent of the efforts of the players not included in the subset $\mathcal{M}$.
6. $p_i(\omega \mathbf{x}) = p_i(\mathbf{x}) \ \forall \omega > 0$ and $\forall i \in \mathcal{N}$.

Axiom 1 ensures that the winning probabilities lie in the interval $[0, 1]$ and sum up to one for all players. It is called the probability axiom. The second axiom states that raising one player's effort strictly increases this player's winning probability and decreases the rivals' winning probabilities. The axiom on the symmetry of players (axiom 3) that makes the players anonymous respectively homogeneous is often criticized in the literature. Axiom 3 implies that any two players who exert equal efforts have equal probabilities of winning the contest. The more important axioms are on invariance properties of the nature of the contest with respect to the number of participants (axioms 4 and 5). Axiom 4 implies that the contest among smaller numbers of players are qualitatively similar to those among a larger number of them. Axiom 5, the independence from irrelevant alternatives axiom, satisfies that whenever the contest is played only among a subgroup of the initial group the winning probabilities of the participating players do not depend on non-participating players. Most importantly, a zero-homogeneity axiom (axiom 6) that makes contest success probabilities invariant with respect to an increase in all contestants' effort by some given factor. Given these six axioms there is just one contest success function which satisfies all of the axioms. It is the Tullock contest success function. This one-to-one relationship between the Tullock contest and these axioms gives strong support for the use of this contest success function in actual applications.

A slightly different set of axioms is stated by Kooreman and Schoonbeek (1997) for the two players case and by Clark and Riis (1998) for n-players. For example Clark and Riis (1998) drop the anonymity axiom while they retain the other axioms. This extension of Skaperdas' work leads to a modified version of the Tullock contest success function that allows for heterogeneity between players. Hence, one may view the crucial properties of Tullock's contest success functions to be homogeneity and independence of irrelevant alternatives.

Microeconomic Underpinnings

Another important reason why the Tullock contest is often used comes from the literature that provides an economic underpinning for this mechanism. In this literature there are mostly probability models described, in which winning is a function of efforts. Such models make a strong case for the lottery contest ($r = 1$). They can be found in the framework of R&D races, innovation tournaments and patent-race games (Hirshleifer and Riley (1992) and Baye and Hoppe (2003)). To give an impression of these microeconomic underpinnings we roughly summarize Hirshleifer and Riley (1992, p.380f.). Starting point is a R&D race between two contestants in form of an all-pay auction for the fixed prize V. Player 1 wins the competition if $q_1x_1 > q_2x_2$, where x_i are the chosen effort levels and q_i are independent draws from an exponential distribution with the cumulative distribution function (c.d.f.) $G(q) = 1 - e^{-aq}$ and the probability density function (p.d.f.) $g(q) = ae^{-aq}$. The parameter a, often called the rate parameter, is a to the exponential distribution inherent real number with $a > 0$. In probability theory, the exponential distributions are a class of continuous probability distributions. Normally, they describe the times between events in a process in which events occur continuously and independently at a constant average rate. The noise that is introduced by this exponential distribution translates this all-pay auction problem into the Tullock contest problem. For a given $\hat{q}_1$ and for given (x_1, x_2), contestant 2 wins if $\hat{q}_1 x_1 < q_2 x_2$ which happens with probability

$$\begin{aligned} prob(\hat{q}_1 x_1 < q_2 x_2) &= prob(\hat{q}_1 x_1 / x_2 < q_2) \\ &= 1 - prob(q_2 \leq \hat{q}_1 x_1 / x_2) \\ &= 1 - (1 - e^{-a(\hat{q}_1 x_1 / x_2)}) \\ &= e^{-a(\hat{q}_1 x_1 / x_2)} \end{aligned}$$

Now, we integrate by parts to consider the unconditional probability for player 2 to win the all-pay auction for (x_1, x_2).

$$\begin{aligned} prob(q_1 x_1 < q_2 x_2) &= \int_0^\infty e^{-a(q_1 x_1 / x_2)} g(q_1) dq_1 \\ &= \int_0^\infty e^{-a(q_1 x_1 / x_2)} [a e^{-a q_1}] dq_1 \\ &= a \int_0^\infty e^{-a q_1 \frac{x_1 + x_2}{x_2}} dq_1 \\ &= a \left[-\frac{1}{a} \frac{x_2}{(x_1 + x_2)} e^{-a q_1 \frac{x_1 + x_2}{x_2}} \right]_0^\infty \\ &= -\frac{x_2}{x_1 + x_2} [0 - 1] \\ &= \frac{x_2}{x_1 + x_2} \end{aligned}$$

This is a straightforward foundation for using the Tullock success function as a probability for winning a contest. In this explanation it is based on the all-pay auction, but with some multiplicative noise that follows from a particular type of distribution.

Another largely unrecognized microeconomic underpinning of the logit win probability function is given in Clark and Riis (1996). They link the win probability to the behavior of the contest designer. Their approach of a discrete choice framework also adopts a random utility formulation in which it is assumed that the contestants view the contest designer as maximizing a random utility function. The players expect that the designer will determine as the winner the player who gives him the most utility. The utility depends on the effort spend by the players and on an unknown stochastic idiosyncratic bias. The interpretation to justify this utility function is that the players do not possess all relevant information about the contest designer and are not exactly certain of his preferences. Assuming that the utility maximizing contest designer has deterministic preferences which are represented by an specific additively separable logarithmic utility function and that the uncertainty of the players is captured by a specific probability distribution lead to precisely the Tullock win probability.

1.1.4 Voluntary Participation

In many contests the choice of whether or not to participate with a positive effort is decided voluntarily by the players. For an early contribution considering entry into Tullock contests see Appelbaum and Katz (1986). If the winner in a contest is awarded a prize that has a positive value for the player but all losers receive nothing, intuition suggests that a player may want to expend at least some effort trying to win the prize if there is no entry fee for the contest. This intuition is right for the case of an decreasing returns to scale technology. Perez-Castrillo and Verdier (1992) show that in the case $r < 1$, for any number of homogeneous agents n, there is always a unique equilibrium in which the expected payoffs are strictly positive. There is therefore always some incentive for a potential entrant to decide to participate in rent-seeking. If $r > 1$, then as long as n is smaller as a threshold value, any potential entrant has some incentive to enter in the game. When the threshold value is reached, an agent contemplating entry will not engage in the process of rent-seeking because by doing so he will receive a negative expected payoff.

Apart from the technology parameter the decision of a player to take actively part in a contest will generally depend on how much other players are willing to expend. This is an interesting question in the context of a heterogeneous players field. For instance, if other players have a higher valuation, and, therefore expend considerable more effort, it need not be worthwhile for a player who values the prize less to expend effort. It may be preferable not to compete at all. The theory states that in Tullock contests, if the group of contestants is large and heterogeneous, players whose valuation of the prize is low or whose ability is low may prefer not to make a positive effort. Participation with asymmetric players in the Tullock contest is addressed by Stein (2002). He focuses on heterogeneity due to different valuation of the prize and due to a different relative ability to win the prize. Whereas we concentrate on the participation decision of asymmetric players in a Tullock contest, in which the heterogeneity arises through differences in the utility functions. They differ in the parameter value of other regarding preferences. In addition we analyze the influence of the technology parameter of the contest on the participation condition (Chapter 3).

1.1.5 Nested Contests - Intra Group Contests

In a real world contest, players expend efforts trying to win some prize. The process by which the efforts translate into success probabilities is often considered as a black box. However, if we look closer at contest games, they often reveal a finer substructure and can be decomposed into a number of smaller battles on different stages of the contest. This is obvious with sports contests. Similarly, researchers in the R&D context noted that research and development is not a single one-shot event (see, e.g., Harris and Vickers (1985) and Harris and Vickers (1987)). Related problems have been analyzed in the context of political campaigns (Klumpp and Polborn (2006)), violent conflicts for territory, resources or power (Mehlum and Moene (2004), McBride and Skaperdas (2007)) and, with a multiplicity of applications in mind, by Gradstein and Konrad (1999), Amegashie (1999, 2000), McAfee (2000), Moldovanu and Sela (2006), Groh, Moldovanu, Sela, and Sunde (2008), Konrad and Kovenock (2005, forthcoming, 2009) and Matros (2006).

One special type of a grand contest with interesting efficiency properties emerges naturally from the analysis of inter-group contests. Conflict often takes place between groups. Once the inter-group contest has determined a winning group such conflict ends if the prize that is awarded to the winning group is a good that all group members can consume (public good) or if the allocation of the good within the group cannot be influenced by group members (exogenous sharing rule). In many cases, the inter-group contest is about private goods, and the conflict does not necessarily end once the contest prize is allocated to one of the groups, or once its shares are allocated to the different groups. Examples come from war, politics, sports, federalism, corporate governance and other areas of conflict. In politics, leading figures inside a political party often join forces prior to an election, trying to get their party into power. Once this goal is achieved, they may start struggling about who will obtain which office, and who will eventually become the party leader (Chapter 4).

It seems plausible that the rules that govern the allocation of the prize will affect the group members' efforts in winning the battle. Those rules can be distinguished in three classes; exogenous sharing rules, groups choice of sharing rules and an intra-group conflict as a sharing rule. We focus in this work on the case of an intra-group contest. It is often plausible that group members will struggle about the intra-group allocation of the prize, regardless how much each of them has contributed in the inter-group conflict. In this case, the intra-group allocation can be seen as the outcome of an intra-group contest in which the members' contribution to the inter-group contest are sunk and irrelevant for the intra-group allocation of the prize. The seminal paper on this topic in which the group prize is contested among the members of the winning group once the winning group has been determined is Katz and Tokatlidu (1996). They analyze this type of problem and provide the comparative statics with a focus on group size. Wärneryd (1998), who studies this problem in the context of conflict between more than two jurisdictions within a federation, and with more than two symmetric players in each jurisdiction, finds that this structure may have advantages compared to a one-stage contest in which no group structures exist and the prize is allocated in a single Tullock contest with the same number of participants. He finds that a more hierarchical structure can be advantageous as it tends to reduce the total effort that is expended in the various contests for allocating the prize. We show in Chapter 4 that the hierarchical structure is not in general preferable to a single one-stage contest.

1.2 (Negatively) Interdependent Preferences

The principal ingredient of our analysis is the assumption of interdependent preferences. Interdependent preferences could be altruistic or spiteful, altruism representing a positive and spite representing a negative interdependency, respectively. Our focus though will be on negatively interdependent preferences in rent-seeking contests. Not because we think that negatively interdependent preferences are the most relevant ones empirically, but rather because we want to point out that negatively interdependent preferences and independent preferences can result in other decisions when comparing and choosing different contest design. We say a player has negatively interdependent preferences if the material payoff of other players negatively enters his utility. In contrast, we say a player has independent preferences if the utility of his actions does not depend on the material consequences for other players. Negatively interdependent preferences reflect the widely acknowledged phenomenon of *keeping up with the Joneses* (i.e. well-being depend on relative standing in the society). The idea that the welfare of an individual depends on the absolute as well as on the relative income goes at least as far back as to Veblen (1899):

"... the desire for wealth can scarcely be satiated in any individual instance ... the ground of [this need] is the desire of every one to excel every one else in the accumulation of goods." *(Veblen (1899), p.32)*[3]

One obvious channel through which an individual's well-being could depend on others' incomes is one's envious feelings towards other. Another, maybe more important and in the Veblen (1899) citation mentioned factor is the desirability of higher status, or more generally speaking, the simple attraction towards being better off than others. We formalize this postulate through negatively interdependent preferences in this thesis.

1.2.1 'Other-Regarding' Preferences

For decades, microeconomic science was built on a specification of independent preferences that implied rational and purely self-interested behavior. Common sense and experimental as well as field evidence point to the limits of this approach. There exist some stylized facts as for example inexplicable rejections in Ultimatum games that cannot be fully accounted for by the standard approach of rational and material payoff maximizing behavior. One way to tackle this problem is to deviate from the assumption of independent preferences while maintaining the rational choice hypothesis, as we do it here. This idea has been used by a number of theorists who integrate all different kinds of other-regarding behavior into the individual preference model. We can classify most of the approaches into four prominent theories. Firstly, the theory of reciprocity, assuming agents to act in a reciprocal way. Reciprocity refers to a tendency to respond to perceived kindness with kindness, to perceived meanness with meanness and to expect

[3] Also cited in Becker (1974) and Ok and Kockesen (2000).

this behavior from others. Authors, who use the reciprocity approach are, among others, Rabin (1993), Levine (1998), Dufwenberg and Kirchsteiger (2004) and Falk and Fischbacher (2006). Secondly, the theory developed by Fehr and Schmidt (1999) and by Bolton and Ockenfels (2000) uses the idea of inequality aversion. It is based on the supposition that agents are not only interested in their own payoff, but also in their relative payoff. Agents compare their own payoff with the payoff of the others people and would like to reduce the inequality in payoffs between people. Thirdly, Charness and Rabin (2002) propose a theory of social preferences that assumes that agents care in their preference function about their own payoff, the others' payoff and, in addition, about efficiency in the allocation process. Fourthly, the theory of altruism is investigated by Andreoni and Miller (2002) and Andreoni, Castillo, and Petrie (2003). They model a concern for altruism and efficiency by defining utility functions for keeping for oneself and giving to others.

There are various assumptions in the models with respect to the interdependency of preferences. The modeler makes the assumptions when specifying the identities of the players. Our special approach of interdependent preferences joins into the other-regarding preference approaches and focuses on those preferences which maintain that well-being depend on material consumption as well as on relative standing in the society. Therefore, we assume that preferences depend on own payoff and on the payoffs of other players in the form that agents are spiteful. This thesis does not claim to have the solely right answer to the question of how to model interdependent preferences, it rather picks one kind of interdependent preference model, envious preferences, and shows that it produces in comparison to independent payoff maximization in different contest situations completely different equilibria.

1.2.2 Empirical, Experimental and Other Evidence

Nevertheless the empirical foundation of interdependent preferences is rather old. A first empirical basis for interdependent preferences is laid by Duesenberry (1949), followed beside others by Layard (1980) and Frank (1985). Frank (1985) (p.5) notes that "…abundant evidence suggests that people do in fact care much more about how their incomes compare with those of their peers than about how large their incomes are in any absolute sense.

Most poor citizens of the United States enjoy an absolute consumption standard that would be the envy of all but the richest citizens of, say, India. Yet the poor here are often said to be much less content with their lot than are the upper-middle class citizens of many poorer nations."

More recent empirical support of the interdependent preference hypothesis is given by Bush (1994a,b) and Kapteyn, van der Geer, van de Stadt, and Wansbeck (1997), who uses field data. In the last few years verification of interdependent preferences is created through laboratory experiments. Andreoni and Miller (2002) and Fishman, Kariv, and Markovits (2007) find that individual behavior can be rationalized to a great extent with a well-behaved utility function which orders different than material payoff maximization. According to them the shape of this function can be of great heterogeneity among subjects. More concrete support for the economic relevance of interdependent preferences based on experimental data is given by Levine (1998). He analyzes experimentally ultimatum games and final rounds of a centipede game and finds a specific distribution of altruism and spite in the population. The surprising fact is that a large mass of individuals are strongly spiteful. This result should be even more pronounced in a more competitive setting like a contest. There are also studies which predict much less spiteful behavior in the population (Andreoni and Miller (2008) and Offerman, Sonnemans, and Schram (1996)). Another experiment indicating negatively interdependent preferences is by Zizzo and Oswald (2000). They included a 'burning money' stage after a betting stage, in which half of the people have an advantage over the others and some of them get an additional gift. There, subjects have the chance to give up some own money to reduce other subjects' money. Zizzo and Oswald find that despite these cost the majority of the subjects choose to destroy at least part of others' money. The same result is found by Herrmann, Thöni, and Gächter (2008) in their experiment on public goods with an added punishment stage. This phenomenon cannot be explained by the standard assumption in economic theory of selfish independent payoff maximizing individuals. It supports the hypothesis of negatively interdependent preferences.

Moreover, evidence for negatively interdependent preferences is generated by the newer field of research of evolutionary game theory. Evolutionary stable behavior in two-player contests is identical to behavior of individuals with negatively interdependent preferences in a Nash equilibrium. Leininger

(2003) shows that stability of behavior according to an evolutionary stable strategy (ESS) incorporates relative concerns of players, which guarantee survival based on spiteful self-defense in the case of a finite population. Whereas Leininger (2009) applies an indirect evolutionary approach (ESP), which combines an evolutionary process at the preference level with rational choice at the action level. He determines negatively interdependent preferences as evolutionarily stable which result in the same aggressive behavior as the direct evolutionary approach. Whether evolution works at the action level or at the preference level the resulting aggressive behavior in equilibrium is same as interdependent, spiteful preferences would create under the Nash-equilibrium concept.

Eaton and Eswaran (2003) and Weibull and Salomonsson (2006) examine how preferences evolve by natural selection in a competitive environment and find that evolutionarily stable preferences exhibit a concern not only for absolute payoffs but also for relative payoffs. Their finding is consistent with the available anthropological evidence. This gives support from the evolutionary context for the hypothesis of interdependent preferences.

Furthermore, there exist empirical evidences for interdependent preferences from the point of view of the neurosciences, of heritability studies (twin and adoption studies), and of cross-cultural and development psychology. Zizzo (2003) gives an overview about the evidence provided in these studies. In particular, the different fields of studies argue about the question whether genes or environment or both together determine what interdependent preferences an economic agent holds. Since we assume interdependent preferences to be fixed in the most part of this thesis, we refer to the hypothesis that interdependent preferences are innately coded in the genes (see for this hypothesis beside others Bergstrom (1995), Hoffman, McCabe, and Smith (1998), and McCabe, Rassenti, and Smith (1998)).

1.3 Literature concerning Contests with Interdependent Preferences

Daily experience as well as experimental and field evidence strongly suggest interdependent preferences to be present in the real world. Little has been done so far to characterize the potential impact of interdependent preferences on contest outcomes in general and on rent-seeking in particular. One of a few contributions is Guse and Hehenkamp (2006). They analyze rent-seeking contests with a heterogeneous population in which part of the players are absolute payoff maximizers while others are also concerned about their relative standing. As a result, it is those players with negatively interdependent preferences who experience a strategic advantage in general two-player contests and in n-player contests with non-increasing marginal efficiency.

Another contribution to this field of research is Shaffer (2006), who examines the effect of preferences that are not independent (altruism or envy) on equilibrium rent-seeking effort and net payoffs in a logit form contest. Like Konrad (2004) Shaffer (2006) restricts her analysis to a one stage, two players contest. Konrad (2004) analyzes altruism and envy in winner-take-all contests, showing that altruistic and envy players together yield higher payoffs than players who pursue independent self-interest.

Absolute and relative payoff maximization in a contest also occurs in an indirect way in Leininger (2003) and Hehenkamp, Leininger, and Possajennikov (2004), who show that an evolutionary stable strategy (ESS) in a contest leads to relative payoff maximization, in contrast to the Nash equilibrium, which is based on absolute payoff maximization. Leininger (2009) shows that the evolutionary stable preferences (ESP) of a finite population determined by behavior in two-player contests turn out to be negatively interdependent.

We want to contribute with this thesis to this strand of literature.

1.4 Negatively Interdependent Preferences in Tullock Contests

As in Bester and Güth (1998) and Konrad (2004), we distinguish between individual's material payoffs in a Tullock contest

$$\Pi_i(x_1, \ldots, x_n) = \frac{x_i^r}{\sum_{j=1}^n x_j^r} V - x_i$$

and utility functions,

$$F_i(x_1, \ldots, x_n) = F_i(\Pi_i(x_1, \ldots, x_n), \Pi_{-i}(x_1, \ldots, x_n)),$$

that describe an ordering of outcomes according to the individuals preferences. An individual's preference depends on the own material payoff Π_i

and on the material payoff of all other players Π_{-i}. The distinction between utility and material payoff is inspired by sociobiology. There, material payoff determines the reproductive fitness of an individual and may differ from the individuals subjective feelings of well-being.

We consider in this thesis a linear shape of the objective function. The utility depends on own as well as on weighted average payoff. Hence, players have a beat-the-average criterion in mind when choosing the effort in the contest.

$$F_i(x_1, \dots, x_n) = \Pi_i + \alpha_i \frac{1}{n} \sum_{j=1}^{n} \Pi_j, \tag{1.1}$$

where $\alpha_i \in [-1, 0]$ is the individual preference parameter and specifies the spitefulness of individual i. Utility and material payoff are identical for a individual with independent preferences, meaning $\alpha_i = 0$ for those individuals. They differ for envious individuals who are concerned about their social standing. Spiteful players have a preference parameter between -1 and 0 ($\alpha_i \in [-1, 0[$). The restriction that α can only take values between -1 and 0 ensures that the utility function of each player depends more on his own payoff than on the averaged payoff.

Alternative specifications of relative payoff are also possible. For example, a player might compare his own payoff to the average payoff of the other players:

$$\tilde{F}_i(x_1, \dots, x_n) = \Pi_i + \alpha_i \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} \Pi_j.$$

To compare the utility functions we get a closer look at the first order conditions of two maximization problems. The first order condition of the utility function (1.1) is given as

$$\frac{\partial F_i}{\partial x_i} = \frac{\sum_{j=1, j \neq i}^{n} x_j}{(\sum_{j=1}^{n} x_j)^2} V - 1 - \frac{\alpha_i}{n} \tag{1.2}$$

whereas the first order condition for the alternative specification is given as

$$\frac{\partial \tilde{F}_i}{\partial x_i} = \frac{\sum_{j=1, j \neq i}^{n} x_j}{(\sum_{j=1}^{n} x_j)^2} V (1 - \frac{\alpha_i}{n-1}) - 1. \tag{1.3}$$

For the extreme case $\alpha_i = 0$ the first order conditions coincide. Solving for the maximum in the other extreme case ($\alpha_i = -1$) yields to an identical condition $(\frac{\sum_{j \neq i} x_j}{(\sum x_j)^2} V = \frac{n-1}{n})$ and therewith to the same maximum value.

Therefore, maximizing the utility function F_i is correlated to the maximization of the utility function $\tilde{F}_i$. Since we concentrate in this work on the distribution of a rent and the arising incentives to spend effort in a contest, we assume players to status seek and try to beat the average. Therefore, we use the utility function (1.1).

1.5 Delegation

Interdependent preferences can be advantageous in the sense that spiteful players earn strictly higher material payoffs than opponents do who seek to maximize their material payoffs. Kockesen, Ok, and Sethi (2000) show this advantage for certain classes of supermodular and submodular games which are symmetric with respect to material payoffs. Guse and Hehenkamp (2006) show the strategic advantage of players with interdependent preferences in general two-player contests and in n-player contests with non-increasing marginal efficiency.

Knowing about the strategic advantage of interdependent preferences this feature can be used via a strategic move in advance to a competition.

"A strategic move is one that influences the other person's choice, in a manner favorable to one's self, by affecting the other person's expectations on how one's self will behave." *(Schelling (1960), p.160.)*

Delegation, for instance in the case of firms that separate management from ownership, can be interpreted as such a strategic move. Owners may benefit from writing contracts with managers in which the compensation of the latter is based, in part, on the performance of their firm relative to that of other firms or relative to some industry average.[4] This would provide an incentive for managers to pursue the maximization of interdependent payoffs. If we abstract from the evolutionary explanation of interdependent preferences, delegation can be cited as another explanation for commitment to a behavioral rule other than material payoff maximization, which induces behavior equivalent to behavior of players with interdependent preferences.

An early analysis of strategic delegation in competitive situations can be found in the context of Cournot and Bertrand models (Fershtman and Judd

[4] See Holmstrom (1982) for this idea.

(1987) and Sklivas (1987)). Often competition can be better characterized by a contest model. Dixit (1987) is the first to analyze strategic behavior in contests. He conveys the results to the case of oligopolistic competition for a homogeneous product with unit-elastic demand, where firms compete for total market revenue by choosing different market shares. Specifically, delegation in the context of Tullock contests, as we consider here, is analyzed in Baik and Kim (1997), Wärneryd (2000), Kräkel (2002), Kräkel and Sliwka (2006) and Baik (2007). In Chapter 2 we extend this strand of literature through the consideration of relative performance contracts and sequential commitment to a contract.

1.6 Structure of the Thesis

The thesis deals with the changes in the contest outcome emerging through the consideration of negatively interdependent preferences. Before concentrating on the main topic, we show in Chapter 2 how contracts can arise, which induce people to act as if they have interdependent preferences. Chapter 3 introduces interdependent preferences into the framework of a one-stage Tullock contest. First, we analyze the participation condition of players with homogeneous interdependent preferences when the contest is characterized by an increasing returns to scale technology. Afterwards, asymmetry due to different interdependent preferences is added, and the reminder of Chapter 3 concentrates on the effort decision of heterogeneous players in a Tullock contest. After introducing interdependent preferences in the standard Tullock contest we concentrate on the effect of interdependent preferences in more complicated contest structures. In Chapter 4 the efficiency of a two-stage grand contest (inter- and intra-group stage) and a single Tullock contest as well as the efficiency of another two-stage contest are compared.[5] The thesis concludes in Chapter 5 with a summary of the main insights, a discussion on the results, and areas for future research.

[5]Content and results of Sections 4.1 - 4.3 are published in Risse (2010). The original publication is available at www.springerlink.com. ©Springer-Verlag

Chapter 2

Delegation in Competitions

2.1 Introduction

The purpose of this chapter is to clarify the incentives to commit to behavioral rules different from pure material payoff maximization in competitions. It is shown that commitment to a behavioral rule can lead to strategic advantages. There are two interpretations of the behavioral rule commitment setting. In the indirect evolutionary approach (Güth and Yaari (1992)) commitment operates through preferences. Whereas we focus in this chapter on the second approach, on delegation. Delegation, for instance in the case of firms that separate management from ownership, can be used as a commitment to a behavioral rule of the firm owners in games. Owners, whose payoff are given by the game payoff function, may benefit from writing contracts with managers in which the compensation of the latter is based, in part, on the performance of their firm relative to that of other firms or relative to some industry average. This would provide an incentive for managers to act more spiteful in the competition than the payoff maximizing owner. This constructs a setting as if the managers pursue the maximization of interdependent utility functions.

An early analysis of strategic delegation in competition situations can be found in the context of Cournot and Bertrand models (Fershtman and Judd (1987) and Sklivas (1987)). They focus on management contracts based on sales and payoffs that are chosen simultaneously. We extend their work by introducing sequential contracting and as a second focus relative performance contracts (Section 2.2).

Beside Cournot and Bertrand models competition is often stylized in a contest model. Dixit (1987) is the first to analyze commitment to a behavioral rule through delegation in contests. Later, Baik and Kim (1997), Baik (2007), Wärneryd (2000), Kräkel (2002) and Kräkel and Sliwka (2006) focus their analysis on delegation in the context of a Tullock contests, as we consider here. The last two analyze linear incentive schemes for managers based on sales and profits, Kräkel (2002) for two players and Kräkel and Sliwka (2006) for the n-player case. Kräkel (2002) shows that there exists a strategic advantage of precommitment in an all-pay auction and that there is none in Tullock contests with two players. The resulting outcome depends on the number of players. In the two-player contest the optimal delegation involves giving agents incentive to maximize owner's payoff while in contests with more than two players commitment to another strategy creates an evolutionary stable equilibrium. Their work is the closest to the Section 2.3 of this thesis, but in contrast to them we analyze relative performance contracts instead of sales and profits as the delegation contract space. Possajennikov (2009, 2008) goes a step further and generalizes these results on delegation in general contests, relates them to properties of indirect evolution and delegation in general symmetric games.

The chapter is organized as follows[1]: It starts with a note on delegation in Cournot competition (Section 2.2). Afterwards it follows a section about delegation in Tullock contests (Section 2.3). The main focus of both sections lies in the calculation of the respective equilibria. We assume the structure of the game as exogenously given. Therefore, the firm owners have to delegate the decision in the competition stage.

2.2 Delegation in Cournot Competitions

2.2.1 Introduction

The aim of this section is to study managerial delegation in Cournot competitions where management contracts are chosen sequentially. In the seminal papers by Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987)

[1] The papers which are the basis for this chapter are joint work with Tobias Wenzel.

attention is restricted to management contracts that are chosen simultaneously. In these papers, the owners of a firm may offer a contract to a manager. There are more than enough managers available on the job market. Hence, they accept the offered contracts. After this contract stage, the manager decides how much output to produce in a Cournot competition. The purpose of our section is to investigate the changes that arise when contracts are chosen sequentially, that is, one firm can commit to a contract before the other firm can do so.

This framework is chosen to study the behavior of market leaders or dominant firms and its impact on the market outcome. Behavior of market leaders is often modeled by assuming that the market leader can commit to an action before smaller competitors can do. Formally, this is done by choosing the sequential time structure with the market leader choosing first and smaller firms following. The previous literature has been concerned with, for instance, investments in capacity, cost reducing-investments, multi-market contact, bundling of goods and so on.[2] This paper complements this literature by analyzing the behavior of a market leader when the leader can commit to management incentive contracts.

The present section shows that if contracts are chosen sequentially, the leader chooses a contract which induces the manager to become more aggressive in the market stage. The follower chooses a contract that leads to less aggressive management behavior. There is a first-mover advantage. The leader earns higher profits than the follower. We consider two classes of incentive contracts: firstly, contracts that are based on profits and sales and secondly, relative performance contracts that are based on own and competitors' profits. Our results holds for both classes of contracts, but are more pronounced for relative performance contracts.

We analyze the welfare implications of our model. Efficiency under sequential contracting is higher than under simultaneous contracting. The reason is that sequential contracting leads to higher output and lower prices.

Delegation games have received widespread attention since the mid-eighties: Among others, the decision whether or not to hire a manager at all has been studied by Basu (1995) and Lambertini (2000). Different classes of contracts have been considered, market share contracts (Jansen, van Lier, and

[2] See, for instance, Fudenberg and Tirole (1984), Tirole (1988) or Etro (2006, 2007).

van Witteloostuijn (2007), Ritz (2008)), and relative performance contracts (Salas Fumas (1992), Miller and Pazgal (2001)). Recently, Mujumdar and Pal (2007) extend contracting to dynamic production environments.

The first part of the chapter proceeds as follows. In Section 2.2.2, we outline a model setup with linear profit-sales contracts and study the simultaneous contracting benchmark. In Section 2.2.3, we analyze the outcome under sequential contracting. Section 2.2.4 extends our results to a three-firm oligopoly. Section 2.2.5 considers relative performance contracts. In Section 2.2.6 we conclude.

2.2.2 The Model

Consider a duopoly market where demand is given by $p = 1 - q_1 - q_2$, where p is the price and q_i is firm i's output. There is a unit-cost of $c < 1$ for producing the output. Firm i's profit function is

$$\Pi_i = (1 - q_1 - q_2 - c)q_i, \tag{2.1}$$

and its sales function is

$$S_i = (1 - q_1 - q_2)q_i. \tag{2.2}$$

Firm owners do not decide on output, but delegate this decision to a manager. The manager is offered an incentive contract which is a linear combination of profits and sales:

$$\tilde{g}_i = w_i \Pi_i + (1 - w_i)S_i, \tag{2.3}$$

where w_i is the weight on profits. Managers strive to maximize income from this contract. As in Fershtman and Judd (1987), the manager's total compensation package is given by $a_i + b_i \tilde{g}_i$, $b_i > 0$ where a_i and b_i are chosen by owner i so that the manager's compensation just equals his reservation value. It is further assumed that this reservation value is zero so that it will always pay for an owner to delegate the decision.

The game we study consists of two stages, the contract stage and the market stage. In the contract stage, firm owners decide on the contract offered to managers. As our benchmark case, we analyze simultaneous contracting as in the Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987) models. After the contracting stage whose outcome is announced publicly, the market stage follows where managers decide on output (Cournot competition). We look for a subgame-perfect equilibrium.

Let us first study our benchmark case with simultaneous contracting. In the market stage, managers choose quantities to maximize income from the incentive contract:

$$\tilde{g}_i = w_i(1 - q_i - q_j - c)q_i + (1 - w_i)(1 - q_i - q_j)q_i. \tag{2.4}$$

This yields the following quantities chosen in the market stage:

$$q_i = \frac{1 + w_j c - 2w_i c}{3}. \tag{2.5}$$

Inserting (2.5) into (2.1), profits can be expressed depending on the contract parameters (w_i, w_j):

$$\Pi_i = \frac{(1 + w_i c + w_j c - 3c)(1 + w_j c - 2w_i c)}{9}. \tag{2.6}$$

Firm owners choose simultaneously the incentive contract to maximize profits, that is, the owner of firm i chooses the weight w_i to maximize (2.6).[3] In equilibrium, firm owners choose the following incentive contracts:

$$\bar{w} = \bar{w}_1 = \bar{w}_2 = \frac{6c - 1}{5c}. \tag{2.7}$$

The weight on sales $(1 - \bar{w})$ is strictly positive since $c < 1$. That is owners have an incentive to deviate manager incentives away from pure profit maximization.

Corresponding profits are:

$$\bar{\Pi}_1 = \bar{\Pi}_2 = \frac{2(1 - c)^2}{25}. \tag{2.8}$$

Firms choose the same incentive contract resulting in the same profits for both firms. Those profits are lower than the resultant profits in the Nash equilibrium without delegation. Hence, the firms would prefer not to delegate if they have the choice. In this framework they do not have the choice. Delegation is exogenous given, because the focus lies in the equilibria differences between simultaneous and sequential contracting and not in the differences between delegation and no delegation.

[3]The second-order conditions are fulfilled.

2.2.3 Sequential Contracting

Suppose now that contracting takes place sequentially. Assume that firm 1 announces the contract with its manager first. The owner of firm 2 observes this contract and then offers a contract to his manager. The outcome from the market stage is identical to simultaneous contracting (Eq.(2.5)). Observing the contract of firm 1, the owner of firm 2 chooses a contract with weight

$$w_2(w_1) = \frac{6c - 1 - w_1 c}{4c}. \tag{2.9}$$

Anticipating the optimal choice by firm 2, profits of firm 1 are:

$$\Pi_1 = \frac{(1 + w_1 c + w_2(w_1)c - 3c)(1 + w_2(w_1)c - 2w_1 c)}{9}. \tag{2.10}$$

Maximization with respect to w_1 yields the optimal weight chosen by the owner of firm 1:

$$w_1^* = \frac{4c - 1}{3c}. \tag{2.11}$$

Plugging into (2.9) gives the optimal weight for the incentive contract of firm 2:

$$w_2^* = \frac{7c - 1}{6c}. \tag{2.12}$$

Equilibrium incentive contracts are asymmetric. As $w_1^* < w_2^*$, firm 1 puts more weight on the sales part, hence this firm induces its manager to be more aggressive in the subsequent market stage. Comparison with the benchmark case shows that $w_1^* < \bar{w} < w_2^*$, that is, compared to simultaneous contracting, the leader (follower) puts more (less) weight on sales. Corresponding profits are:

$$\Pi_1^* = \frac{(1 - c)^2}{12}, \tag{2.13}$$

$$\Pi_2^* = \frac{(1 - c)^2}{18}. \tag{2.14}$$

As $\Pi_1^* = \frac{3}{2}\Pi_2^*$, firm 1 earns higher profits than firm 2. Comparing the equilibrium profits of the sequential contracting with those of the simultaneous contracting shows $\Pi_1^* > \bar{\Pi} > \Pi_2^*$. Hence, there is a first-mover advantage.

Summarizing our results:

Result 2.1 *When incentive contracts are chosen sequentially*

i) the leader puts more weight on sales and earns higher profits than the follower, and

ii) compared to simultaneous incentive contracts, the leader (follower) puts more (less) weight on sales and earns higher (lower) payoff.

We can now turn to a welfare comparison between simultaneous and sequential contracting. Output under sequential contracting exceeds output under simultaneous contracting which in turn leads to lower prices, $(1+4c)/5$ when contracts are announced simultaneously and $(1+5c)/6$ when contracts are announced sequentially. As prices are now closer to marginal costs efficiency increases. Consumer and total welfare increases while industry profits decrease.

Result 2.2 *Sequential contracting generates greater output, lower prices, higher total welfare and lower industry profits than simultaneous contracting.*

2.2.4 The Three-Firm Case

We are interested whether our results from the two-firm case are robust with respect to the number of firms in the market. Therefore, we study an oligopoly market with three firms. Assume that firm 1 announces its contract first, firm 2 second, and firm 3 last. As the analysis follows along the same lines as the duopoly case, we skip derivations and present immediately optimal contract weights:

$$\hat{w}_1 = \frac{13c - 4}{9c}, \tag{2.15}$$

$$\hat{w}_2 = \frac{11c - 2}{9c}, \tag{2.16}$$

$$\hat{w}_3 = \frac{10c - 1}{9c}. \tag{2.17}$$

As $\hat{w}_1 < \hat{w}_2 < \hat{w}_3$, our main result is robust: A firm announcing its contract earlier chooses a contract design which induces more aggressive behavior in the market stage. Again profits are higher the earlier a firm commits to its incentive contract: $\hat{\Pi}_1 > \hat{\Pi}_2 > \hat{\Pi}_3$.

2.2.5 Relative Performance Contracts

Until now we have focused on profit-sales contracts. Whereas in strategic management and marketing relative market share is often used as an indicator for the market position of a company. Therefore, it is reasonable to compensate managers through a relative comparison of performance figures. Especially in the duopoly case, in which the direct competition is distinct, relative performance contracts for managers are worthy of consideration. Here, we turn our attention to the class of relative performance contracts, in which manager compensation depends on own profits and competitor's profits:

$$g_i = \Pi_i + \alpha_i \Pi_j, \tag{2.18}$$

where $\alpha_i \in [-1, 0]$ is a weight on competitor's profits.

We follow the same steps as in Sections 2.2.2 and 2.2.3. Given incentive contracts characterized by α_1 and α_2, the outcome in the market stage is given by

$$q_i = \frac{(1-\alpha_i)(1-c)}{3-\alpha_i-\alpha_j-\alpha_i\alpha_j}, \tag{2.19}$$

leading to profits of

$$\Pi_i = \frac{(1-\alpha_i)(1-\alpha_i\alpha_j)(1-c)^2}{(3-\alpha_i-\alpha_j-\alpha_i\alpha_j)^2}. \tag{2.20}$$

Simultaneous Contracting

In the case of simultaneous contracting, both firm owners choose simultaneously the contract parameters α_1 and α_2 as to maximize Eq.(2.20). There exists a symmetric equilibrium in which both firm owners choose $\bar{\alpha}_1 = \bar{\alpha}_2 = -\frac{1}{3}$. This leads to firm profits of $\bar{\Pi} = \frac{3}{32}(1-c)^2$.

Sequential Contracting

In the case of sequential contracting, in the second stage of the game firm 2 chooses its incentive parameter α_2 for given choice by firm 1:

$$\alpha_2 = \frac{\alpha_1 + 1}{3\alpha_1 - 1}. \tag{2.21}$$

Anticipating the choice of firm 2, profits of firm 1 at the relevant stage can be expressed as:

$$\Pi_1 = \frac{(1-3\alpha_1)(1-c)^2}{16(1-\alpha_1)}. \tag{2.22}$$

This expression strictly decreases in α_1. Hence, the owner of firm 1 chooses the minimum value possible, that is, $\alpha_1^* = -1$. Plugging into the choice function by firm 2 gives, $\alpha_2^* = 0$. That is the leader chooses to be a relative profit maximizer, and the follower chooses to be a pure profit maximizer. The leader then earns profits of $\Pi_1^* = \frac{2}{16}(1-c)^2$ and the follower earns profits of $\Pi_2^* = \frac{1}{16}(1-c)^2$. That is $\Pi_1^* = 2\Pi_2^*$.

The main result of the case with profit-sales contracts survives: Compared to simultaneous contracting, with sequential contracting the leader chooses the more aggressive contract, offers a larger quantity, and earns higher profits.

In general, firms do always earn higher profits when relative performance contracts are employed than compared to the case with profit-sales contracts. That is under simultaneous (sequential) contracting firms own higher profits when relative performance contracts are employed. However, it should be noticed that the advantage of the leader is larger when relative performance contracts are employed. Here, the leader earns twice as much as the follower compared to 1.5 as much in the case of profit-sales contracts.

2.2.6 Conclusion

The aim of this section is to study sequential contracting in a delegation game with Cournot competition. Firms may engage in sequential contracting if one has some sort of advantage enabling it to move first. We show that optimal contracts are asymmetric. The leader puts more weight on sales or relative profits in the contract proposed to its manager. Hence, he chooses a contract that induces more aggressive behavior in the market stage. This leads to an asymmetric oligopoly outcome with the leader at the contracting stage earning higher profits than the follower. Contracting first gives a strategic advantage.

2.3 Delegation in Tullock Contests

2.3.1 Introduction

Competition can often be better characterized by a contest model. Contest in the basic form means the expenditure of resources in order to influence the allocation of a fixed demand of a certain product. Dixit (1987) analyzes strategic behavior in contests and conveys the results to the case of oligopolistic competition for a homogeneous product with unit-elastic demand, where firms compete for total market revenue V by choosing different market shares. Furthermore, he mentions rent-seeking as another application of delegation in the contest model. There the awarded prize is for example a contract or an important license and the expended effort is wasted from a social welfare viewpoint. In an analogous manner it is valid for a more natural setting, patent races.

Specifically, delegation in the context of Tullock contests, as we consider here, is analyzed in Baik and Kim (1997), Baik (2007), Wärneryd (2000), Kräkel (2002) and Kräkel and Sliwka (2006). Closest to our research are the last two, but in contrast to us they analyze linear incentive schemes for managers based on sales and profits, Kräkel (2002) for two players and Kräkel and Sliwka (2006) for the n players case. Whereas, we concentrate on relative performance contracts in this section.

The chapter proceed as follows. In Section 2.3.2, we outline a model setup with relative performance contracts and study shortly the results of the benchmark case of no delegation. In Section 2.3.3, we analyze the outcome under simultaneous contracting. If we have more than two firms in the market, there will exist a unique subgame perfect equilibrium which is symmetric and different from the equilibrium without delegation. Afterwards, in Section 2.3.4 we give an outlook on the results in the sequential contracting case. In Section 2.3.5, we conclude.

2.3.2 The Model

Model Setup

We consider a market with $n \geq 3$ risk neutral firms. The number of firms is exogenously given. Those n firms compete in a contest through expending effort for a common prize $V > 0$. Firm owners do not decide on contest effort directly. They have a shortage of time due to many other duties, so that they have to delegate this decision to a manager. Delegation is exogenous in the model and happens on a first, namely the contract stage of the game. The firm owners influence the managers behavior in the contest by designing an incentive contract based on relative market performance at the first stage.

An alternative interpretation of this contract stage, which would not change the analysis and the results, would be that there are managers with different intrinsic preferences. Firm owners can observe these exogenously given preferences and choose one manager out of this group. There are enough managers, so that each firm owner can freely choose. Hence, the contract stage would be to choose delegates with the desirable corresponding preference.

After the contract stage follows a second stage, the contest stage, in which the n duly accredited managers, one delegate per firm, decide about the effort to spend in the contest. The contest is an imperfectly discriminating rent-seeking contest for the rent V. In such an imperfectly discriminating contest it is not guaranteed that the highest effort wins. This feature is captured by the following contest success function (CSF) proposed by Tullock (1980)

$$p_i(x_1, \ldots, x_n) = \frac{x_i}{\sum_{j=1}^{n} x_j},$$

where x_i is the effort spend by the manager of firm i.

Consequently, the profit to the owner of firm i is

$$\Pi_i(x_1, \ldots, x_n) = \frac{x_i}{\sum_{j=1}^{n} x_j} V - x_i. \tag{2.23}$$

Managers choose their effort in order to maximize their utility. The utility function arises from the designed contract with the firm owners. As in Fershtman and Judd (1987) and the previous section, the manager's compensation consists of two parts and is given by $a_i + b_i g_i$, where $b_i > 0$.

The first part a_i is performance independent and $b_i g_i$ is the performance dependent part of the contract, which is chosen by the owners. a_i and b_i are chosen by owner i so that the manager's compensation just equals his reservation value. It is further assumed that this reservation value is zero so that it will always pay for an owner to delegate the decision. Unlike existing models we assume that owners offer delegates a contract based on relative performance evaluation:

$$g_i(x_1, \ldots, x_n) = \Pi_i + \alpha_i \frac{\sum_{j=1}^n \Pi_j}{n}, \tag{2.24}$$

where $\alpha_i \in [-1, 1]$. The parameter α_i is the weight put on average industry profits. We assume $-1 \leq \alpha_i \leq 1$ to ensure that the utility of each manager depends stronger on the payoff of the own firm than on the average industry payoff. Managers with a contract with $\alpha_i \in [-1, 0[$ do not only care about their own firms material payoff, they care more about the firm's material payoff compared to the average material payoff of all active firms in the market. The contract induces managers to behave as if they have negatively interdependent preferences. Their utility increases with own material success and decreases with the material success of competitors. Therefore, they are envious. The degree of enviousness depends on the chosen weight α_i in the contract. When $\alpha_i = 0$, this corresponds to pure profit maximization, and hence no delegation. Managers with a contract weight of $\alpha_i \in]0, 1]$ behave like altruists. They are concerned with the profit of the other firms.

Delegates attempt to maximize income from this incentive contract. The contract is a linear combination of own profits and average industry profits. We call the resulting preference interdependent, because it depends beside the own payoff on the rivals' payoffs. Substituting Eq.(2.23) into Eq.(2.24) the contract can be expressed as:

$$g_i(x_1, \ldots, x_n) = \frac{x_i}{\sum_{j=1}^n x_j} V - x_i + \frac{\alpha_i}{n} \left(V - \sum_{j=1}^n x_j \right). \tag{2.25}$$

Summarizing, we consider the following game. In the first stage, the contract stage, firms decide simultaneously on the incentive contracts for their delegates and announce them publicly. In the second stage, the contest, the delegates choose their effort levels simultaneously. We look for a subgame-perfect equilibrium of the game.

Benchmark without Delegation

To filter out the impact of delegation, we briefly report the results of the model without delegation. The game without the contract stage would result in a standard one-stage Tullock contest with the absolute payoff maximizing firm owners as a homogeneous players field. This reduced game results in a symmetric Nash-equilibrium in which each contestant expends $x_i^* = \frac{n-1}{n^2}V$ effort and earns $\Pi_i^* = \frac{1}{n^2}V$ expected payoff. The participation condition, that each contestant has a positive expected payoff, is fulfilled and all potential players take part in the contest. Hence, the aggregate effort in the contest is given by $\frac{n-1}{n}V$ and therewith the ratio between aggregated effort and rent, the rent dissipation, by $RD = \frac{\sum x_i^*}{V} = \frac{n-1}{n}$. From a welfare point of view these efforts are unproductive waste in the framework of rent-seeking. The arising question now is whether delegation leads to the same or to another equilibrium.

2.3.3 Simultaneous Contracting

We solve the two-stage game using the technique of backward induction. Therefore, we start the analysis with the second, the contest stage.

Contest Stage

In the contest stage, we have to consider that players can be heterogeneous due to different incentive contracts negotiated in the contract stage. We sort the field of players via the weight put on average industry profits in their contracts $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n$ that are endogenously determined in the first stage. To obtain a pure strategy Nash equilibrium solution for this contest we follow and extend the framework of Stein (2002). He investigates the properties of a single stage n players rent-seeking contest with different valuations and/or different abilities among the players and obtains a pure strategy Nash equilibrium solution. We modify this framework by introducing heterogeneity due to different incentive contracts. Different incentive contracts mean, that the underlying contest success function and the value of the rent are still the same for all players but they have diverse utility functions.

It is possible to characterize the form of the resulting effort vector in the equilibrium $(x_1^* \ldots, x_n^*)$. In this rent-seeking contest we may claim that there are n potential players, but there is no reason to assume that all will make positive expenditures. Therefore, assume that the number of active players is $k \leq n$, $k \in \mathbb{N}$. Since the expected payoff of player i is concave in x_i for all $\alpha_i \in [-1, 1]$, it is optimal for a player to use an interior point solution if one exists. Guse (2006) shows that the participation problem in a contest with interdependent preferences of this type can be translated in an incentive problem with heterogeneous rent valuations but independent preferences. According to Guse (2006) and Fang (2002) a player in a contest with heterogeneous valuations $\frac{k}{k+\alpha_i}V$ will only exert positive effort in equilibrium if the aggregate effort of all active players does not exceed the own valuation. That is

$$\frac{k}{k+\alpha_i}V > \sum_{j=1}^{k} x_j. \tag{2.26}$$

Hence, assume $k \leq n$ as the largest number for which this condition holds. Players with small values of α_i, meaning more spiteful players, will make positive expenditures whereas those with sufficiently high values will choose to make no expenditure and thus will not be part of the contest. The condition depends on the equilibrium efforts. After finding the equilibrium we can state this participation condition only in dependence to the preference parameter.

To simplify the analysis we introduce now the following auxiliary-parameter

$$\lambda_i = \frac{k+\alpha_i}{k}.$$

From $\alpha_i \in [-1, 1]$ it follows that $\lambda_i \in \left[1 - \frac{1}{k}, 1 + \frac{1}{k}\right] \subseteq \left[\frac{1}{2}, \frac{3}{2}\right]$. Note that the most spiteful player who has the smallest α_i is assigned to the smallest λ_i value. Hence, the transformation of α_i to λ_i is just an adjustment of the intervals. From $\alpha_1 \leq \cdots \leq \alpha_k$ it follows that $\lambda_1 \leq \cdots \leq \lambda_k$.

In the contest stage of the game, delegates decide on effort expended in the contest. They try to maximize their income out of the incentive contract which they got in the first stage. Therefore, the first-order condition of player i for the incentive Eq.(2.25) is

$$\frac{\partial g_i}{\partial x_i} = \frac{\sum_{j \leq k} x_j - x_i}{(\sum_{j \leq k} x_j)^2}V - 1 - \frac{\alpha_i}{k}. \tag{2.27}$$

It can be verified that the expected payoff and therewith the utility is concave in each x_i. Then, $\frac{\partial g_i}{\partial x_i} \overset{!}{=} 0$ implies that for any $x_i > 0$:

$$x_i = \sum_{j \leq k} x_j - \frac{(\sum_{j \leq k} x_j)^2 \lambda_i}{V} \tag{2.28}$$

This equation states that the x_i with $x_i > 0$ are ordered in the opposite way as the λ_i; $x_1^* \geq x_2^* \geq \cdots \geq x_k^* > 0$ for some index k and $x_i^* = 0$ for $i > k$.

Now sum (2.28) over all $i = 1, \ldots, k$ and solve:

$$\begin{aligned}
& \sum_{i \leq k} x_i = \sum_{i \leq k} \left[\sum_{j \leq k} x_j - \frac{(\sum_{j \leq k} x_j)^2 \lambda_i}{V} \right] \\
\Leftrightarrow \quad & \sum_{i \leq k} x_i = \sum_{j \leq k} x_j \left[k - \frac{(\sum_{j \leq k} x_j) \sum_{i \leq k} \lambda_i}{V} \right] \\
\Leftrightarrow \quad & 1 = k - \frac{(\sum_{j \leq k} x_j) \sum_{i \leq k} \lambda_i}{V} \\
\Leftrightarrow \quad & \sum_{i \leq k} x_i = \frac{(k-1)V}{\sum_{i \leq k} \lambda_i} \\
\Leftrightarrow \quad & \sum_{i \leq k} x_i = \frac{k-1}{k} V \Gamma_k
\end{aligned} \tag{2.29}$$

where $\Gamma_k = \left[\frac{1}{k} \sum_{i \leq k} \lambda_i \right]^{-1}$ is the inverse of the arithmetic mean of the first k values of λ_i or in other words the harmonic mean of the first k values of the sequence $(\frac{1}{\lambda_i})$.

Equation (2.29) can be used with (2.28) in order to yield an expression for the equilibrium strategy for player i for all $i \leq k$:

$$x_i^* = \frac{k-1}{k} V \Gamma_k \left[1 - \frac{(k-1)\Gamma_k \lambda_i}{k} \right] \tag{2.30}$$

Given the optimal effort levels we can develop the participation condition depending on the preference parameter. Player $i \in \{1, \ldots, k\}$ is active, if:

$$x_i^* > 0 \quad \Leftrightarrow \quad \lambda_i < \frac{\sum_{j \leq k} \lambda_j}{k-1} \tag{2.31}$$

From (2.30) we can also compute the probability that player i, $i \leq k$, wins the contest:

$$p_i = \frac{x_i}{\sum x_j} = 1 - \frac{(k-1)\Gamma_k \lambda_i}{k}$$

Since $0 < \lambda_1 \leq \cdots \leq \lambda_k$ it follows that $p_1 \geq \cdots \geq p_k > 0$. Player i is said to be more successful than player j if $p_i > p_j$.

The rent dissipation (RD), the total expenditures by all the players divided by the rent V, is given by

$$RD = \frac{k-1}{k}\Gamma_k.$$

The rent dissipation depends on the harmonic mean of the sequence $(\frac{1}{\lambda_i})$. If one of the preference parameter λ_i is increased and another one λ_j is decreased by the same amount, then the rent dissipation does not change as long as the number of participating players, k, is unaffected.

By direct computation, $\frac{\partial RD}{\partial \lambda_i} = -\frac{k-1}{(\sum \lambda_i)^2} < 0$ and $\frac{\partial^2 RD}{\partial \lambda_i^2} = \frac{2(k-1)}{(\sum \lambda_i)^3} > 0$.

Assuming k is not affected, this leads to the following result:

Result 2.3 *The rent dissipation decreases with an increase in λ_i resp. in α_i.*

An increase in the preference parameter means that player i has got a less spiteful contract in the first stage. If a player acts less spiteful it implies that he expends less effort and therewith the rent dissipation decreases.

Example Assume a contest with $n = 3$ and $V = 1$, where the players are characterized by $\lambda_1 = \frac{2}{3}$, $\lambda_2 = \frac{5}{6}$ and $\lambda_3 = 1$. In competitive equilibrium they will choose $x_1^* = \frac{28}{75} \approx 0.373$, $x_2^* = \frac{4}{15} \approx 0.267$ and $x_3^* = \frac{4}{25} = 0.16$, resulting in a rent dissipation of $RD = \frac{4}{5}$. Now, increase the preference parameter for one player, e.g. for the second player by assuming $\lambda_2 = 1$. In competitive equilibrium the new set of players will choose $\tilde{x}_1^* = \frac{3}{8} = 0.375$, $\tilde{x}_2^* = \frac{3}{16} = 0.1875$ and $\tilde{x}_3^* = \frac{3}{16} = 0.1875$, resulting in a rent dissipation of $\tilde{RD} = \frac{3}{4}$. The rent dissipation in the latter setting is smaller than in the former as Result 2.3 states.

Contract Stage

Giving the optimal behavior of the managers in the second stage we can analyze the optimal relative performance contracts between firm owners and managers on the first stage. Firm owners choose the contract such that they maximize indirectly their profit. Given the chosen effort on the second stage the payoff of the firm owners is given by

$$\Pi_i = \frac{\left[1 - \frac{(k-1)\Gamma_k \lambda_i}{k}\right]}{\sum_{j\leq k}\left[1 - \frac{(k-1)\Gamma_k \lambda_j}{k}\right]} V - \frac{k-1}{k} V \Gamma_k \left[1 - \frac{(k-1)\Gamma_k \lambda_i}{k}\right]. \qquad (2.32)$$

The first-order condition of the maximization of Eq.(2.32) with respect to the auxiliary contract parameter λ_i is given by:

$$\frac{\partial \Pi_i}{\partial \lambda_i} = -(k-1)V\left[\frac{\sum \lambda_j - (\lambda_i + 1)}{(\sum \lambda_j)^2}\right] + (k-1)^2 V \frac{(\sum \lambda_j)^2 - 2\lambda_i \sum \lambda_j}{(\sum \lambda_j)^4} \qquad (2.33)$$

Solving (2.33) under the assumption of symmetric, rational behavior of the firm owners we get

$$\lambda^* = \frac{k^2 - 2k + 2}{k(k-1)}. \qquad (2.34)$$

Now, reversion of the substitution of $\lambda_i = \frac{k+\alpha_i}{k}$ leads to the following result.

Result 2.4 *The optimal incentive contract satisfies* $\alpha^* = -\frac{k-2}{k-1}$ *for* $k \geq 2$.

Result 2.4 states that the optimal incentive contract depends on the number of contestants.[4] For $k \geq 3$ the optimal contract puts negative weight on the rival's payoff and creates always spiteful, aggressive behavior. For $k = 2$ we get a multiplicity of equilibria. All equilibria fulfill the following condition: $\alpha_1 = -\alpha_2$. Since we restrict the analysis to symmetric equilibria there exist only one equilibrium for $k = 2$ that satisfies the symmetric condition, namely $\alpha_1 = \alpha_2 = 0$.[5] This means that in two-player contests the optimal delegation involves giving the manager an independent profit maximizing contract. As Possajennikov (2008) shows the incentive for delegation depends on the slope of the reaction function at equilibrium. He explains that if the slope of the reaction function is zero, a change in the strategy does not have a first-order effect on the rival's strategy. This means that delegation with $\alpha_i \neq 0$ cannot be beneficial as this will lead to a change in own action to which the rival will not react.

[4] We are aware of the restriction that the number of contestants, k, can possibly change with the chosen α_i in this stage. For that we go back afterwards and show, that under the chosen optimal α_i always all n players take part in the contest.

[5] See Kräkel (2002) for the problem within the field of delegation in contests with two players. He analyzes in contrast to our framework contracts based on sales and payoffs.

Associated with the equilibrium preference parameter α^* equilibrium efforts are:

$$x_i^* = \frac{(k-1)^2}{k(k^2-2k+2)}V, \tag{2.35}$$

and the corresponding equilibrium profits

$$\Pi_i^* = \frac{V}{k(k^2-2k+2)}. \tag{2.36}$$

Interestingly, equilibrium effort and profits coincide with those from incentive contracts which are a linear combination of profits and sales as shown by Kräkel and Sliwka (2006). They choose a contract with a linear combination of profits and sales $\hat{g}_i = \lambda_i \Pi_i + (1-\lambda_i)S_i$, where $S_i = \frac{x_i}{\sum x_j} S$ are the sales of firm i and S is the total market volume. Restating our relative performance contract leads to $g_i = \lambda_i \Pi_i + (1-\lambda_i)S_i - (1-\lambda_i)(S - \sum_{j\neq i} x_j) = \hat{g}_i - (1-\lambda_i)(S - \sum_{j\neq i} x_j)$. Since the second term in the contract is independent of x_i both contracts lead to the same first-order conditions in the contest stage $\frac{\partial g_i}{\partial x_i} = \frac{\partial \hat{g}_i}{\partial x_i} = \frac{\sum x_j - x_i}{(\sum x_j)^2} V - \lambda_i$ and therewith to the same equilibrium effort and profits. Hence, the result by Kräkel and Sliwka (2006) that all owners together are worse off in case of delegation compared to non-delegation is assignable to our setting. Lower profits in the delegation game coincide with a higher rent dissipation, which we analyze next.

As we have seen, in the case of simultaneous contracting all managers get the same incentive contract on the first stage and therewith are equal spiteful on the second stage. Giving this symmetric equilibrium the participation condition (2.26) is always fulfilled and all potential players take part in the contest, which means $k = n$. Hence, the rent dissipation in the simultaneous contracting game amounts to

$$RD_{del} = \frac{(n-1)^2}{(n^2-2n+2)}.$$

Result 2.5 *The rent dissipation with simultaneous delegation is strictly higher for $n > 2$ or equal for $n = 2$ than the rent dissipation without delegation.*

Proof.

$$\begin{aligned} \frac{(n-1)^2}{(n^2-2n+2)} &\geq \frac{(n-1)}{n} \\ \Leftrightarrow n &\geq 2 \end{aligned}$$

This condition is always fulfilled since we assume $n \geq 2$ in our calculations.
■

Given the finding that all potential players take part in the contest, it is worthwhile to examine the contract when the number of contestants becomes large. An increase in the number of participants results in a decreasing contract parameter α^*. The limit case is given in the following result:

Result 2.6 *As $n \to \infty$: $\alpha^* \to -1$ and therewith $g_i \to \Pi_i$.*

Proof. Since $\sum_{j=1}^{n} \Pi_j \leq V$ is bounded it follows

$$\lim_{n\to\infty} g_i = \lim_{n\to\infty} \left(\Pi_i + \frac{\alpha_i}{n} \sum \Pi_j \right) = \lim_{n\to\infty} \Pi_i + \underbrace{\lim_{n\to\infty} \frac{\alpha_i}{n} \sum \Pi_j}_{=0}.$$

■

This result states that the optimal contract parameter α^* approaches -1 as the number of competitors in the contest goes toward infinity. With unlimited number of contest participants owners choose pure payoff maximizing contracts because the interdependent part in the contract, the average of all payoffs, goes toward zero. Our finding is in line to Kräkel and Sliwka (2006). They consider the same basic setup as ours but study profit-sales contracts. Their finding is that an unlimited number of competitors induces firm owners to choose pure profit contracts for their managers.

2.3.4 Sequential Contracting

Until now we have focused on simultaneous contracting, that is, all firm owners announce their incentive contracts at the same time. This section studies the changes when firm owners announce contracts sequentially. First, we analyze the situation when there is one first mover who can commit to an action and then all other choose simultaneously their action. Second, one firm can commit to a contract before the second firm can do so, the second firm can do before the third one can do, and so forth.

It is hard to get an analytical solution, hence we provide numerical results up to five players. We determine by computation the subgame perfect equilibria.[6] Constructing the optimally chosen contract weights and resultant payoffs for the case of a Stackelberg leader and simultaneous committing followers gives us the following Table 2.1:

	α_1 (Π_1/V)	α_2 (Π_2/V)	α_3 (Π_3/V)	α_4 (Π_4/V)	α_5 (Π_5/V)
$n = 3$	-1 (0.0799)	-0.3542 (0.0486)	-0.3542 (0.0486)		
$n = 4$	-1 (0.0295)	-0.5925 (0.0209)	-0.5925 (0.0209)	-0.5925 (0.0209)	
$n = 5$	-1 (0.0137)	-0.7039 (0.0105)	-0.7039 (0.0105)	-0.7039 (0.0105)	-0.7039 (0.0105)

Table 2.1: Sequential contracting: Stackelberg leader, homogeneous followers

The table indicates that the first moving firm owner choses the most spiteful and therewith the most aggressive contract that is possible in this framework. This is a boundary result since the weight put on average industry profits α is restricted to the interval $[-1, 1]$. If the restriction did not exist the first moving firm would choose an even more aggressive contract. The most spiteful contract goes along with the highest payoffs. Comparing the payoffs with those of the simultaneous contracting shows that there exists a strategic first mover advantage for owners when choosing their incentive schemes first. The Stackelberg leader can gain from sequential contracting whereas all other firms lose.[7] As the number of followers increases the equilibrium approaches the same solution in which all firm owners chose $\alpha = -1$ as the simultaneous contracting equilibrium (Result 2.6).

[6] The numerical results presented here are computed with Maple. The analysis uses the backward induction technique and therefore started with the second stage, the contest stage analysis. The optimal equilibrium effort and the profits of the firm owners, depending just on the chosen weights of competitor's profits ($\alpha_i, i = 1, \ldots, n$), were conducted. Given the first order conditions for the optimal profit we computed the best response of the homogeneous followers and of the Stackelberg leader given the knowledge of the best responses of the followers. Having found on this way the optimal chosen weight of the first mover we solved forward for the weights of later moving firm owners in equilibrium.

[7] This result is generally proved for commitment in symmetric contests by Possajennikov (2009).

Result 2.7 *When incentive contracts are chosen first by the Stackelberg leader and then simultaneously by all other competing firm owners*

i) the leader puts more negative weight on the average market payoff and earns higher profits than the followers and

ii) the Stackelberg leader earns higher profits than in the completely simultaneous setting.

In Table 2.2 there are in extracts the results calculated by computation for the optimally sequentially chosen contract weights and the resultant payoffs in the subgame perfect equilibrium when all firms commit sequentially to their actions.[8]

	α_1 (Π_1/V)	α_2 (Π_2/V)	α_3 (Π_3/V)	α_4 (Π_4/V)	α_5 (Π_5/V)
$n=3$	-1 (0.0609)	-0.8066 (0.0531)	-0.1980 (0.0283)		
$n=4$	-1 (0.0188)	-1 (0.0188)	-0.8423 (0.0165)	-0.3480 (0.0091)	
$n=5$	-1 (0.0079)	-1 (0.0079)	-1 (0.0079)	-0.8775 (0.0071)	-0.4012 (0.0039)

Table 2.2: Sequential contracting: leader, heterogeneous followers

Table (2.2) indicates that all owners choose equal or even stronger aggressive contracts than in the scenario before and therewith earn less expected payoff in the contest. The earlier moving firm owners chose the most spiteful and therewith the most aggressive contract that is possible in this framework. This is a boundary result since the weight put on average industry profits α is restricted to the interval $[-1, 1]$. If the restriction did not exist the earlier moving firms would choose an even more aggressive contract. The

[8]The numerical results presented here are computed with Maple. The analysis uses the backward induction technique and therefore started with the second stage, the contest stage analysis. The optimal equilibrium effort and the profits of the firm owners, depending just on the chosen weights of competitor's profits ($\alpha_i, i = 1, \ldots, n$), were conducted. Given the first order conditions for the optimal profit we computed the best response of the last moving firm owner until the first moving one, given the knowledge of all best responses of later moving owners. Having found on this way the optimal chosen weight of the first mover we solved forward for the weights of later moving firm owners in equilibrium.

expected payoff falls for all firm owners below the equilibrium payoff in a completely symmetric game. In this scenario none can gain from committing sequentially. The spitefulness decreases with each position in the rank order of announcing the contract. This means that the later moving firms earn less expected payoff in the contest than the earlier moving. In the end, all the firm owners earn a positive payoff. That is the reason why all of them take part in this game under the assumption that delegation is exogenous. Again, as the number of followers increases the equilibrium approaches the same solution in which all firm owners chose $\alpha = -1$ as the simultaneous contracting equilibrium and even faster as in the former Stackelberg setting.

Result 2.8 *When incentive contracts are chosen first by the leader and then sequentially by all other competing firm owners*

i) the earlier committing owners put more negative weight on the average market payoff in the subgame perfect equilibrium and earn higher profits than the later ones and

ii) all owners earn lower profits than in the completely simultaneous setting.

2.3.5 Conclusion

The aim of this section is to study delegation in advance to a Tullock contest. Firm owners delegate the decision of how much to spend in the contest to a manager. Managers are employed through a relative performance contract which the firm owner can determine. We show that optimal contracts in the simultaneous contracting case have a weight on the relative standing component and do not maximize pure payoffs. Therefore, the players in the contest are envious and spiteful. As the number of players increases the spitefulness increases. Spitefulness induces more aggressive behavior and therewith more expended effort in the contest stage. The rent dissipation is higher in the case of delegation.

Focusing on sequential contracting in a delegation game, we find that optimal contracts are asymmetric. The leader puts more weight on relative profits in the contract proposed to its manager. Hence, he chooses a contract that induces more aggressive behavior in the contest stage. This leads to an asymmetric oligopoly outcome with the leader at the contracting stage earning higher profits than the follower.

2.4 Conclusion

As we have seen in this chapter, it is possible to create spiteful behavior or in other words to generate negatively interdependent preferences in a competition through a two-stage game. Competition can mean Cournot competition or contests. In the case of simultaneous contracting the games have a unique symmetric equilibrium in which all participants exhibit spiteful aggressive behavior. To convey this feature in the framework of other regarding preferences, we would say all participants have the same degree of negatively interdependent preferences. To create a heterogeneous player field we study sequential contracting in a delegation game with Cournot competition as well as with contests. Due to the time structure the optimal chosen contracts are different for players in both situations. The leader puts more weight on relative profits in the contract proposed to its manager. Hence, he chooses a contract that induces more aggressive behavior in the market stage. This leads to an asymmetric oligopoly outcome with the leader at the contracting stage earning higher profits than the follower. Contracting first gives a strategic advantage comparable with the strategic advantage arising through negatively interdependent preferences in an evolutionary framework.

Chapter 3

Rent-Seeking Contests of Heterogeneous Players with Interdependent Preferences

3.1 Introduction

There is a already large and still growing literature on the theory and application of contests. A contest is a game where all players simultaneously exert effort, and only one of them wins a prize, while the others get nothing. Effort determines the players' chance of winning the prize. Some prominent examples of contests in the every day life are R&D and patent races or lobbying.

One special field of the contest literature is the area of rent-seeking.[1] Rent-seeking generally implies the extraction of uncompensated value from players without making any contribution to productivity. For example it is held to be associated with efforts to cause a redistribution of wealth by shifting the government tax burden or government spending allocation. In this chapter we explore rent-seeking contests, where the set of players is heterogeneous, as Stein (2002), Cornes and Hartley (2005) and Ryvkin (2007) did. In contrast to them we focus on heterogeneity concerning the preferences of the play-

[1] For surveys about rent-seeking see Nitzan (1994), Tollison (1997), Konrad (2007) or Corchon (2007).

ers. Persistent experimental findings in game theory[2] suggest that players assess their well-being not entirely in terms of their absolute payoff; some weight is put on relative payoffs. That means that the players in reality have positively or negatively interdependent preferences. In our analysis we focus on the envious characteristic of persons. Therefore, the players have negatively interdependent preferences in different parameter values. Specifically, all players are status seekers whose preferences differ with respect to spitefulness.

For a long time the assumption of preferences being independent from the other players' payoff has been unquestioned in economics. In the more recent literature, especially the literature about evolutionary game theory, it became more common to relax this assumption of purely self-interested preferences. Leininger (2003) and Hehenkamp, Leininger, and Possajennikov (2004) with an evolutionary approach at the level of strategies and Leininger (2009) with an evolutionary approach at the level of preferences show that in rent-seeking contests, as analyzed here, it is evolutionarily stable to behave like a player with interdependent preferences in Nash equilibrium would behave.

Nevertheless, the idea of interdependent preferences is rather old. One of the first findings of aspiration for relative success can be found in Veblen (1899). Much later Duesenberry (1949) laid a first empirical basis for interdependent preferences. More recent empirical support of the interdependent preference hypothesis is given by Bush (1994a,b) and Kapteyn, van der Geer, van de Stadt, and Wansbeck (1997), who used field data. Furthermore, Levine (1998) and Davis and Holt (1993) give support for the economic relevance of interdependent preferences based on experimental data. Moreover, our experience in the daily business strongly suggest interdependent preferences to be present in the real world. For example people regard colleagues in the job with envy as well as teammates in leisure time sports.

Therefore, we assume players to have interdependent preferences and study the structure of the equilibria that emerge in those rent-seeking contests. We analyze whether the equilibria should be symmetric or not. We stress the importance of the exact preferences in order to understand the impact of the technology of rent-seeking on the outcome of the game. In the case

[2]See for example Levine (1998), Bolton and Ockenfels (2000) or Zizzo and Oswald (2000).

of a constant returns to scale technology we explore asymmetric equilibria. In addition, we approximate asymmetric equilibria for arbitrary technology parameters when preference heterogeneity is weak. The question studied is whether weak heterogeneity of players due to different preferences is capable of causing significant changes in the equilibrium structure. A second question is which influence the chosen technology has on the number of active players in a rent-seeking contest. Therefore, we calculate the threshold values of the discriminatory power for which players sequentially drop out.

The chapter is organized as follows: Section 3.2 presents the general setup of the rent-seeking contest. In Section 3.3, we analyze the benchmark case with homogeneous players with negatively interdependent preferences and explore the condition for an asymmetric Nash equilibrium. Then, in Section 3.4, we turn to heterogeneous players and examine the Nash equilibria for the case of a constant returns to scale technology. To approximate equilibria in the case of an increasing returns to scale technology with all players participating (Section 3.5) and with less than all players participating (Section 3.6), we restrict players to be weakly heterogeneous concerning their preferences. We demonstrate the results in an numerical example in Section 3.7 and, finally, in Section 3.8, we conclude.

3.2 General Setup of the Contest

The general setup consists of an imperfectly discriminating rent-seeking contest for the common rent $V > 0$. n risk neutral players, denoted by the elements of the set $\mathcal{N} = \{1, \ldots, n\}$, potentially take part in the contest. The number of players $n \geq 2$ is exogenously given and fixed. A fixed number of players n is reasonable for situations where the group of participants is restricted for example through specific characteristics or informational advantages.

The contest, which will be analyzed, is a one-stage contest, which means that players expend simultaneous efforts $x_i \geq 0$. We call player i active in the contest if his effort is positive ($x_i > 0$), otherwise ($x_i = 0$) we say that player i has dropped out of the contest. Then, the subset of active players $\mathcal{K} = \{1, \ldots, k\} \subseteq \mathcal{N}$ is defined. In an imperfectly discriminating contest it is not guaranteed that the highest effort wins. This feature is captured by the following contest success function (CSF) which is proposed by Tullock (1980)

$$p_i(x_1, \ldots, x_n) = \begin{cases} \frac{x_i^r}{\sum_{j=1}^n x_j^r} & \text{if} \quad \max\{x_1, \ldots, x_n\} > 0 \\ 1/n & \text{otherwise} \end{cases} \tag{3.1}$$

where $0 \leq r \leq 2$ is the technology parameter. This technology parameter r represents the efficiency of the given contest technology. r is also called the discriminatory power of the contest, because it determines how much impact a player's own effort has on his winning probability. We restrict r to the interval $[0, 2]$ because for $r > 2$ the existence of equilibria in pure strategies is not guaranteed as Perez-Castrillo and Verdier (1992) found out. In the following sections the interval is readjusted for the applying framework. Among other things the dependence of the equilibrium structure on different levels of discriminatory power is analyzed in this chapter.

Players of the contest gain expected material payoff which depends on their own effort and the efforts of their active competitors. The dependency is captured in the following material payoff function, which determines the material success of player i:

$$\Pi_i(x_1, \ldots, x_n) = \frac{x_i^r}{\sum_{j=1}^n x_j^r} V - x_i. \tag{3.2}$$

Players choose their effort in order to maximize their utility. They have negatively interdependent preferences. For simplification, we assume that their utility function is linear in their own and in the average material payoff of active players.

$$F_i(x_1, \ldots, x_n) = \Pi_i + \alpha_i \frac{1}{n} \sum_{j=1}^{n} \Pi_j \tag{3.3}$$

where $-1 < \alpha_i \leq 0$ is the preference parameter. Players not only care about their own payoff, they rather care about their payoff compared to the average payoff of all active players. Their utility increases with own material success and decreases with the material success of competitors. In addition we assume $\alpha_i > -1$ to ensure that the utility of each player depends stronger on his own payoff than on the average payoff. Let the preference parameter $\alpha_1, \ldots, \alpha_n$ for convenience be ordered so that $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n$. Hence, the players are ordered by their preferences: player 1 has the most spiteful preference and player n acts least spiteful.

3.3 The Case of Identical Players

This section summarizes the analysis of the Tullock contest with identical players. Especially the main results, adopted to our particular setting, of the model presented in Perez-Castrillo and Verdier (1992), Ryvkin (2007) and Guse and Hehenkamp (2006) are repeated, compared and extended for players with interdependent preferences.

Assume that all players have identical negatively interdependent or independent preferences ($\alpha_1 = \cdots = \alpha_n = \alpha$) and that all n potentially interested players participate in the contest. The number of players n is exogenously given and fixed during the contest. In the unique Nash equilibrium of the game among identical players effort levels have to solve the following first-order conditions:

$$\frac{\partial}{\partial x_i}\left(\frac{x_i^r}{\sum x_j^r}\right) = \frac{1+\frac{\alpha}{n}}{V}. \tag{3.4}$$

Searching for symmetric equilibria in which n players participate we substitute $\sum_{j=1}^{n} x_j^r = nx^r$ in the above first-order conditions and get the equilibrium levels: The optimal effort level is given by

$$x^*(n) = \frac{(n-1)}{n^2} rV \frac{1}{(1+\frac{\alpha}{n})}. \tag{3.5}$$

If we assume players to be identical the effort level of players with interdependent preferences ($\alpha < 0$) is higher than the effort level in a game among players with independent preferences ($\alpha = 0$).

Using the optimal effort level, Eq.(3.5), we can calculate the material payoff in the equilibrium

$$\Pi^*(x_1^*, \ldots, x_n^*) = \frac{1}{n}V - x^*(n) = \frac{V}{n}\left(1 - \frac{r(n-1)}{n}\frac{1}{(1+\frac{\alpha}{n})}\right) \tag{3.6}$$

and the utility

$$F^*(x_1^*, \ldots, x_n^*) = (1+\alpha)\Pi^*(x_1^*, \ldots, x_n^*). \tag{3.7}$$

The material payoff as well as the utility is smaller for players with interdependent preferences due to the higher effort they spend.

Now, we relax the assumption that all n players participate in the contest. The number of active players depends on the technology parameter r holding all other factors constant. For those asymmetric equilibria we derive the upper threshold values of the technology parameter for which k active players scrape through. First, we characterize a Nash equilibrium with k active players out of n ($\mathcal{K} \subseteq \mathcal{N}$). In such an equilibrium all active players spend the same effort in rent-seeking.[3]

$$x^*(k) = \frac{(k-1)}{k^2} rV \frac{1}{(1+\frac{\alpha}{k})} \tag{3.8}$$

Second, we derive general conditions under which players participate in the contest and under which they stay out of the contest. Player i participates in the contest whenever this gives him a higher utility than non-participation would give. In contrast to players with independent preferences, who get zero utility for non-participating in the contest, players with negatively interdependent preferences evaluate their relative payoff that would result from non-participation. Being inactive as a player corresponds to zero effort, zero material payoff and a negative utility. For a given discriminatory power r of the contest success function the participation condition (PC) is given by :

$$\begin{aligned}
\forall i \in \mathcal{K}: \quad & F_i(x^*(k)) \geq F_i(0, x^*_{-i}(k-1)) \\
\Leftrightarrow \quad & (1+\alpha)\left(\frac{V}{k} - x^*(k)\right) \geq \alpha\left(\frac{V}{k-1} - x^*(k-1)\right) \\
\Leftrightarrow \quad & \frac{(k+\alpha)(k-1-\alpha)(k-1+\alpha)}{(k-1)^3 + \alpha(k+\alpha)} \geq r. \qquad (3.9)
\end{aligned}$$

No player in the active subset $\mathcal{K}$ has an incentive to drop out as long as these inequalities hold. Participating with the optimal effort in an equilibrium with k players has a higher utility for the players than not being actively involved in a $(k-1)$ players contest, meaning player i choses zero effort whereas the other $k-1$ players chose their optimal equilibrium effort. This condition, Eq.(3.9), gives the upper bound on r for the existence of an intermediate equilibrium with k active players.

$$r_{max}(k) = max\left\{r | \forall i \in \mathcal{K} : F_i(x^*(k)) \geq F_i(0, x^*_{-i}(k-1))\right\} \tag{3.10}$$

It is interesting to see that the number of active players k does not depend on the number of potential players n. It only depends on two exogenously given

[3] See Perez-Castrillo and Verdier (1992) for the prove of the case with independent preferences.

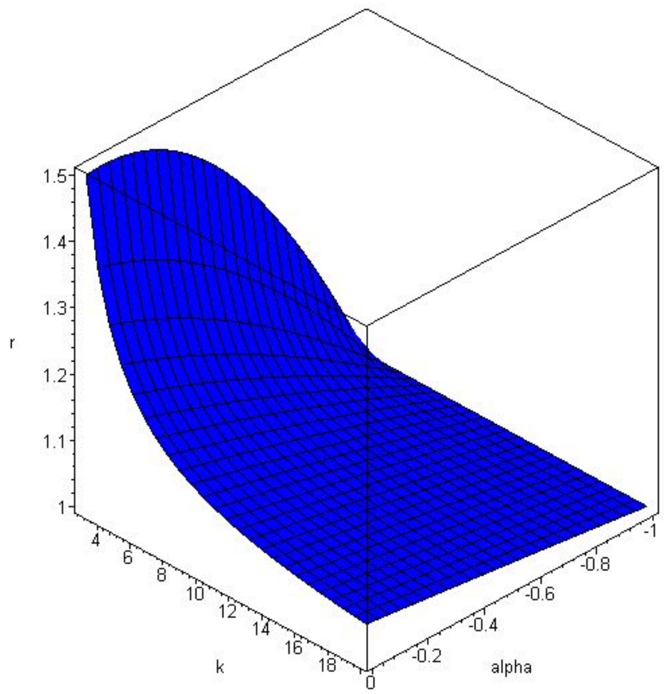

Figure 3.1: Maximum levels of discriminatory power r for α.

values, on the preference parameter α and on the technology parameter r of the contest.

Result 3.1 *In the case of identical players with interdependent preferences the discriminatory power r has to fulfill the following condition to secure the existence of an equilibrium with k active players:*

$$r \leq r_{max}(k) = \frac{(k+\alpha)(k-1-\alpha)(k-1+\alpha)}{(k-1)^3 + \alpha(k+\alpha)}.$$

Proof. See calculation of Eq.(3.9). ■

In the case where the technology of rent-seeking shows constant or decreasing returns to scale ($r \leq 1$), for a fixed number of players the above derived condition is always fulfilled and therefore there is a unique Nash equilibrium which is symmetric. In this equilibrium all n players participate in the rent-seeking process since the participation condition (Eq.(3.9)) is fulfilled.

In the case of increasing returns to scale, if the number of possible players is not too high, the condition is not binding. Again, there is a unique symmetric equilibrium with all players participating actively in the contest. However, when the number of players potentially interested in the contest is higher than a certain level and thereby the condition for the technology parameter is binding, we find a multiplicity of equilibria. Those equilibria are asymmetric; there are k players devoting the same amount of effort and $n-k$ staying inactive. The reason for the multiplicity of equilibria is the indeterminacy of who will be the active players and who the inactive ones.

By computation we prove that the partial derivative of the maximal technology parameter with respect to the preference parameter is positive independent of the number of actively players ($\frac{\partial r_{max}}{\partial \alpha} > 0$). r_{max} decreases for decreasing values of the preference parameter α. Hence, the upper bound for the technology parameter r to secure the existence of an asymmetric Nash equilibrium with k active players is lower for a contest of players with negatively interdependent preferences than for independent preferences. This finding creates the following result:

Result 3.2 *For the same technology parameter r the set of active players under the assumption of negatively interdependent preferences is smaller or equal than the set of active players under the assumption of independent preferences.*

However, for higher values of the technology parameter r, competition between players intensifies. For players with negatively interdependent preferences this intensified competition ends up with negative material payoff. Notice that opting out is no option for these players (provided the technology parameter r is not too large). Opting out by an individual would raise the others' absolute payoffs above zero payoff level, while it fixes his own utility to a negative value. If r is very large, opting out becomes profitable. This threshold value of r is faster reached for the case of more spiteful players.

Our results point in the same direction as the result of Eaton and Eswaran (2003). They find that agents with preferences for relative payoffs are shown to behave more aggressively and that this leads to a reduction in the population size when this size is endogenous. As seen in Figure 3.1 equilibria with the active subset k exist if $r \leq r_{max}(k)$. This is a necessary condition for the existence of such equilibria. The number of active players depends on the discriminatory power.

3.4 Heterogeneous Spiteful Players and Constant Returns to Scale

Now, assume that players in the rent-seeking contest are heterogeneous due to different interdependent preferences. We sort them via $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n$. They face a constant returns to scale technology $r = 1$. This condition is necessary to permit explicit solutions. To obtain a pure strategy Nash equilibrium solution for this contest we follow and extend the framework of Stein (2002). He investigates the properties of a single stage n players rent-seeking contest with different valuations and/or different abilities among the players and obtains a pure strategy Nash equilibrium solution. We modify his framework by introducing heterogeneity with respect to different preferences.

To simplify the analysis we introduce the following transformation parameter

$$\lambda_i = \frac{n + \alpha_i}{n}. \tag{3.11}$$

From $\alpha_i \in [-1, 0]$ it follows that $\lambda_i \in \left[1 - \frac{1}{n}, 1\right] \subseteq \left[\frac{1}{2}, 1\right]$. Note that the most spiteful player who has the smallest α_i is assigned to the smallest λ_i value. Hence, the transformation of α_i to λ_i is just an adjustment of the interval from the negative numbers into the positive ones. From now on we call λ_i the preference parameter of individual i, regard the λ's as endogenously given and use them for all following calculations. We can sort the individuals via $\lambda_1 \leq \cdots \leq \lambda_n$.

Under this condition, the partial derivative of the utility maximization problem of individual i changes to

$$\frac{\partial F_i(\mathbf{x})}{\partial x_i} = \frac{\sum_{j=1}^{n} x_j - x_i}{(\sum_{j=1}^{n} x_j)^2} V - \lambda_i. \tag{3.12}$$

The first-order conditions for a maximum to exist fulfill $\frac{\partial F_i(\mathbf{x})}{\partial x_i} = 0$ if $x_i > 0$ and $\frac{\partial F_i(\mathbf{x})}{\partial x_i} \leq 0$ if $x_i = 0$. It can be verified that the expected payoff and therewith the utility is concave in each x_i. Then, $\frac{\partial F_i(\mathbf{x})}{\partial x_i} = 0$ implies that for any $x_i > 0$:

$$x_i = \sum_{j=1}^{n} x_j - \frac{(\sum_{j=1}^{n} x_j)^2 \lambda_i}{V}. \tag{3.13}$$

It is now possible to calculate the equilibrium solution $\mathbf{x}^* = (x_1^*, \ldots, x_n^*)$. Therefore, we first characterize which players have an incentive to take part with positive effort. In this rent-seeking contest we may claim that there are n potential players, but there is no reason to assume that all will make positive expenditures. From Eq.(3.13) it follows that the sequence $(x_i : x_i > 0)$ is ordered in the opposite way as the sequence (λ_i). The more spiteful players expend more effort in the contest. Therefore, $x_1 \geq x_2 \geq \cdots \geq x_k > 0$ for some index k and $x_i = 0$ for $i > k$. Since the expected payoff is concave in x_i, it is optimal for a player to use an interior point solution if one exists. Players with small values of λ_i, who are more spiteful players, will make positive expenditures whereas those with sufficiently high values will choose to make no expenditure.

Now, calculate the sum of Eq.(3.13) over $i = 1, \ldots, k$ and solve:

$$\sum_{i \leq k} x_i = \frac{k-1}{\sum_{i \leq k} \frac{\lambda_i}{V}} = \frac{k-1}{k} V \Gamma_k \tag{3.14}$$

where $\Gamma_k = \left[\frac{1}{k} \sum_{i \leq k} \lambda_i\right]^{-1}$ is the inverse of the arithmetic mean of the first k values of λ_i or - in other words - the harmonic mean of the first k values of the sequence $\left(\frac{1}{\lambda_i}\right)$.

Eq.(3.14) can be used with Eq.(3.13) to yield an expression for the equilibrium strategy for player i for all $i \leq k$:

$$x_i^* = \frac{k-1}{k} V \Gamma_k \left[1 - \frac{(k-1)\Gamma_k \lambda_i}{k}\right]. \tag{3.15}$$

The individual effort x_i^* consists of two parts. The first one is independent of the individual preference and the same for all active players. The second part shows the preference dependent effort. As we see later all potential players take part in the contests. Therefore, Eq.(3.15) coincides for the case of homogeneous players and a constant returns to scale with Eq.(3.5) and Eq.(3.8) from the former section.

With Eq.(3.15) we can compute the probability that player i, $i \leq k$, wins the contest:

$$
\begin{aligned}
p_i &= \frac{x_i^*}{\sum x_j^*} \\
&= \frac{\frac{k-1}{k} V \Gamma_k \left[1 - \frac{(k-1)\Gamma_k \lambda_i}{k}\right]}{\sum \frac{k-1}{k} V \Gamma_k \left[1 - \frac{(k-1)\Gamma_k \lambda_j}{k}\right]} \\
&= \frac{1 - \frac{(k-1)\Gamma_k \lambda_i}{k}}{k - \frac{k-1}{k} \frac{k}{\sum \lambda_j} \sum \lambda_j} \\
&= 1 - \frac{(k-1)\Gamma_k \lambda_i}{k}. \qquad (3.16)
\end{aligned}
$$

The winning probability of individual i depends on the number of active players k, the inverse of the arithmetic mean of the preference parameters Γ_k and his own preference parameter λ_i. k and Γ_k do not vary between individuals. They are identical for all players. Since $0 < \lambda_1 \leq \cdots \leq \lambda_k$ it follows that $p_1 \geq \cdots \geq p_k > 0$. Player i is said to be more successful than player j if $p_i > p_j$.

From Eq.(3.15) it is possible to determine the value of k. Given the equilibrium effort we can calculate the utility from participating in the contest and the utility from non-participation. The individuals take part in the contest whenever the former is higher than the latter.

$$
\begin{aligned}
\forall i \in \mathcal{K} : F_i(x^*(k)) &\geq F_i(0, x^*_{-i}(k-1)) \\
p_i V - x_i^*(k) + (\lambda_i - 1)(V - \sum x_j^*(k)) &\geq (\lambda_i - 1)(V - \sum x_j^*(k-1))
\end{aligned}
$$

Result 3.3 *The number of active players k out of all potential players n is found as the largest index $k \geq 2$ such that*

$$\frac{1}{\lambda_i} \geq 1 - \frac{(1 - \frac{k-1}{k}\Gamma_k)^2}{1 - \frac{k-2}{k-1}\Gamma_{k-1}}.$$

This result can be used to show that all potential players be active in the rent-seeking contest if we restrict the preference parameter λ_i to the interval $[\frac{n-1}{n}, 1]$, as we did before.

Result 3.4 *All potential players n take part in the contests with constant returns to scale ($r = 1$) as long as $\lambda_i \in [\frac{n-1}{n}, 1]$, $\forall i = 1, \ldots, n$.*

Proof. To show: $\frac{1}{\lambda_i} \geq 1 - \frac{(1-\frac{n-1}{n}\Gamma_n)^2}{1-\frac{n-2}{n-1}\Gamma_{n-1}}$ valid for all $\lambda_i \in [\frac{n-1}{n}, 1]$.

From $\lambda_i \in [\frac{n-1}{n}, 1]$ it follows that $1 \leq \frac{1}{\lambda_i}$ and therewith we can prove the even stronger inequality

$$1 - \frac{(1 - \frac{n-1}{\sum_{j\leq n}\lambda_j})^2}{1 - \frac{n-2}{\sum_{j\leq n-1}\lambda_j}} \leq 1$$
$$\Leftrightarrow \quad \frac{(1 - \frac{n-1}{\sum_{j\leq n}\lambda_j})^2}{1 - \frac{n-2}{\sum_{j\leq n-1}\lambda_j}} \geq 0.$$

The quadratic term in the nominator is always bigger than zero. Hence, it is still to show that the denominator is also positive.

$$1 - \frac{n-2}{\sum_{j\leq n-1}\lambda_j} \geq 0$$
$$\Leftrightarrow \quad 1 - \frac{n-2}{\sum_{j\leq n-1}\lambda_j} \geq 0$$
$$\Leftrightarrow \quad \sum_{j\leq n-1} \lambda_j \geq n-2$$

Using the condition $\lambda_j \geq \frac{n-1}{n}$ we yield

$$\sum_{j\leq n-1} \lambda_j \geq (n-1)\frac{(n-1)}{n} = \frac{n^2-2n+1}{n} = n-2+\frac{1}{n} > n-2.$$

■

The rent dissipation (RD), which is total expenditures by all players divided by the rent V, is given from the Eq.(3.14) and the Result 3.4 by

$$RD = \frac{n-1}{n}\Gamma_n. \tag{3.17}$$

The rent dissipation depends on the harmonic mean of the sequence $(\frac{1}{\lambda_i})$. If one of the preference parameter λ_i is increased and another one λ_j is decreased by the same amount, then the rent dissipation does not change.

By direct computation, $\frac{\partial RD}{\partial \lambda_i} = -\frac{n-1}{(\sum \lambda_j)^2} < 0$ and $\frac{\partial^2 RD}{\partial \lambda_i^2} = \frac{2(n-1)}{(\sum \lambda_j)^3} > 0$. This leads to the following result:

Result 3.5 *The rent dissipation decreases with an increase in an individual preference parameter* λ_i, $\forall i = 1, \ldots, n$..

An increase in the preference parameter λ_i means that this player i gets less spiteful. If a player is less spiteful it implies that he expends less effort and therewith the rent dissipation decreases.

3.5 Weakly Heterogeneous Players: Equilibrium with all Players Participating

In the last section we have shown that the rent-seeking problem with heterogeneous players who exhibit negatively interdependent preferences is exactly solvable for $r = 1$. For the case of two players, $n = 2$, and/or a constant returns to scale technology, $r = 1$, the problem of rent-seeking contests with heterogeneous players leads to first-order conditions which are exactly analytical solvable.[4] In this section we use the technique of linearization which refers to finding the linear approximation to a function at a given point to find a numerical solution. Linearization is a method for assessing the local stability of an equilibrium point of a system of nonlinear differential equations. We approximate the solutions for a contest with an arbitrary number of players $n \geq 2$ and a discriminatory power $r \in [0, \frac{n}{n-1}]$ in a small neighborhood around the homogeneous equilibrium for the case when the heterogeneity of players is weak.[5] Therefore, we assume in this first section that all n players participate in the contest. Afterwards, we analyze the case with less than all players participating.

To define weak heterogeneity, we first introduce the following average parameters. The average preference parameter is given by $\bar{\lambda} = \frac{1}{n}\sum_{j=1}^{n}\lambda_j$. The preferences are concentrated around this average value which is indicated by $\lambda_i = \bar{\lambda}(1-\beta_i)$, where $|\beta_i| \ll 1$ shows how weak the heterogeneity is. β_i shows how spiteful player i is compared to the average preference. $\beta_i > (<) 0$ means player i is less (more) spiteful than the average player. The average equilibrium effort of all players is given by $\bar{x}_n = \frac{1}{n}\sum_{j=1}^{n} x_j$. With the average effort we can describe each single effort as $x_i = \bar{x}_n(1+e_i)$ where $|e_i| \ll 1$. e_i is the unknown difference in the effort levels due to the heterogeneity. The obvious idea behind the following analysis is that small heterogeneity in preferences can only lead to small differences in efforts. That this is true, cannot be shown a priori. We first have to find the relative effort levels to derive conditions under which these levels remain small. For the moment we can use the Taylor approximation for the analysis.

[4]For example, see Malueg and Yates (2004) for the case of $n = 2$ players; Stein (2002), Epstein and Nitzan (2002), and Stein and Rapoport (2004) for the case of $r = 1$.

[5]See Ryvkin (2007) for the analysis of weakly heterogeneous players with differences in the cost parameter.

To find the equilibrium efforts we calculate the first-order condition:

$$\frac{x_i^{r-1}(\sum x_j^r - x_i^r)}{(\sum x_j^r)^2} = \frac{\lambda_i}{rV}, \quad i = 1, \dots, n. \tag{3.18}$$

Now, we linearize this generally nonlinear equation at the equilibrium effort levels. Therefore, we substitute the representations for λ_i and x_i in the equation

$$\begin{aligned} & \frac{\bar{x}_n^{r-1}(1+e_i)^{r-1}(\sum \bar{x}_n^r(1+e_j)^r - \bar{x}_n^r(1+e_i)^r)}{(\sum \bar{x}_n^r(1+e_j)^r)^2} = \frac{\bar{\lambda}(1-\beta_i)}{rV} \\ \Leftrightarrow \quad & \frac{(1+e_i)^{r-1}(\sum(1+e_j)^r - (1+e_i)^r)}{(\sum(1+e_j)^r)^2} = \frac{\bar{x}_n\bar{\lambda}(1-\beta_i)}{rV}. \end{aligned} \tag{3.19}$$

Then, we use a Taylor expansion around the homogeneous equilibrium. Taylor expansion is applicable whenever payoffs are sufficiently smooth functions of players' actions in the neighborhood of the homogeneous equilibrium. The homogeneous equilibrium results whenever players choose their preference dependent part of effort, e_i, equal to zero. For this reason, we use the Taylor expansion of $(1+e_i)^{r-1} = 1+(r-1)e_i+O(\mu^2)$ and $(1+e_i)^r = 1+re_i+O(\mu^2)$ up to the first-order around zero. As a measure of the accuracy of the linear approximation, we introduce a single small parameter $\mu = max_{i\in\mathcal{N}}|\beta_i|$. From now on all equations involving the relative equilibrium effort levels e_i just hold with accuracy $O(\mu^2)$ due to the Taylor approximation. Substituting these two expressions into Eq.(3.19) yields

$$\frac{[1+(r-1)e_i][\sum(1+re_j) - (1+re_i)]}{[\sum(1+re_j)]^2} = \frac{\bar{x}_n\bar{\lambda}(1-\beta_i)}{rV}. \tag{3.20}$$

After some further transformations we get

$$\begin{aligned} & \frac{[1+(r-1)e_i][n+rE-1-re_i)]}{[n+rE]^2} = \frac{\bar{x}_n\bar{\lambda}(1-\beta_i)}{rV} \\ \Leftrightarrow \quad & \frac{n-1+rE-re_i+(n-1)(r-1)e_i+(r-1)re_iE-(r-1)re_i^2}{n^2+2nrE+r^2E^2} \\ & = \frac{\bar{x}_n\bar{\lambda}(1-\beta_i)}{rV} \end{aligned}$$

and neglecting all terms of order higher than one in e_i and β_i results in

$$n-1+rE-re_i+(n-1)(r-1)e_i = \frac{\bar{x}_n\bar{\lambda}}{rV}(n^2+2nrE-\beta_i n^2), \tag{3.21}$$

where $E = \sum_{i=1}^n e_i$.

Using the definitions of average effort $\bar{x}_n$ and single effort e_i we get $x_i = \bar{x}_n + \bar{x}_n e_i = \frac{1}{n}\sum x_j + \frac{1}{n}\sum x_j e_i$. This equation consists of two parts, one part which is independent of preferences and one dependent part. The independent part composes the stationary steady state. As long as all players participate, the equilibrium effort is a smooth function in the heterogeneity of players. Therefore, we can take Eq.(3.5) as the stationary steady state around which we approximate. We split the above equation in parts of the steady state, the symmetric equilibrium level, and the dependent individual preference part to find the resultant deviation from the homogeneous equilibrium. Hence, $\bar{x}_n$ must be chosen such that

$$n - 1 = \frac{\bar{x}_n \bar{\lambda} n^2}{rV} \Leftrightarrow \bar{x}_n = \frac{n-1}{n^2}\frac{1}{\bar{\lambda}} rV. \tag{3.22}$$

Using the remaining terms in Eq.(3.21), which demonstrate the differences arising from the heterogeneity, yields

$$rE + ((n-1)(r-1) - r)e_i = (n-1)\left(\frac{2rE}{n} - \beta_i\right), \quad \forall i \in \mathcal{N}. \tag{3.23}$$

In the following, this is the equation of interest. It determines the preference dependent part of efforts. We sum up all single equations for every i and introduce $\bar{\beta}_n = \frac{1}{n}\sum \beta_i$. This gives us

$$nrE + ((n-1)(r-1) - r)E = (n-1)(2rE - n\bar{\beta}_n) \tag{3.24}$$

which leads after some further transformations to $E = n\bar{\beta}_n$. Inserting this into the above Eq.(3.23) yields

$$e_i = \frac{\beta_i - r\frac{n-2}{n-1}\bar{\beta}_n}{1 - r\frac{n-2}{n-1}}. \tag{3.25}$$

The relative effort e_i is a weighted average of the relative preference of player i, β_i, and the average relative preference, $\bar{\beta}_n$. Note that $\bar{\beta}_n = \frac{1}{n}\sum \beta_i$ is constructed to be zero for the case with all players participating. It becomes different from zero if the players start to drop out of the contest.

Relative equilibrium effort levels e_i, Eq.(3.25), must be checked for self-consistency as mentioned before. First, they are supposed to remain small. This is fulfilled if

$$|\beta_i| \ll 1 - r\frac{n-2}{n-1} \quad \forall i \in \mathcal{N}. \tag{3.26}$$

When the discriminatory power r is small, this constraint is not binding. When the discriminatory power r becomes larger, the range of allowed heterogeneity shrinks. For the case $r = \frac{n-1}{n-2}$ no heterogeneity is allowed to satisfy the existence of an equilibrium in pure strategies. Hence, we assume for more than two players $r < \frac{n-1}{n-2}$ as an additional condition in our model for weakly heterogeneous players. The equilibrium with n players participating satisfies this condition. We already showed for the case with n players that the stronger inequality $r \leq \frac{(n+\alpha)(n-1-\alpha)(n-1+\alpha)}{(n-1)^3+\alpha(n+\alpha)}$ must hold independent of $\alpha \in [-1, 0]$. Second, none of the active players should have an incentive to deviate from the equilibrium strategy and leave the contest. Suppose individual i considers to drop out of the contest and the other players believe that individual i will actively participate. The active players then maintain their strategies as indicated. We need to compare the utility of player i for the cases of participation and non-participation to test whether the equilibrium is stable. This comparison gives the following participation condition

$$\begin{aligned} \forall i \in \mathcal{N} : F_i(x^*(n)) &\geq F_i(0, x^*_{-i}(n)) \\ \Leftrightarrow \quad \frac{V}{n}(1 + re_i - \frac{r}{n}E) - \bar{\lambda}\bar{x}_n(1 - \beta_i + e_i) &\geq 0. \end{aligned} \tag{3.27}$$

After some transformations and the substitution of the expressions for $\bar{x}_n$ and e_i, Eq.(3.27) turns to the following inequality which we call participation condition $PC(\mathcal{N})$

$$\frac{V}{n}\left[1 - \frac{r(n-1)}{n} + \frac{r(\beta_i - \bar{\beta}_n)(1 - r\frac{n-2}{n})}{1 - r\frac{n-2}{n-1}}\right] \geq 0. \tag{3.28}$$

The term, which accounts for the heterogeneity, is proportional to the difference $(\beta_i - \bar{\beta}_n)$ between the relative preference of player i and the average preference.

The material payoff of player i after the linear approximation is given by

$$\Pi_i(\mathcal{N}) = \frac{V}{n}\left[1 - \frac{r(n-1)}{\bar{\lambda}n} + \frac{1 - \frac{n-1}{\bar{\lambda}n}}{1 - r\frac{n-2}{n-1}}r\beta_i - \frac{1 - r\frac{n-2}{\bar{\lambda}n}}{1 - r\frac{n-2}{n-1}}r\bar{\beta}_n\right]. \tag{3.29}$$

Result 3.6 *If a player's preference is stronger, $\beta_i > 0$ (weaker, $\beta_i < 0$) than the average, his payoff is above (below) the homogeneous value.*

The requirement Eq.(3.28) ($PC(\mathcal{N})$) implies that necessarily the resulting constraint from the homogeneous participation condition, Eq.(3.9), also holds, which is stronger than the inequality $r < \frac{n-1}{n-2}$. In the presence of weak heterogeneity the participation condition for homogeneous players, Eq.(3.9), is not sufficient. The player with the weakest preference resp. smallest β_i, player n, has the lowest payoff. Hence, he is the first potential candidate to drop out of the contest. His participation condition $PC(\mathcal{N})$ does not hold at first. Therefore, the correct constraint is of the form $r \leq r_{max}(\mathcal{N})$, where $r_{max}(\mathcal{N})$ is a solution of the participation condition $PC(\mathcal{N})$ for the player with the weakest preferences respective highest preference parameter λ. $r_{max}(\mathcal{N})$ is close to $r_{max}(n)$, therefore we assume that the solution has the form $r = r_{max}(n)(1 + \delta(\mathcal{N}))$ where $|\delta(\mathcal{N})| << 1$. The result is given by

$$\delta(\mathcal{N}) = \frac{\frac{1}{n} + \tilde{\beta}_n(n-3) - \left[(\frac{1}{n} + \tilde{\beta}_n(n-3))^2 - 4\tilde{\beta}_n(1-\tilde{\beta}_n)(n-2)\right]^{1/2}}{2(n-2)(1-\tilde{\beta}_n)}. \tag{3.30}$$

Result 3.7 *Weakly heterogeous players start dropping out at a smaller technology parameter r compared to the homogeneous contest.*

Proof. In Eq.(3.30) $\tilde{\beta}_n = \beta_n - \bar{\beta}_n$ is the difference of the weakest player from the average strength. If all players participate, as assumed, $\tilde{\beta}_n < 0$ and therewith $\delta(\mathcal{N}) \leq 0$. ■

3.6 Weakly Heterogeneous Players: Equilibrium with less than all Players Participating

It has been shown formally that payoffs are higher for players with more spiteful preferences. Consequently, the equilibrium of n actively participating players ceases to exist when the participation condition of the player with the weakest preference does not hold anymore. At that point the n-th player drops out of the contest. The remaining $n-1$ players then find themselves in a situation similar to the one discussed in the previous section, but this time with $n-1$ instead of n players. As the technology parameter r increases further, some other player drops out, and so on. Consider now an intermediate equilibrium after $n-k$ players dropped out, $2 < k \leq n$, in which players $\mathcal{K} = \{1, \ldots, k\}$ are active, while players $\{k+1, \ldots, n\}$ have dropped out. For players in the subset $\mathcal{K}$, the average homogeneous effort $\bar{x}_k$ should be used as the reference effort. Then, the unknown equilibrium effort should be written in the form $x_i(\mathcal{K}) = \bar{x}_k(1 + e_i(\mathcal{K}))$. Further, let $\bar{\beta}(\mathcal{K}) = \frac{1}{k}\sum_{j\in\mathcal{K}} \beta_j$ denote the average relative preference in a subset $\mathcal{K}$, with $\bar{\beta}(\mathcal{N}) \equiv \bar{\beta}_n$. Then, following the same logic as in the previous section, the equilibrium relative effort of the active players will be

$$e_i(\mathcal{K}) = \frac{\beta_i - r\frac{k-2}{k-1}\bar{\beta}(\mathcal{K})}{1 - r\frac{k-2}{k-1}}, i \in \mathcal{K}. \tag{3.31}$$

The material payoffs of the active players become

$$\Pi_i(\mathcal{K}) = \frac{V}{k}\left[1 - \frac{r(k-1)}{\bar{\lambda}k} + \frac{(1 - r\frac{k-1}{\bar{\lambda}k})r\beta_i - (1 - r\frac{k-2}{\bar{\lambda}k})r\bar{\beta}_k}{1 - r\frac{k-2}{k-1}}\right]. \tag{3.32}$$

Let us now identify the conditions under which subset $\mathcal{K}$ will be active in equilibrium. As before, the technical condition to be able to use a linear approximation is $|\beta_i| << 1 - r\frac{k-1}{k-2}, i \in \mathcal{K}$, which implies in particular $r < \frac{k-1}{k-2}$. It also must be fulfilled that no player in $\mathcal{K}$ has an incentive to drop out. Let k denote the player with the least spiteful preference in the subset $\mathcal{K}$. Using Eq.(3.32), the expression for the upper bound on r can be written as $r_{max}(\mathcal{K}) = r_{max}(k)(1 + \delta(\mathcal{K}))$, with

$$\delta(\mathcal{K}) = \frac{\left[\frac{1}{k} + \tilde{\beta}_k(k-3)\right] - \left[\left(\frac{1}{k} + \tilde{\beta}_k(k-3)\right)^2 - 4\tilde{\beta}_k(1-\tilde{\beta}_k)(k-2)\right]^{1/2}}{2(k-2)(1-\tilde{\beta}_k)}. \tag{3.33}$$

Here again $\tilde{\beta}_k = \beta_k - \bar{\beta}(\mathcal{K})$ is the difference of the weakest player from the average relative preference. Since $\tilde{\beta}_k < 0$ it can be easily seen that the correction $\delta(\mathcal{K})$ is always negative, as before. The second bracketed term in the numerator is always bigger than the first bracketed term. It means that players start dropping out on a smaller technology parameter compared to the homogeneous contest.

3.7 Numerical Illustration

In this section we illustrate via a numerical example the results obtained above. Assume a small rent-seeking contest of four ($n = 4$) heterogeneous

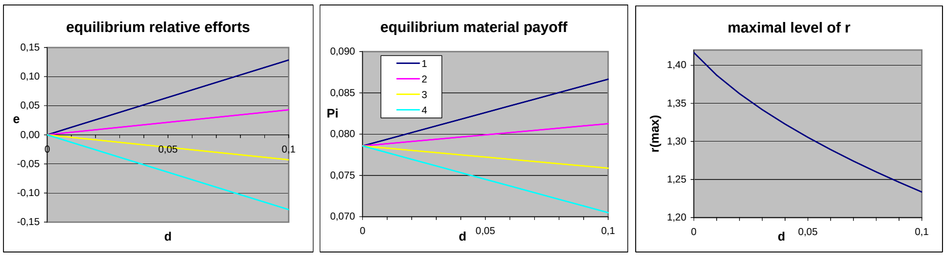

Figure 3.2: The equilibrium relative efforts (left), the equilibrium payoffs (middle) and the maximal level of r for $n = 4$ weakly heterogeneous players, as a function of heterogeneity parameter d. Discriminatory power $r = 0.8$ is used in the left and middle panels.

players, who have averaged negatively interdependent preferences of the value $\alpha = -0.5$ and compete for prize $V = 1$. The heterogeneity is considered to be weak. To simplify the analysis we assume the relative preferences β_i to be distributed along the interval $(-d, d)$, where $d \ll 1$. Specifically, we set $\beta_i = d\frac{n+1-2i}{n+1}$. If $d = 0$ all players have the same negatively interdependent preferences leading us back to the homogeneous case. We calculate all relevant values dependent on the heterogeneity measure d up to a value of $d = 0.1$. The results for the case when all four players are active are shown in Figure (3.2). The left panel shows the equilibrium relative efforts e_i as a function of d for $r = 0.8$. They start from $e_i = 0$ for $d = 0$. They increase (decrease) with an increasing d for the two players, whose preferences lie above (below) the average preference. This means that more spiteful players spend a higher relative effort. The middle panel shows the equilibrium payoffs Π_i as a function of d. They behave in the same kind as the equilibrium relative efforts, starting from the homogeneous value of the payoff. With an increasing heterogeneity, increasing d, players with more (less) spiteful preferences exert higher (lower) payoffs. The equilibrium of all players participating exists for $0 \le r \le r_{max}(\mathcal{N})$. This threshold value as a function of d is shown in the right panel. It decreases monotonically with increasing heterogeneity.

3.8 Conclusion

Players with identical preferences are rarely observed in real-world contests. In the present chapter we analyze contests between heterogeneous players. Heterogeneity exists due to different preferences of players. The preference functions are all interdependent, which means that the utility of player i depends on his own payoff as well as on the averaged payoff of all players. The preference functions are of different interdependence because they vary in the individual weight set to the averaged payoff. For a constant returns to scale technology we calculate the asymmetric equilibrium arising in the contest. To generalize the result for decreasing and increasing returns to scale we have to use the trick of a Taylor expansion because first-order conditions are not longer exactly analytical solvable. We Taylor expand around the equilibrium arising with homogeneous players. Therefore, we have to assume weak heterogeneity among the players.

In the present chapter we analyze contests with arbitrary marginal efficiency, in which preferences of players are close but still distinguishable. We study weak heterogeneity using a simple but powerful technique of linear approximation. Its basic idea is that efforts of weakly heterogeneous players in equilibrium are close to those of identical players who have the same averaged preference parameter. The equilibrium of identical players is often well characterized, and the behaviors of weakly heterogeneous players can then be analyzed by Taylor-expanding the corresponding function around the homogeneous equilibrium. This generic idea is not limited to contests, it is applicable whenever payoffs are sufficiently smooth functions of players' actions in the neighborhood of the homogeneous equilibrium. It is methodologically important that the limits of validity of the linear approximation can be obtained within the approximation itself. The explicit analytical expressions derived in the linear approximation produce results for equilibrium effort levels and payoffs of players. This expression for equilibrium effort levels and payoffs in the linear approximation only contains the player's own preference, β_i, and the average preference in the population, $\bar{\beta}_n$. These expressions are also relevant to contests with private values (Malueg and Yates (2004)), where only the distribution of preferences in the population is known. It is interesting to see how robust the symmetric equilibrium is with respect to weak heterogeneity. In contests, the elasticity of effort with respect to a player's own preference is large, which implies that weakly heterogeneous players respond strongly to a change in their relative advantage or disadvantage. In contrast, the elasticities of effort with respect to the other player's preferences are small, which means that the player's reaction to information about changes in the weak heterogeneity of others is small. These findings are independent of parameter values.

For Tullock contests of identical players with independent preferences Perez-Castrillo and Verdier (1992) identified the sequence of equilibria that arise as the discriminatory power increases. We find that this sequence is shifted downwards through the introduction of negatively interdependent preferences and is changed through the introduction of heterogeneity. In the case of heterogeneity between the players the upper bound on r is downward-sloping and destroys some of the equilibria. Weak heterogeneity qualitatively alters the structure of the equilibria compared to the homogeneous case with respect to the question who drops out first. In the homogeneous equilibria it does not matter which player drops out of the contest first. However, even weak heterogeneity determines the weakest player who drops out first.

A remark can be made on situations in which the heterogeneity becomes strong. Clearly, the equilibrium effort levels and payoffs will deviate significantly from those predicted by the linear theory, but the overall tendency should remain the same: players with more spiteful preferences exert higher efforts and enjoy higher payoffs (Guse (2006)). The upper bound on r for the existence of a particular equilibrium configuration should still drop with increasing heterogeneity, as disparity between the player who is the least spiteful and the rest of the active subset increases.

Chapter 4

Two-Stage Between-Group Rent-Seeking with Negatively Interdependent Preferences[1]

4.1 Introduction

Various economic, political and social interactions can be viewed as rent-seeking contests. One component in the theoretical literature on rent-seeking tournaments are group rent-seeking contests. An example for these group contests are struggles for government support such as request for subsidies, lobbying for relaxed regulations, imposing tax reductions between different industries. Given that many interest groups of players are involved, the competition is an example of group decision making. This chapter examines group rent-seeking contests in which rent-seeking activities take place in two stages.[2] In the first stage two groups of homogeneous players compete for a single rent V. Aggregate group effort determines the probability that the group wins the rent. No predetermined distribution rule exists in the groups like in Nitzan (1991). Instead, the members of the winning group compete

[1]Content and results of Sections 4.1 - 4.3 are published in Risse (2010). The original publication is available at www.springerlink.com. ©Springer-Verlag

[2]For an overview of the literature on rent-seeking and contests see Tollison (1982), Nitzan (1994), Corchon (2007) or Konrad (2009).

in a second stage for the prize, which may, but need not, be divisible in this stage. Individual spending determines the probability that a particular individual receives the rent or the proportion of the rent allocated to each member. The results derived in this chapter apply to both situations. We call this analyzed model two-stage Between-Group contest. Afterwards, we compare those results of the two-stage Between-Group contest with the results of a standard one-stage individual contest and the results of another two-stage group contests, the Semi-Finals Contest. In the Semi-Finals model one player is chosen from each of the two groups through competing for the position and then these two players compete for the rent by expending effort in a second stage. Semi-Finals contests are analyzed beside others by Parco, Rapoport, and Amaldoss (2005), Moldovanu and Sela (2006), Baik and Lee (2000), Lee (2003), Easterlin (2009) and Sheremeta (2009).

Earlier work in the literature for two-stage Between-Group rent-seeking is provided by Katz and Tokatlidu (1996). In a model with symmetric rent valuation, they focus on the relation between group size and aggregate rent-seeking. They show that social waste does not only depend on total number of players but also on the distribution across groups. In this case group size asymmetry facilitates the reduction of rent dissipation. This chapter extends the Katz and Tokatlidu model by introducing a technology parameter and by relaxing the common assumption of purely self-interested preferences of the players. In order to allow for status seeking interests we introduce the concept of negatively interdependent (relative) preferences (Section 4.2). Relative preferences can be seen in individual decisions as well as in group decision making. They have been shown to play an important role in both individual and group decision making. Porac, Wade, and Pollock (1999) and Alexopolos and Sapp (2006) analyze the impact on the observed economic behavior of firms. Executive compensation in firms is based on a comparison of firm performance to the performance of other, similar firms. Hence, managers will focus their actions on improving the performance of their firm relative to a set of competitor firms. Therefore, many firms appear to maximize their market share at the expense of profits, a strategy which corresponds to relative behavior. Individuals may also care about their own payoffs relative to those of others and not just about the absolute level of their payoffs. These individuals have relative rather than absolute preferences. Empirical evidence on relative preferences is given in

Bewley (1995), Clark and Oswald (1996), Sobel (2005), Kapteyn, van der Geer, van de Stadt, and Wansbeck (1997), Solnik and Hemenway (1998) and Levine (1998). Their results indicate that there are many situations in which individuals exhibit relative preferences.

Introducing agents with negatively interdependent preferences in a two-stage rent-seeking contest translates into adding another strategic component to the theoretical decision-making process. The relative maximizer's aim is to beat the average payoff and this is not only achieved by increasing one's own payoff, but also by reducing the payoff of the other players. Behavior that lowers the own payoff and the rival's payoff even more has been termed "spiteful" (Hamilton (1970)). In the context of two-stage contests, it can occur via lowering the opponents (group) probability of winning the contest.

Wärneryd (1998) uses the two-stage Between-Group contest to discuss the implications of the possibility of costly influence activities for the endogenous formation of different jurisdictional systems. Through a comparison with a one-stage contest, Wärneryd (1998) shows that less resources are spent in aggregate on appropriative activities under a federalist system of two-stage rent-seeking than in a unified jurisdiction with a single central government. Using the concept of interdependent preferences we can show that the two-stage contest model does not always lead to less rent dissipation as Wärneryd (1998) predicts (Section 4.3).

Built upon the model of Katz and Tokatlidu, Stein and Rapoport (2004) compare two variations of a two-stage contest. They consider more than two groups and focus on asymmetries between groups and players. They find that the percentage of rent dissipation - the ratio of total expenditures to the expected value of the rent - is higher under the Semi-Finals than under the Between-Group model. The result on rent dissipation in two-stage contests provided by Stein and Rapoport (2004) is point of departure for Section 4.4. We also compare rent dissipation in the two variations of a two-stage contest. However, we go a step further by allowing for an alternative preference structure of the players, i.e. relaxing the common assumption of purely self-interested preferences. Using the concept of interdependent preferences we can show that the Semi-Finals model does not always lead to higher rent dissipation.

To date, there is hardly any theoretical work on the characteristics and

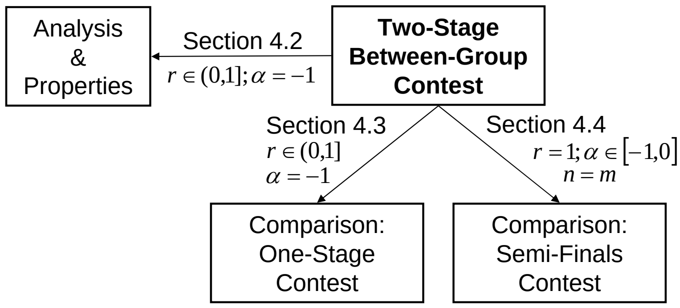

Figure 4.1: Section-Survey of Chapter 4.

the potential impact of negatively interdependent preferences on contest outcomes, even though Leininger (2009) has shown that evolutionary stable preferences in contests are negatively interdependent. One exception is Guse and Hehenkamp (2006) who analyze rent-seeking contests with a heterogeneous population in which part of the players are absolute payoff maximizers while others are also concerned about their relative position. As a result, it is those players with negatively interdependent preferences who experience a strategic advantage in general two-players contests and in n-players-contests with non-increasing marginal efficiency. In contrast, we focus in this chapter on homogeneous populations in which the players have either absolute preferences or strongly negatively interdependent preferences, in order to find the influence of preference types on the aggregate rent-seeking and therewith on the contest-structure choice.

As noted by Gradstein and Konrad (1999), in certain contexts the contest rule is exogenously determined. In other applications it is the outcome of a judicious design intended to attain a variety of objectives. According to experiences in political rent-seeking, politicians who allocate rents through a contest value high outlays. Would they have the choice to design a contest, they would choose the one that results in outlay maximization. Therefore, one objective is the maximization of the outlays by the contestants in a rent-seeking contest. To achieve the best possible objective, Gradstein and Konrad (1999) compare a one-stage contest with a pairwise multistage contest. Whereas, we analyze the outlays of players in a one-stage contest, and in two different two-stage contests and compare the resulting rent dissipation.

The remainder of the chapter is graphed in Figure 4.1 and is structured as follows: In Section 4.2, we analyze the properties of our basic contest-structure, the two-stage Between-Group contest. Therefore, we introduce the model (4.2.1) and analyze it with a technology parameter under the assumption of absolute payoff maximizers following Katz and Tokatlidu (1996) (4.2.2). In addition we extend the analysis to players with negatively interdependent preferences. Afterwards, we examine the relationship between group size and rent-seeking (4.2.3) and the relationship between preference type and rent-seeking in the two-stage contest (4.2.4). In Section 4.3 we compare the results of the two-stage Between-Group contest with those of an individual one-stage contest to find the influence of the assumed preferences on the contest-structure. The theoretical results are applied to the case of the formation of jurisdiction, unification and federalism (4.3.3). In the first two sections (4.2 and 4.3) the basic contest is called two-stage contest to pronounce the difference to the compared structure, the one-stage contest. In Section 4.4 we compare the results of the basic two-stage Between-Group contest with those of the Semi-Finals contest. Both analyzed structures are two-stage contests. Therefore, we call the basic contest Between-Group contest. First, we present the Nash equilibria in the Between-Group contest adjusted for the case with constant returns to scale technology, equal sized groups and an additional preference parameter. Afterwards, we analyze the Semi-Finals model separately for players with absolute payoff preferences and players with negatively interdependent preferences (4.4.2). Then, we compare the rent dissipations of the two contests to investigate the influence of the assumed preferences on the choice of the contest-structure (4.4.3). The theoretical results of this section are applied to the case of USA and German elections (4.4.4) and to the case of awarding prestige sport events (4.4.5). Finally, we conclude in (4.4.6).

4.2 Two-Stage Between-Group Contest: Analysis and Properties

4.2.1 The Model

Assume that the government intends to adopt a policy that will provide a rent V to group X or to group Y. The rent V can be monetary or non monetary. Group sizes are exogenously given. Groups X and Y consist respectively of n and m homogeneous risk neutral members. Each player has the same valuation for the rent. A member of a group is given a chance to compete directly for the rent V only if its group is awarded the rent. In the first round members of each group attempt to obtain the rent for their own group. The two groups play an inter-group Tullock contest, which means that a group of players can make an investment in order to participate in a lottery. The higher a group's investment, the higher its chance of winning the prize. Hence, their initial rent-seeking is group-oriented. Denote the first round rent-seeking pursued by member i of group X and member j of group Y by x_{1i} and y_{1j} respectively, and denote the probability that group X (Y) is awarded the rent by p_X (p_Y). We assume the probability that a given group will win the rent depends on the value of this group's outlays relative to the outlays of both groups: Extending Tullock (1980) for groups, the probability that group X wins the rent is given by the ratio

$$p_X = \frac{\sum_{i=1}^{n} x_{1i}^r}{\sum_{i=1}^{n} x_{1i}^r + \sum_{j=1}^{m} y_{1j}^r} \tag{4.1}$$

where r is the technology parameter.[3] It determines whether the contest is characterized by a decreasing $(r < 1)$, constant $(r = 1)$ or increasing $(r > 1)$ returns to scale technology. The technology will not be changed during the whole contest and is the same for all players.

The probability that group Y wins the rent, p_Y, is equal to $1 - p_X$. After the completion of the first round, each member of the winning group engages in rent-seeking activities in order to win the rent V or a part of the rent V for himself. The members of the winning group play an intra-group Tullock contest. Denote the second round rent-seeking investment pursued by member i of group X and member j of group Y by x_{2i} and y_{2j} respectively. The first stage investment is sunk. If group X has won the first round, then its i-th member will win in the second round with probability

$$p_{xi}(x_{21}, \ldots, x_{2n}) = \frac{x_{2i}^r}{\sum_{l=1}^{n} x_{2l}^r}. \tag{4.2}$$

If group Y has won the first round, its j-th member will win the rent in the second round with probability

$$p_{yj}(y_{21}, \ldots, y_{2m}) = \frac{y_{2j}^r}{\sum_{l=1}^{m} y_{2l}^r}.$$

[3]For an axiomatic foundation of group contest success functions see Münster (2009).

The parameter r can be interpreted as a measure of how decisive relative effort is. For instance, if r approaches zero, the probability to win the rent tends to $1/n$ respective to $1/m$, indicating that the designation of the winner is independent of effort. If r approaches infinity, the contest becomes fully discriminatory. Then the prize is always awarded to the contestant who exerts the most effort, and the random elements of the contest outcome disappear.

We could see p_{xi} and p_{yj} not as winning probabilities of the whole rent but as shares of the rent, which we could divide in the second round into shares for the different participants of this stage. This feature does not change the analysis.

The model follows Katz and Tokatlidu (1996), but we introduce the technology parameter r in the winning probability function and, in addition, agents with relative preferences. We restrict the technology parameter to decreasing or constant returns to scale technologies, which means $r \leq 1$. The analysis of the contest with increasing returns to scale technology ($r > 1$) goes beyond the scope of this chapter, because then the existence of a pure strategy Nash equilibrium is not guaranteed.

The first round of the contest does not offer individuals an ultimate payoff. It only determines the probability that a contestant will participate in the second round. The marginal benefit of an individual's first round investment is the increased probability to win the ultimate rent. Hence, the rent-seeking activities of individuals in the first round are determined by optimizing their expected payoff, given that all individuals act rationally and optimally in the second round.

First we analyze the model under the assumption that players are absolute payoff maximizers following Katz and Tokatlidu (1996). In a second step, we assume rent-seekers to be relative payoff maximizers. Like in Kockesen, Ok, and Sethi (2000) and Guse (2006) the focus lies on intra-group symmetric equilibria, where all players choose the same effort level in each group. We solve for a subgame perfect equilibrium outcome of this game via backward induction. Therefore we start the analysis with the second stage. All previous rent-seeking expenditures are sunk at this stage.

4.2.2 Analysis

Analysis with Absolute Payoff Maximizers

Without loss of generality assume that group X with n players wins the contest in the first stage. Therefore, we analyze the second stage of the contest from the perspective of the players of group X. For group Y the analysis applies equivalently.

The Second Stage In the second round the i-th member of group X maximizes his payoff of the second stage

$$\begin{aligned} F_{2i}^{abs} &= \Pi_{2i}(x_1, \ldots, x_n) \\ &= p_{xi}V - x_{2i} \end{aligned} \tag{4.3}$$

where p_{xi} is the success probability of player i chosen as in (4.2).

Assuming a symmetric Nash equilibrium within each group and a regular interior solution, the first-order condition for member i of group X is

$$\frac{\partial F_{2i}^{abs}}{\partial x_{2i}} = \frac{r x_{2i}^{r-1} \sum_{l \neq i} x_{2l}^r}{(\sum x_{2l}^r)^2} V - 1 \overset{!}{=} 0. \tag{4.4}$$

Since we assume agents to be homogeneous and search for symmetric equilibria, we set $x_{2i} = x_{2l} = x_2$. This gives $\frac{r x_2^{2r-1}(n-1)}{(n x_2^r)^2} V - 1 = 0$ which is solved by

$$x_2^{abs} = \frac{n-1}{n^2} rV. \tag{4.5}$$

This is our unique candidate for an interior solution. The second-order conditions proving the maximum are given as

$$\begin{aligned} \frac{\partial^2 F_{2i}^{abs}}{\partial x_{2i}^2} &= \frac{(r(r-1)x_{2i}^{r-2} \sum x_{2l}^r + r^2 x_{2i}^{2r-2} - x_{2i}^{2r-2}(2r-1)r)V}{(\sum x_{2l}^r)^2} \\ &\quad - \frac{2(r x_{2i}^{r-1} \sum x_{2l}^r - x_{2i}^{2r-1} r) V x_{2i}^{r-1} r}{(\sum x_{2l}^r)^3} \overset{!}{<} 0. \end{aligned}$$

Simplification under the assumption that all players have identical preferences leads to

$$\frac{Vr(rn^2 - n^2 - 3rn + n + 2r)}{n^3 x^2} < 0.$$

This holds in the symmetric case $x_2^{abs} = \frac{n-1}{n^2} rV$ if

$$\begin{aligned} & \frac{n(rn-n-2r)}{Vr(n-1)} < 0 \\ \iff & rn - 2r - n < 0 \\ \iff & r(n-2) - n < 0. \end{aligned}$$

Hence, the second-order conditions for local maximization holds for $r < \frac{n}{n-2}$.

Note that the second-order conditions implies global concavity of the payoff function if $r \leq 1$: The bracketed term in the numerator is always negative, the one in the denominator always positive. Hence, the local optimum is a global one for all $r \leq 1$. We concentrate on the case of a decreasing returns to scale technology, which means $r < 1$. Therefore, the second-order conditions for local maximization is always fulfilled: $r \leq 1 < \frac{n}{n-2}$.

Since the second-order condition is satisfied for all $r \leq 1$, we have found the global unique Nash equilibrium effort for the second stage of our model with absolute payoff maximizers as given in Eq.(4.5).

Substituting the optimal effort (4.5) of each member of group X in the utility function (4.3), we find that the individual's valuation of entering round two is $\frac{1}{n}V - \frac{n-1}{n^2} rV = \frac{n(1-r)+r}{n^2} V$ for a member of group X. If a player enters the second round of the contests he can earn this amount of the rent. The amount is the difference between the share of the rent he gets minus his effort chosen in this stage. Every player will bear this optimal payoff in mind when finding an optimal effort level for the first stage. Hence, in the first round each group-member solves the rent-seeking maximization for a reduced rent:
$V_n^{abs} = \frac{n-nr+r}{n^2} V$ for group X and $V_m^{abs} = \frac{m-mr+r}{m^2} V$ for group Y.

The value of entering the second round is lower for a member of the larger group, because the prize is divided by the number of members.[4]

The First Stage In the first stage of the contest with the given group success function (4.1), an absolute payoff maximizing individual i of group X tries to maximize his own payoff Π_{1i}, which is given as

$$F_{1i}^{abs} = \Pi_{1i} = \frac{\sum x_{1l}^r}{\sum x_{1l}^r + \sum y_{1j}^r} V_n^{abs} - x_{1i}. \tag{4.6}$$

[4]See Münster (2007) for further insights on the group-cohesion effect.

Calculating the first-order condition gives

$$\frac{\partial F_i^{abs}}{\partial x_{1i}} = \frac{r x_{1i}^{r-1} \sum y_{1j}^r}{(\sum x_{1l}^r + \sum y_{1j}^r)^2} V_n^{abs} - 1 \stackrel{!}{=} 0. \tag{4.7}$$

Following our assumption that members in each group have identical preferences, we can write that $\sum x_{1l}^r = n x_1^r$ and $\sum y_{1j}^r = m y_1^r$. Substituting this in the Eq.(4.7), we get

$$\frac{r x_1^{r-1} m y_1^r}{(n x_1^r + m y_1^r)^2} V_n^{abs} - 1 = 0. \tag{4.8}$$

This holds equivalently for a player of group Y:

$$\frac{r y_1^{r-1} n x_1^r}{(n x_1^r + m y_1^r)^2} V_m^{abs} - 1 = 0.$$

After some calculations and equating both equations we get

$$n x_1 V_m^{abs} = m y_1 V_n^{abs}. \tag{4.9}$$

Substituting this relation of x_1 and y_1 in the above Eq.(4.8) we get the solution

$$x_1^{abs} = \frac{r m V_n^{abs} (\frac{n V_m^{abs}}{m V_n^{abs}})^r}{(m (\frac{n V_m^{abs}}{m V_n^{abs}})^r + n)^2}. \tag{4.10}$$

This is the optimal effort of a player of group X in the first stage if all players have absolute preferences. The optimal effort of a player of group Y in the first stage, y_1^{abs}, can be calculated analogously.

$$y_1^{abs} = \frac{r n V_m^{abs} (\frac{m V_n^{abs}}{n V_m^{abs}})^r}{(n (\frac{m V_n^{abs}}{n V_m^{abs}})^r + m)^2}. \tag{4.11}$$

Analysis with Relative Payoff Maximizers

We now repeat the analysis of the two-stage contest under the assumption of relative payoff maximizers. We choose the simplest and at the same time strongest kind of social comparison. In our preference function the averaged payoff of the rivals has the same valency as the own payoff. We choose it that way to make the differences in the equilibrium behavior considerable visible. If we choose a lower weight of the rivals payoff in the preference function, the results would be the same but weaker in the valency.

We assume that group X with n players wins the contest in the first stage. Therefore, we concentrate the analysis of the second stage of the contest on the players of group X.

The Second Stage In the second round the i-th member of group X maximizes

$$\begin{aligned} F_{2i}^{rel} &= \Pi_{2i}(x_1, \ldots, x_n) - \bar{\Pi}_2(x_1, \ldots, x_n) \\ &= \frac{x_{2i}^r}{\sum_{l=1}^n x_{2l}^r} V - x_{2i} - \frac{1}{n}\Big(V - \sum_{l=1}^n x_{2l}\Big) \end{aligned} \tag{4.12}$$

where $\bar{\Pi}_2$ is the population mean payoff of the second stage. The success probability of player i is chosen as in Eq.(4.2). In this stage the individual is concerned of his payoff compared to the payoff of his own group members. The other group is not involved in this stage and hence not included in his utility function.

Assuming a symmetric Nash equilibrium within each group and a regular interior solution, the first-order condition for member i of group X is

$$\frac{\partial F_{2i}^{rel}}{\partial x_{2i}} = \frac{r x_{2i}^{r-1} \sum_{l \neq i} x_{2l}^r}{(\sum x_{2l}^r)^2} V - 1 + \frac{1}{n} \stackrel{!}{=} 0. \tag{4.13}$$

Since we assume identical agents and search for symmetric equilibria, we set $x_{2i} = x_{2l} = x_2$. This gives $\frac{x_2^{r-1}(n-1)x_2^r}{n^2 x_2^{2r}} rV - 1 + \frac{1}{n} = 0$ which is solved by

$$x_2^{rel} = \frac{r}{n} V. \tag{4.14}$$

(4.14) is our unique candidate of an interior solution of the second stage. We again concentrate on the case of a decreasing or constant returns to scale technology ($r \leq 1$). The second-order condition for a local maximum becomes

$$\frac{\partial^2 F_{2i}^{rel}}{\partial x_{2i}^2} = rV \left[\frac{(r-1)x_{2i}^{r-2} \sum\limits_{l \neq i} x_{2l}^r (\sum x_{2l}^r)^2 - 2r x_{2i}^{2r-2} \sum\limits_{l \neq i} x_{2l}^r \sum x_{2l}^r}{(\sum x_{2l}^r)^4} \right] < 0, \tag{4.15}$$

which under the assumption that all players are identical, simplifies to:

$$rV\frac{n-1}{n^3}\left[\frac{(r-1)n-2r}{x_2^2}\right] < 0. \tag{4.16}$$

(4.16) holds in the symmetric solution $x_2^{rel} = \frac{r}{n}V$ if $\frac{n-1}{n}\frac{1}{rV}\left[(r-1)n-2r\right] < 0$, which holds if $rn - 2r - n < 0$. Hence, the second-order condition for local maximization holds for $r < \frac{n}{n-2}$.

Note that the second-order condition implies global concavity of the relative payoff function, if $r \leq 1$ holds: The term in parentheses in the numerator is always negative, the one in the denominator always positive. Hence, the local optimum is a global one for all $r \leq 1$.[5]

Result 4.1 *Aggregate rent dissipation in x_2^{rel} of the second stage is independent of the number of contestants in the rent-seeking contest:*
$n \cdot \frac{r}{n}V = rV$

An equilibrium in a contest displays overdissipation (full dissipation, underdissipation) if and only if $\sum_{i=1}^{n} x_i^{rel} > (=, <)V$, respectively. In this case the technology parameter r determines the dissipation rate of this stage.[6] The dissipation rate r represents the efficiency of the given contest technology. For $r < 1$ there is underdissipation of the rent, for $r = 1$ full dissipation and for $r > 1$ overdissipation as long as the existence of the Nash equilibrium is secured.

Calculating the benefit of entering the second round requires to take a broader view on the whole game. In the second stage the individuals with relative preferences want to maximize their payoff in comparison to those of their own group-members, because those are still the only active players in the contest. Under this assumption the optimal effort is given as $\frac{r}{n}V$ in the second stage. Now, analyzing the first stage, individuals additionally compete with the members of the rival group. Players want to maximize their payoff compared to those of all other players. The benefit of entering the second round is now the difference in the material payoff between individual

[5]For $r > 1$ the proof is technically beyond the scope of this chapter. See Hehenkamp, Leininger, and Possajennikov (2004) for it.

[6]Hehenkamp, Leininger, and Possajennikov (2004) show that this is also the unique evolutionarily stable strategy (ESS) in an evolutionary game with absolute payoffs.

i and the average over all other players. We first substitute the optimal effort ($\frac{r}{n}V$) into Eq.(4.3) and find that the individual's material payoff is $\frac{1-r}{n}V$ for a member of group X. Second, regarding all players we calculate the utility of entering the second stage assuming group X won the rent in the first stage.

$$V_n^{rel} = \frac{1-r}{n}V - \frac{n\frac{1-r}{n}V + m \cdot 0}{m+n} = \frac{m}{m+n}\frac{1-r}{n}V \tag{4.17}$$

Due to the sunk costs each player solves the rent-seeking maximization for a reduced rent in the first round: $V_n^{rel} = \frac{m}{m+n}\frac{1-r}{n}V$ for members of group X and $V_m^{rel} = \frac{n}{m+n}\frac{1-r}{m}V$ for members of group Y.

For constant returns to scale technologies ($r = 1$) this reduced rent is zero. None of the players has an incentive to expand any effort in the first stage. The competition on the second stage is so strong that there is full rent dissipation as it can be easily seen from Result 4.1.

The First Stage The utility of a relative payoff maximizing individual i of group X is $F_{1i}^{rel} = \Pi_i - \bar{\Pi}$, where $\bar{\Pi}$ is the population mean payoff:

$$\begin{aligned} \bar{\Pi} &= \frac{\sum_{l=1}^{n}\Pi_l + \sum_{j=1}^{m}\Pi_j}{n+m} \\ &= \frac{p_X n V_n^{rel} - \sum_{l=1}^{n} x_{1l} + p_Y m V_m^{rel} - \sum_{j=1}^{m} y_{1j}}{n+m} \end{aligned}$$

and the success function of the groups p_X respective p_Y chosen as in Eq.(4.1). With these we get

$$\begin{aligned} F_{1i}^{rel} = & \frac{\sum x_{1l}^r}{\sum x_{1l}^r + \sum y_{1j}^r}V_n^{rel} - x_{1i} - \frac{1}{n+m}\cdot \\ & \left(\frac{\sum x_{1l}^r}{\sum x_{1l}^r + \sum y_{1j}^r}nV_n^{rel} - \sum x_{1l} + \frac{\sum y_{1j}^r}{\sum x_{1l}^r + \sum y_{1j}^r}mV_m^{rel} - \sum y_{1j}\right). \end{aligned}$$

The first-order condition is given by

$$\begin{aligned} \frac{\partial F_i^{rel}}{\partial x_{1i}} &= \frac{r x_{1i}^{r-1}\sum y_{1j}^r}{(\sum x_{1l}^r + \sum y_{1j}^r)^2}V_n^{rel} - 1 - \frac{1}{n+m}\cdot \\ & \quad \left(\frac{r x_{1i}^{r-1}\sum y_{1j}^r}{(\sum x_{1l}^r + \sum y_{1j}^r)^2}nV_n^{rel} - 1 - \frac{(\sum y_{1j}^r) r x_{1i}^{r-1}}{(\sum x_{1l}^r + \sum y_{1j}^r)^2}mV_m^{rel}\right) \\ &\overset{!}{=} 0. \end{aligned}$$

Following our assumption that group members are identical within each group, we have $\sum x_{1l}^r = nx_1^r$ and $\sum y_{1j}^r = my_1^r$. Substituting this in the first-order condition, we yield

$$\frac{rx_1^{r-1}my_1^r}{(nx_1^r+my_1^r)^2}V_n^{rel} - 1 - \frac{1}{n+m}\cdot$$
$$\left(\frac{rx_1^{r-1}my_1^r}{(nx_1^r+my_1^r)^2}nV_n^{rel} - 1 + \frac{-my_1^r rx_1^{r-1}}{(nx_1^r+my_1^r)^2}mV_m^{rel}\right) = 0.$$

Equivalently, we get for a player of group Y:

$$\frac{ry_1^{r-1}nx_1^r}{(nx_1^r+my_1^r)^2}V_m^{rel} - 1 - \frac{1}{n+m}\cdot$$
$$\left(\frac{ry_1^{r-1}nx_1^r}{(nx_1^r+my_1^r)^2}mV_m^{rel} - 1 + \frac{-nx_1^r ry_1^{r-1}}{(nx_1^r+my_1^r)^2}nV_n^{rel}\right) = 0.$$

After some manipulations and equating both equations we get

$$m^2 y_1^{rel} = n^2 x_1^{rel}. \tag{4.18}$$

Substituting (4.18) into the FOC, we yield

$$x_1^{rel} = \frac{1}{n+m-1}\cdot\frac{rm^2(\frac{m^2}{n^2})^r(V_n^{rel}+V_m^{rel})}{(n(\frac{m^2}{n^2})^r+m)^2}. \tag{4.19}$$

4.2.3 Relationship between Group Size and Rent-Seeking

The First Stage Rent-Seeking

The effect of an increase in the size of the player's own group on the individual effort spend in the first round is negative ($\frac{\partial x_1}{\partial n} < 0$), no matter what preference type we assume. Players of bigger groups spend as an individual less effort than players of smaller groups. They have a lower expected payoff at the second stage. The effect of an increase in the size of group Y on the first stage effort done by players of group X is ambiguous ($\frac{\partial x_1}{\partial m} \gtreqless 0$). It depends on the assumption of preferences and on the distribution of the players across groups. If group X is the smaller group ($n < m$), the effect is negative under the assumption of absolute payoff maximizers. This means that effort spend by group X decreases as group Y becomes even larger. If group X is the bigger group ($n > m$) it is a positive effect. An increase in the size of the smaller group raises first-round effort by the larger group. Whereas for players with negatively interdependent preferences the effect is the reverse. If the already bigger group increases even more, the effect is positive. This means that effort spend by the smaller group X increases as group Y becomes even larger. If group X is the bigger group ($n > m$) it is a negative effect. An increase in the size of the smaller group abates first-round effort by the larger group.

Summarized these findings give:

Result 4.2 *In the unique subgame perfect equilibrium of the contest it holds true that*

i.) individual expenditures of the first stage decrease with an increasing size of a contestants own group independent of preference type.

ii.) under the assumption of absolute preferences individual expenditures of the first stage decrease (increase) with an increasing size of the rivals group, if the group asymmetry increases (decreases).

iii.) under the assumption of relative preferences individual expenditures of the first stage decrease (increase) with an increasing size of the rivals group, if the group asymmetry decreases (increases).

The Overall Rent-Seeking

If we take a look at the contest from a welfare perspective, we focus on the aggregate expenditures R^{abs} and R^{rel} in the whole contest to determine the social waste. Since the first stage outcome assigns a probability of winning to each group rather than determining a winner, the amount of aggregate effort will depend on which group enters the second round. Therefore, we aggregate the expenditures of the single stages and add them up multiplied by their probability to occur; once for the contest with absolute payoff maximizers and once for the contest with relative payoff maximizers.

$$
\begin{aligned}
x_1^{abs} &= \frac{rmV(\frac{n^3(m-mr+r)}{m^3(n-nr+r)})^r(n-nr+r)}{n^2(m(\frac{n^3(m-mr+r)}{m^3(n-nr+r)})^r+n)^2} \\
y_1^{abs} &= \frac{rnV(\frac{m^3(n-nr+r)}{n^3(m-mr+r)})^r(m-mr+r)}{m^2(n(\frac{m^3(n-nr+r)}{n^3(m-mr+r)})^r+m)^2} \\
x_2^{abs} &= \frac{n-1}{n^2}rV \\
y_2^{abs} &= \frac{m-1}{m^2}rV
\end{aligned}
$$

$$
R^{abs} = nx_1^{abs} + my_1^{abs} + \pi_x^{abs}nx_2^{abs} + \pi_y^{abs}my_2^{abs} \tag{4.20}
$$

where $\pi_x^{abs} = \frac{nx_1}{nx_1+my_1}$ and $\pi_y^{abs} = \frac{my_1}{nx_1+my_1}$ are the probabilities that group X or group Y reaches the second stage of the contest, respectively.

The aggregate effort of the contest with relative payoff maximizers is given as

$$
\begin{aligned}
x_1^{rel} &= \frac{1}{(n+m-1)(n+m)} \cdot \frac{rm^2(\frac{m^2}{n^2})^r(\frac{n}{m}+\frac{m}{n})(1-r)V}{(n(\frac{m^2}{n^2})^r+m)^2} \\
y_1^{rel} &= \frac{1}{(n+m-1)(n+m)} \cdot \frac{rn^2(\frac{n^2}{m^2})^r(\frac{n}{m}+\frac{m}{n})(1-r)V}{(m(\frac{n^2}{m^2})^r+n)^2} \\
x_2^{rel} &= \frac{r}{n}V \\
y_2^{rel} &= \frac{r}{m}V
\end{aligned}
$$

$$
\begin{aligned}
R^{rel} &= nx_1^{rel} + my_1^{rel} + \pi_x^{rel}nx_2^{rel} + \pi_y^{rel}my_2^{rel} \\
&= nx_1^{rel} + my_1^{rel} + rV. \qquad (4.21)
\end{aligned}
$$

Under the assumed decreasing returns to scale technology ($r < 1$) there is always underdissipation of the rent in the contest independent of the preference assumption. With an increasing number of players in both groups $\left(\lim_{n\to\infty}\lim_{m\to\infty} \text{ or } \lim_{m\to\infty}\lim_{n\to\infty}\right)$ the aggregate expenditure converges to rV, but never reaches rV as long as the number of players is finite.[7] The difference between the two scenarios is that in the case of players with absolute preferences, it converges from below and in the case with relative preferences from above.

[7]Iterated limits are computed with Maple.

If the number of players in just one group increases, there are three effects on the aggregate effort. Assume the number of players in group Y, m, increases whereas n the number of members of group X remains constant. The effect of m on R^{abs} is given by

$$\frac{\partial R^{abs}}{\partial m} = \frac{\partial(nx_1^{abs})}{\partial m} + \frac{\partial(my_1^{abs})}{\partial m} + \pi_y^{abs}\frac{\partial(my_2^{abs})}{\partial m} + \frac{\partial \pi_x^{abs}}{\partial m}(nx_2^{abs} - my_2^{abs}). \tag{4.22}$$

The sum of the first two terms in (4.22) is negative. It captures the effect in the first stage effort done by both groups. Whenever one group becomes larger the cumulative effort of both groups in the first stage shrinks, because the competition in the second stage increases. The third term in (4.22) is positive and outweighs the first sum. It measures the actual anticipated increase in second round aggregate effort. Then the overall effect is positive. These first three terms determine the direct effect of an increasing number of players. The last fraction measures the indirect effect through the changing asymmetry between the groups. It depends on the distribution of the players across groups. Under the assumption of absolute payoff maximizers the effect is negative if the group asymmetry increases and it is positive otherwise.

In the contest with status seekers the effect of m on R^{rel} is given by

$$\frac{\partial R^{rel}}{\partial m} = \frac{\partial(nx_1^{rel})}{\partial m} + \frac{\partial(my_1^{rel})}{\partial m}. \tag{4.23}$$

This sum consists of two effects and can be negative or positive. Under the assumption of status seekers the second stage effect is not relevant, because the effort of the second stage does not change with the number of participants. The terms in (4.23) capture the effect in the total first stage effort done by both groups to an increase of the number of members of group Y. The first effect arises through the changing asymmetry between the groups. It is positive if the group asymmetry increases and it is negative otherwise. The other one arises in group Y through the increase in the own group size and it is always negative. If group asymmetry is strong, the former exceed the latter. The effort of the second stage does not change with an increase in the size of one group. It is constant and independent from the distribution of players. Therefore, the overall effect is ambiguous.

4.2.4 Relationship between Preference Type and Rent-Seeking

We have seen that the second stage equilibrium with relative payoff maximizers is given by $x_2^{rel} = \frac{r}{n}V$. In comparison, the unique pure strategy Nash equilibrium with absolute payoff maximizers is given by $x_2^{abs} = \frac{n-1}{n^2}rV$. When comparing the effort levels of the two Nash equilibria of the second stage we find that the agent interested in relative payoffs shows a higher effort gaining lower payoffs than the agent maximizing absolute payoffs $(\frac{r}{n}V > \frac{r}{n}V - \frac{r}{n^2}V)$. Hence, the incentive of the relative maximizers to reach the second round is smaller.

In a private good contest for the rent V, as in the second stage of our model, the contest with relative payoff maximizers always leads to a higher expenditure than the one with absolute payoff maximizers.[8] The Nash equilibrium under the assumption of absolute preferences yields a positive value of the first derivative of the relative maximization function. A marginal increase of expenditures beyond the optimal absolute Nash level increases relative payoff. First, a marginal increase of expenditure has second order negative effects on a player's own payoff because the first derivative of his payoff function is zero at the absolute Nash level. Second, a marginal increase has a first order negative effect on the other players' payoff, because cross-derivatives in the absolute Nash equilibrium are always negative. Resulting in the case that the difference between one's own and others' payoffs increases.

At the first stage the effect is the reverse. The aggregate expenditure of the first stage in the equilibrium with relative preferences is always lower than those of the Nash equilibrium of players with independent preferences. Nevertheless, the explanation given above is still valid. How can it be? This result is driven by the different premises. Players valuations to enter the second round of the contest are unequal. If we compare the benefit of entering the second stage of absolute payoff maximizers $(\frac{1-r}{n}V + \frac{r}{n^2}V)$ with those of relative payoff maximizers $(\frac{m}{m+n} \cdot \frac{1-r}{n}V)$, we find that absolute maximizers have a higher valuation of the rent in the first stage. This creates a much stronger effect than the effect arisen from the different preferences and explains the observed asymmetry in the player's behavior. The relative payoff maximizers still act more aggressively, but given the different premises, the

[8] See Leininger (2003) and Hehenkamp, Leininger, and Possajennikov (2004).

much lower rent valuation, they spent less effort compared to the absolute payoff maximizers. The difference in the chosen effort tends toward zero as the number of players increases.

The effects of both stages counteract each other, but since the effect of the second stage is stronger, it dominates the first-stage effect.

Result 4.3 *The total expenditure in equilibrium of the overall two-stage contest with relative preferences is higher than the overall expenditure in the contest with absolute maximizers.*

Proof. To show:

$$\begin{aligned}
& RD^{rel} > RD^{abs} \\
\Leftrightarrow \quad & \frac{(nx_1^{rel} + my_1^{rel} + rV)}{V} > \frac{(nx_1^{abs} + my_1^{abs} + \pi_x^{abs} nx_2^{abs} + \pi_y^{abs} my_2^{abs})}{V}.
\end{aligned}$$

First, we know that the rent dissipation in the two-stage contest with relative payoff maximizers is always bigger than the technology parameter ($RD^{rel} > r$). Second, the rent dissipation under the assumption of a fixed number of absolute payoff maximizing individuals is bounded above by the rent dissipation of the contest with the same number of players which are ordered in groups with the same size. Katz and Tokatlidu (1996) analyze in a two-stage group contest how the magnitude of aggregate rent seeking depends on the total number and on the distribution of population across groups. They find that a given population distributed equally among groups will tend to yield more rent seeking than an unequally distributed one. Therefore, it is enough to show that $r > ((n+m)x_1^{abs} + \frac{n+m}{2} x_2^{abs})/V$ where now x_1^{abs} and x_2^{abs} are the chosen effort levels with $\frac{n+m}{2}$ players in each group.

$$\begin{aligned}
& r > \left[(n+m)x_1^{abs} + \frac{n+m}{2} x_2^{abs}\right] /V \\
\Leftrightarrow \quad & r > (n+m) \frac{r(\frac{n+m}{2} - \frac{n+m}{2} r + r)}{\frac{n+m}{2}(n+m)^2} + \left(1 - \frac{1}{\frac{n+m}{2}}\right) r \\
\Leftrightarrow \quad & 1 > \left[\frac{n+m}{2}(1-r) + r\right] \frac{2}{(n+m)^2} + 1 - \frac{2}{n+m} \\
\Leftrightarrow \quad & 0 > 1 - r + \frac{2r}{n+m} - 2 \\
\Leftrightarrow \quad & 1 > \underbrace{\left(\frac{2}{n+m} - 1\right)}_{<0} r
\end{aligned}$$

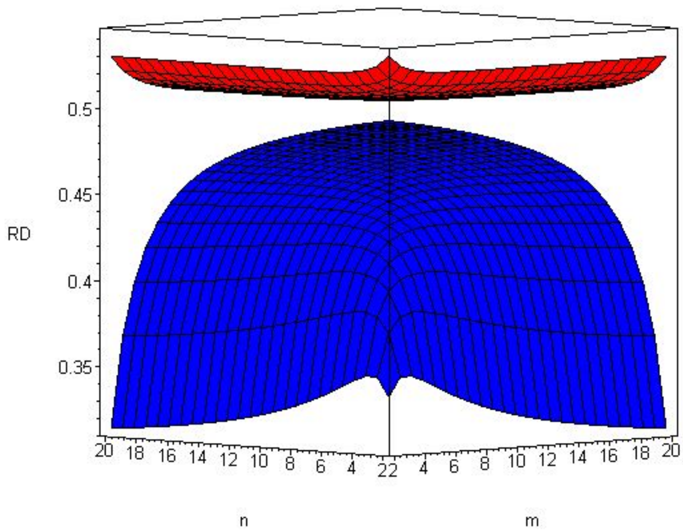

Figure 4.2: Rent dissipation in the two-stage contest with absolute payoff maximizers (blue) and with relative payoff maximizers (red). Parametrization: $r = 0.5$.

We have shown that $RD^{rel} > r > RD^{abs}_{(n+m)/2} \geq RD^{abs}$. For an impression of the rent dissipation of the two-stage contest scenario see Figure 4.2. ■

In a two stage group contest the incentive to increase one's own expenditure is counteracted by the free-rider problem. An increase above the Nash level still advances one's own position in relation to members of the other group, but it puts oneself also at a disadvantage in relation to one's own group members. They can free-ride on the additional spiteful effort. Free-riding is in the context of relative payoff maximizers sensible spiteful behavior against members of the own group. Free-riding increases ex post the payoff of members of the other group, which brings along a decrease in one's own relative payoff, but it increases one's own payoff by even more.[9]

4.3 Comparison with One-Stage Contests

4.3.1 Analysis One-Stage Contest

A natural alternative to a two-stage contest like the one outlined above is an individual one-stage contest, in which all members of both groups are pooled

[9]See for this problem Leininger (2002).

together and fight in a one-stage Tullock contest for the rent or for a part of the rent V. This is the simplest way to model a contest and therefore used here as a first comparison-contest. To compare these two different structures of a contest, we calculate the efforts of the players in a one-stage contest: once for the case with absolute payoff maximizers and once for the case of relative payoff maximizers.

The effort of an individual in a one-stage contest for $(n+m)$ players with the contest success function proposed by Tullock can be calculated analogously to the second stage of the two-stage contest. Differences are just in the number of players, $n+m$ instead of n or respective m, and in the amount of the rent. The equilibrium effort for the case of players with absolute preferences is described as:

$$x^*_{abs} = \frac{n+m-1}{(n+m)^2} rV. \tag{4.24}$$

The dissipation rate is given by $RD^{one}_{abs} = \frac{n+m-1}{(n+m)} r$.

The expenditure for payoff maximizers with relative preferences is characterized by:

$$x^*_{rel} = \frac{r}{(n+m)} V, \tag{4.25}$$

whereas the dissipation rate is given by $RD^{one}_{rel} = r$.

4.3.2 Comparison of Two-Stage Contest and One-Stage Contest

Comparing the results of a one-stage contest with those of a two-stage contest, we find the following:

Result 4.4

i.) In the case of absolute payoff maximizers total expenditure in a one-stage contest is always higher or equal than in a two-stage contest.

ii.) For relative payoff maximizers total expenditure in a one-stage contest is always lower or equal than the one in a two-stage contest.

Proof. ad i.) To prove: $RD^{one}_{abs} \geq RD^{two}_{abs}$
Since group size asymmetry acts to reduce the rent dissipation in a two-stage

contest[10] the rent dissipation in a contest with $\frac{n+m}{2}$ players in each group is higher than the rent dissipation of the two-stage contests with n players in group X and m players in group Y. Assuming the same group size without changing the total number of participating players the rent dissipation is given as $RD^{two}_{abs} = (n+m)x^{abs}_2 + \frac{n+m}{2}x^{abs}_1$ where x^{abs}_1 and x^{abs}_2 are calculated with $\frac{n+m}{2}$ players in group X. Using this result it is enough to show

$$\begin{aligned}
& \frac{n+m-1}{n+m} r \geq (n+m) \frac{r(\frac{n+m}{2}(1-r)+r)}{\frac{n+m}{2}(2\frac{n+m}{2})^2} + \frac{\frac{n+m}{2}-1}{\frac{n+m}{2}} r \\
\Leftrightarrow \quad & 1 - \frac{1}{n+m} \geq (n+m)\left(\frac{n+m}{2}(1-r)+r\right)\frac{2}{(n+m)^3} + 1 - \frac{2}{n+m} \\
\Leftrightarrow \quad & 1 \geq \left(\frac{n+m}{2}(1-r)+r\right)\frac{2}{(n+m)} \\
\Leftrightarrow \quad & 1 \geq (1-r) + \frac{2r}{(n+m)} \\
\Leftrightarrow \quad & 0 \geq \frac{2r}{(n+m)} - r.
\end{aligned}$$

The last inequality is always fulfilled since $n+m > 2$.

The left panel in Figure 4.3 shows the comparison of the rent dissipation in the framework of absolute payoff maximizers.

ad ii.) In the framework of relative payoff maximizers the rent dissipation of the one-stage contest is equal to the technology parameter r and equal to the rent-dissipation of the second stage in the two-stage contest. In the two-stage contest there is additionally the effort of the first stage. Therefore, the rent dissipation of the two-stage contest overtops the rent dissipation of the one-stage contest.

The right panel in Figure 4.3 shows the comparison of the rent dissipation in the framework of relative payoff maximizers. ■

Adding an additional stage before the one-stage contest results in a higher rent dissipation in the framework of players with negatively interdependent preferences. In this framework the second stage of the two-stage contest and the one-stage contest exhibit the same aggregated amount of effort. The aggregated rent dissipation is independent of the number of contestants which is the only difference between the second stage of the two-stage contest and the one-stage contest. Whereas, adding an additional stage in the

[10] See for this result the former section or Katz and Tokatlidu (1996).

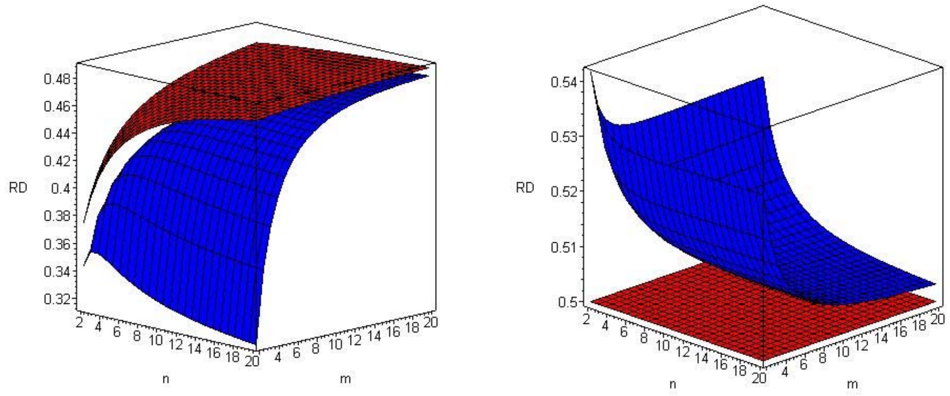

Figure 4.3: Left panel: Rent dissipation of absolute payoff maximizers in the two-stage contest (blue graph) compared to a one-stage contest (red graph). Right panel: Rent dissipation of relative payoff maximizers in the two-stage contest (blue graph) compared to a one-stage contest (red graph). Parametrization: $r = 0.5$.

framework of players with independent preferences influences the aggregate expenditures in the former existing stage. An additional stage reduces the number of contestants in the second stage since only the winning group of the first stage takes actively part in the second stage. The reduction of the expenditure in the second stage due to the lower number of contestants in this stage outweighs the increase due to the effort expanded in the additional stage.

Let's now endogenize the contest structure. One might imagine a contest organizer who is interested in maximizing the efforts expended by the players.[11] If the contest organizer believes that the players have absolute preferences, he should prefer a one-stage contest to elicit lower total aggregate effort. However, if he thinks they are relative payoff maximizer, he should favor the two-stage contest. The differences decrease with an increasing number of players and the dissipation rate of all contest structures converges to r.

[11] See Gradstein and Konrad (1999) for endogenizing contest structures.

4.3.3 Application: Federalism or Unitary State

An important responsibility of a government is to distribute parts of the national income through regulations. There are two different endogenous formations of jurisdiction for a country with two regions. On the one hand, there is the system of federalism. A federation is a union comprising a number of partially self-governing regions united by a central government. Hence, sovereignty is constitutionally divided between a central governing authority and constituent political units like states or regions. One example is the German political federal system. Seen from a more abstract perspective, Germany's political system resembles our theoretical two-stage Between-Group model. Political decisions are made on two hierarchical stages. First, the countrywide government decides about the magnitude of GNP to be distributed across regions. Once the level of GNP has been decided the local regions can influence the share they will receive (first stage). In a second step the pie is divided within the regions. This corresponds to our two-stage model discussed above. In the first stage, there are two (or more) groups, the different federal states, competing against each other in a contest organized by the government. In the second stage, the individuals of the winning province start an intra group contest to split up the rent. The individuals of the second stage could also be interest groups composed of many individual agents like cities, single political parties or lobbyists, but we abstract in this chapter from the problem of enforcing cooperation within such a group.[12]

In contrast to the model of a decentralized state, we could think of a unitary state. Both regions are unified into a single jurisdiction. The incentives of this jurisdiction are analogous to the one-stage contest we presented in this chapter. We assume that the unification does not have any (technological) efficiency gains or losses effecting GNP. Hence, the total rent and the appropriative activities are the same in both jurisdictions.

Wärneryd (1998) analyzes these two models of rent-seeking contests on the endogenous formation of jurisdictions. Under the assumption that the political process is a costly battle to acquire shares of GNP, he finds that less resources are spent in aggregate on appropriative activities under a hierar-

[12] See Olson (1965), Nitzan (1991) and Abbink, Brandts, Herrmann, and Orzen (forthcoming).

chical system of federalism than in a unified jurisdiction with a single central government. Hence, federalism weakens the competition in the distribution of scarce resources. The rate of rent dissipation is strictly lower than under unification.

Under the assumption of relative preferences of the players, our result contrasts Wärneryd (1998) who shows that it is more efficient for an aggregate effort maximizing principal respective the government to induce unification, because then the rent dissipation rate is higher. In our model with relative payoff maximizers and a decreasing returns to scale technology the federal system would induce a strictly higher rate of rent dissipation than the unified jurisdiction. For this reason, unification does not always increase competition among individuals in the sense that the global rate of dissipation increases such as Wärneryd (1998) stated. Therefore, the decision about the preferable jurisdiction has to go in line with the assumption on the preference structure of the contestants.

4.3.4 Conclusion

This section examines the rent dissipation in a group rent-seeking contest in which rent-seeking activities take place in two stages compared to the rent dissipation of a one-stage individual contest. Changing the common assumption of purely self-interested preferences in rent-seeking contests, it is shown that the choice of the contest structure is ambiguous and depends on the preference-type. We find that rent dissipation given absolute payoff maximizers in a two-stage contest is lower than in a one-stage contest and given relative seekers it is the reverse.

This result carries over to the case of endogenous formation of jurisdictions: It depends on the assumption of preferences whether resources are spent more efficiently in a decentralized or a centralized government structure. In contrast to the perceived literature we find that a system of federalism is not always significant in ameliorating distributional competition and conflict.

4.4 Comparison with Semi-Finals Contests

4.4.1 The Adapted Between-Group Model

In this section we present the main results, adopted to our particular setting, of the two-stage Between-Group model analyzed in the former section. We use the special case of two equal sized groups ($n = m$) and a constant returns to scale technology ($r = 1$). But we extend the interdependent preference function for an additional parameter, α, which displays the weight of how much players care for the averaged population payoff.

Between-Group Contest with Absolute Payoff Maximizers

Introducing a constant returns to scale technology and equal sized groups converts the probability that group X wins the rent in the first stage, Eq.(4.1), into

$$p_X = \frac{\sum_{i=1}^{n} x_{1i}}{\sum_{i=1}^{n} x_{1i} + \sum_{j=1}^{n} y_{1j}} \tag{4.26}$$

and the probability that its i-th member will win in the second round, Eq.(4.2), into

$$p_{xi}(x_{21}, \ldots, x_{2n}) = \frac{x_{2i}}{\sum_{l=1}^{n} x_{2l}}. \tag{4.27}$$

Given these new contest success functions we maximize the utility functions (4.3) of the second stage and (4.6) of the first stage which results in the optimal equilibrium effort

$$x_2^{abs} = \frac{n-1}{n^2} V \tag{4.28}$$

and

$$x_1^{abs} = y_1^{abs} = \frac{1}{4n^3} V. \tag{4.29}$$

This is the optimal effort of a player of group X as well as of group Y if all players have absolute preferences.

From a welfare perspective, we can now determine the degree of social waste. The degree of the social waste, the rent dissipation RD_{BG}^{abs}, is defined as the aggregate expenditures made in both stages of the contest divided by the amount of rent.

$$RD_{BG}^{abs} = (2nx_1^{abs} + nx_2^{abs})/V = \left(\frac{1}{2n^2} + \frac{n-1}{n}\right) \tag{4.30}$$

Between-Group Contest with Relative Payoff Maximizers

We now repeat the analysis of the Between-Group contest under the assumption of relative payoff maximizers. The material payoff function of the second stage is again given as in Eq.(4.3) and in Eq.(4.6). The players maximize their expected utility given relative preferences: The utility function is of the form

$$F_{2i}^{rel} = \Pi_i(\mathbf{x}) + \alpha \frac{1}{n} \sum_{j=1}^{n} \Pi_j(\mathbf{x})$$

where $\alpha \in [-1, 0]$. The individual i maximizes a sum of his own material payoff and a weighted average payoff of all active players. The restriction that the preference parameter α can only take values between -1 and 0 ensures that the utility function of each player depends more on his own payoff than on the averaged payoff. The value $\alpha = 0$ conforms to the above analyzed case of absolute payoff maximizers.

We assume again that group X with n players wins the contest in the first stage. Therefore, we concentrate the analysis of the second contest stage on the players of group X. In the second round the i-th member of group X maximizes

$$\begin{aligned} F_{2i}^{rel} &= \Pi_{2i}(\mathbf{x}) + \alpha \frac{1}{n} \sum_{l=1}^{n} \Pi_{2l}(\mathbf{x}) \\ &= \frac{x_{2i}}{\sum_{l=1}^{n} x_{2l}} V - x_{2i} + \alpha \frac{1}{n} \Big(V - \sum_{l=1}^{n} x_{2l} \Big). \end{aligned} \tag{4.31}$$

In this stage the individual i is concerned with his payoff relative to the payoff of his own group members. Members of the other group have lost the first stage competition and therefore dropped out of the contest. The individual i does not include the other group members in his utility function.

Assuming a symmetric Nash equilibrium within the group and a regular interior solution, the first-order condition for member i of group X is

$$\frac{\partial F_{2i}^{rel}}{\partial x_{2i}} = \frac{\sum x_{2l} - x_{2i}}{(\sum x_{2l})^2} V - 1 - \alpha \frac{1}{n} \stackrel{!}{=} 0. \tag{4.32}$$

Since we assume identical agents and search for symmetric equilibria, we set $x_{2i} = x_{2l} = x_2$. This gives the optimal effort of the second stage

$$x_2^{rel} = \frac{n-1}{n(n+\alpha)} V. \tag{4.33}$$

Eq.(4.33) is our interior equilibrium solution of the second stage. To find the equilibrium solution of the first round we calculate the benefit of entering the second stage and use this value as a reduced rent which the individuals can imaginary win in the first stage. Regarding all players and the optimal effort of the second stage as defined in Eq.(4.33), we calculate the utility of entering the second stage.

$$\tilde{V}^{rel}_{BG} = \frac{1+\alpha}{n(n+\alpha)}V + \alpha\frac{n\frac{1+\alpha}{n(n+\alpha)}}{2n}V = (1+\frac{\alpha}{2})\frac{1+\alpha}{n(n+\alpha)}V \tag{4.34}$$

Hence, in the first round each player solves the rent-seeking maximization for a reduced rent. For the parametrization $\alpha = -1$, extreme spitefulness, this reduced rent is zero. The players dissipate the complete rent in the competition on the second stage. They do not have an incentive to spent any effort in the first stage.

The utility of a relative payoff maximizing individual i of group X in the first stage is $F^{rel}_{1i} = \Pi_i + \alpha\bar{\Pi}$, where $\bar{\Pi}$ is the population mean payoff:

$$\begin{aligned}\bar{\Pi} &= \frac{\sum_{l=1}^{n}\Pi_l + \sum_{j=1}^{n}\Pi_j}{2n} \\ &= \frac{p_X n\tilde{V}^{rel}_{BG} - \sum_{l=1}^{n}x_{1l} + p_Y n\tilde{V}^{rel}_{BG} - \sum_{j=1}^{n}y_{1j}}{2n} \end{aligned}\tag{4.35}$$

and the success function of the groups p_X respective p_Y chosen as in (4.26).

With these we get

$$F^{rel}_{1i} = \frac{\sum x_{1l}}{\sum x_{1l} + \sum y_{1j}}\tilde{V}^{rel}_{BG} - x_{1i} + \alpha\left(\frac{n\tilde{V}^{rel}_{BG}}{2n} - \frac{\sum x_{1l} + \sum y_{1j}}{2n}\right). \tag{4.36}$$

The first-order condition is given by

$$\frac{\partial F^{rel}_{1i}}{\partial x_{1i}} = \frac{\sum y_{1j}}{(\sum x_{1l} + \sum y_{1j})^2}\tilde{V}^{rel}_{BG} - 1 - \frac{\alpha}{2n} \stackrel{!}{=} 0. \tag{4.37}$$

Following our assumption that players are identical, we have $\sum_{l=1}^{n} x_{1l} = nx_1 = \sum_{j=1}^{n} y_{1j}$. Substituting this in the first-order condition, we yield

$$x^{rel}_1 = \frac{1}{(1+\frac{\alpha}{2n})4n}\tilde{V}^{rel}_{BG} = \frac{(1+\frac{\alpha}{2})(1+\alpha)}{2n(2n+\alpha)(n+\alpha)}V. \tag{4.38}$$

If we take a look at the contest with status seekers from a welfare perspective, we focus on the rent dissipation RD^{rel}_{BG} in the whole contest to determine

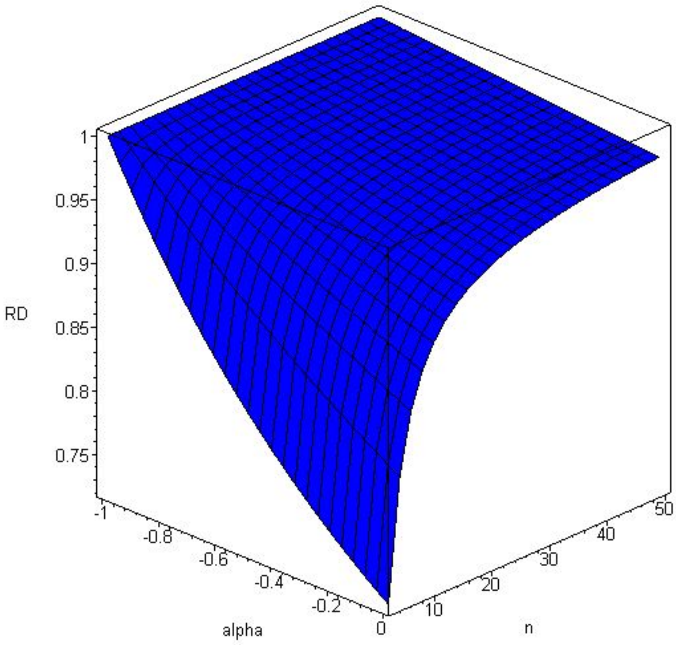

Figure 4.4: Rent dissipation of the Between-Group contest.

the social waste. Therefore, we aggregate the expenditures of the single stages multiplied by their absolute frequency. The aggregate effort of the contest with relative payoff maximizers in proportion to the rent (Figure 4.4) is given as

$$RD_{BG}^{rel} = (2nx_1^{rel} + nx_2^{rel})/V = \left(\frac{(1+\frac{\alpha}{2})(1+\alpha)}{(2n+\alpha)(n+\alpha)} + \frac{n-1}{n+\alpha}\right). \qquad (4.39)$$

Relationship between Preference Type and Rent-Seeking in the Between-Group Contest

We again want to analyze the relationship between the preference type and the rent dissipation in the two-stage Between Group contest. The arising question is if Result 4.3 still holds for all $\alpha \in [-1, 0]$.

We have seen, the second stage Nash equilibrium with relative payoff maximizers is given by $x_2^{rel} = \frac{n-1}{n(n+\alpha)}V$. In comparison, the unique pure strategy Nash equilibrium of this stage when players have absolute payoff maximizing preferences is given by $x_2^{abs} = \frac{n-1}{n^2}V$. When comparing the effort levels of the second stage in the Nash equilibria , we find that independent of the preference parameter α the players interested in relative payoffs exert more effort gaining lower payoffs than the players maximizing absolute payoffs. Focusing on the first stage of the contest, the aggregate expenditure in the equilibrium with relative payoff maximizers is always lower than those of the Nash equilibrium of players with absolute independent preferences. The effects of both stages counteract each other, but since the effect of the second stage is stronger, it dominates the first-stage effect. We can conclude

Result 4.5 *The total expenditure and along with it the rent dissipation in equilibrium of the overall Between-Group contest with negatively interdependent preferences is higher than the overall expenditure and rent dissipation in the contest with absolute payoff maximizers.*

Proof. The idea of this proof is to show that the minimum of the rent dissipation with respect to α under the assumption of relative preferences lies at $\alpha = 0$ and that this coincides with the rent dissipation under the assumption of absolute preferences. Partial derivative of the rent dissipation of the Between-Group contest with respect to the preference parameter is given as

$$\frac{\partial RD_{BG}^{rel}}{\partial\alpha} = \frac{(1+\alpha)+2(1+\frac{\alpha}{2})}{2(2n+\alpha)(n+\alpha)} - \frac{(1+\frac{\alpha}{2})(1+\alpha)}{(2n+\alpha)^2(n+\alpha)} - \frac{(1+\frac{\alpha}{2})(1+\alpha)}{(2n+\alpha)(n+\alpha)^2} - \frac{n-1}{(n+\alpha)^2}.$$

For $n \geq 2$, we show

$$\frac{\partial RD_{BG}^{rel}}{\partial\alpha} < 0.$$

For $\alpha \in (-1, 0]$ the second and third term are negative and for $\alpha = -1$ they are zero. Analyzing all four terms of the derivative for $\alpha = -1$ gives:

$$\frac{\partial RD_{BG}^{rel}}{\partial\alpha} = \frac{3-4n}{2(2n-1)(n-1)} < 0, \quad \forall n \geq 2.$$

Given the negative second and third term for $\alpha \in (-1, 0]$, it is sufficient to show that the negative fourth term overtops or equalizes the positive first term:

$$\begin{aligned} & \frac{(1+\alpha)+2(1+\frac{\alpha}{2})}{2(2n+\alpha)(n+\alpha)} \leq \frac{n-1}{(n+\alpha)^2} \\ \Leftrightarrow \quad & (3+2\alpha)(n+\alpha) \leq 2(n-1)(2n+\alpha). \end{aligned}$$

Now, using the allowed range $[-1, 0]$ for α we can displace $(3+2\alpha)(n+\alpha) \leq 3n$ and $2(n-1)(2n+\alpha) \geq (n-1)(4n-2)$. Therewith, it is sufficient to show that

$$\begin{aligned} & 3n \leq (n-1)(4n-2) \\ \Leftrightarrow \quad & 4n^2 - 9n + 2 \leq 0. \end{aligned}$$

Given the condition $n \geq 2$ this is obviously fulfilled.

We have proved that the partial derivative of RD_{BG}^{rel} with respect to α is always smaller than zero ($\frac{\partial RD_{BG}^{rel}}{\partial \alpha} < 0$) and approximate zero for $n \to \infty$. The rent dissipation function is strictly monotonic decreasing in α. A decrease of the preference parameter α strictly increases the rent dissipation in the Between-Group contest. The minimum of the rent-dissipation function is situated at $\alpha = 0$, because α is restricted to the interval $[-1, 0]$. If $\alpha = 0$ the rent-dissipations are equal $RD_{BG}^{rel} = RD_{BG}^{abs}$. Knowing that a decrease in α results in an increase of the rent-dissipation we can state that for $\alpha \in [-1, 0]$ the inequality $RD_{BG}^{rel} > RD_{BG}^{abs}$ holds. ■

4.4.2 Analysis Semi-Finals Model

In contrast to the Between-Group model, in the Semi-Finals model the idea is that group members of the same group have to compete for a mandate in the first stage. Once, one individual wins the mandate, he must compete in a second stage against the winner of the first stage of the rival group for the rent V. This is appropriate when the competition consists of a semi-finals round and final round with the semi-final winners selected from a partition of the players. A classic example are sport events in which first stage events are used for qualifying for the final round. Another example is an election process consisting of primaries and final elections.

The Semi-Finals model assumes that $2n$ risk neutral and homogeneous players compete in a two-stage contest for a single indivisible rent V. Each player has the same valuation for the rent. The $2n$ players are arranged exogenously in two groups of same size, group X and group Y. Each player competes within his group. Member i of group X invests effort x_{1i} in the first stage. One winner is selected from each group using Tullock's contest success function $\frac{x_{1i}}{\sum_{l=1}^{n} x_{1l}}$ as the probability that player i of this group advances to the second stage. On this stage the two winners of the semi-finals stage compete against each other with the winner again chosen according to the Tullock contest rule $\frac{x_2}{x_2+y_2}$. We assume again constant returns to scale technology. The technology will not be changed during the whole contest and is the same for all players.

The first round of the contest does not offer individuals an ultimate payoff; it only determines the probability that a contestant will participate in the second round. The marginal benefit of an individual's first round investment is the increased probability to win the ultimate rent. Hence, the rent-seeking activities of individuals in the first round are determined by optimizing their expected payoff, given that all individuals act rationally and optimally in the second round.

First we analyze the model under the assumption that players are absolute payoff maximizers following Stein and Rapoport (2004). In a second step, we assume rent-seekers to be relative payoff maximizers. This extends the analysis of a Semi-Finals two-stage rent-seeking contest to an additional strategic component. Like in the last section the focus lies on intra-group symmetric equilibria, where all players choose the same effort level. We solve for a subgame perfect equilibrium outcome of this game via backward induction. Therefore, we start the analysis with the second stage. All previous rent-seeking expenditures are sunk at this stage.

Semi-Finals Contest with Absolute Payoff Maximizers

In this subsection we summarize the main results presented in Stein and Rapoport (2004) for the case of absolute payoff maximizing preferences, two groups and identical rent valuations. In the second round the winning member i of group X maximizes his utility, which is equal to the payoff.

$$F_{2i}^{abs} = \Pi_{2i} = \frac{x_2}{x_2 + y_2} V - x_2 \tag{4.40}$$

Assuming a symmetric Nash equilibrium $x_2 = y_2$ and a regular interior solution we can solve the first-order condition. This gives the equilibrium effort

$$x_2^{abs} = \frac{1}{4} V. \tag{4.41}$$

Substituting the optimal effort defined in Eq.(4.41) in the utility function Eq.(4.40), we find that the individual's valuation of entering round two

$\tilde{V}_{SF}^{abs}$ is $\frac{1}{4}V$. If a player enters the second round of the contests he earns this expected amount of the rent. Every player will bear this optimal payoff in mind when finding an optimal effort level for the first stage. Hence, in the first round each player solves the rent-seeking maximization for a reduced rent: $\tilde{V}_{SF}^{abs} = \frac{1}{4}V$.

In the first stage of the contest, an absolute payoff maximizing individual i of group X tries to maximize his expected payoff Π_{1i}, which is at the same time the utility F_{1i}^{abs} and is given as

$$F_{1i}^{abs} = \Pi_{1i} = \frac{x_{1i}}{\sum x_{1l}} \tilde{V}_{SF}^{abs} - x_{1i}. \tag{4.42}$$

Solving the first-order condition under the assumption that all players have identical payoff maximizing preferences, we get the solution

$$x_1^{abs} = \frac{1}{4}\frac{n-1}{n^2}V. \tag{4.43}$$

This is the optimal effort of a player of group X as well as of group Y in the first stage if all players have absolute preferences.

From a welfare perspective, we can now determine the degree of social waste. Social waste or rent dissipation RD_{SF}^{abs} is defined as the aggregated expenditure by all players in proportion to the rent V.

$$RD_{SF}^{abs} = \frac{(2nx_1^{abs} + 2x_2^{abs})}{V} = \frac{2n(n-1)}{4n^2} + \frac{2}{4} = 1 - \frac{1}{2n} \tag{4.44}$$

Semi-Finals Contest with Relative Payoff Maximizers

We now repeat the analysis of the Semi-Finals contest under the assumption of relative payoff maximizers. The material payoff function is again given as

$$\Pi_i = \frac{x_i}{\sum_{l=1}^{n} x_l} V - x_i. \tag{4.45}$$

The players maximize their expected utility. The utility function is of the form $F_i^{rel} = \Pi_i(x) + \alpha\frac{1}{n}\sum_{l=1}^{n}\Pi_l(x)$ where $\alpha \in [-1, 0]$. The individual i maximizes a sum of his own material payoff and a weighted average of payoffs of all active players. The restriction that the preference parameter α can only take values between -1 and 0 ensures again that the utility function of each player depends more on his own payoff than on the averaged payoff.

The value $\alpha = 0$ conforms to the above analyzed case of absolute payoff maximizers.

We assume that member i of group X and member j of group Y win the contest in the first stage. In the second round the i-th member maximizes

$$\begin{aligned} F_{2i}^{rel} &= \Pi_{2i} + \alpha \frac{\Pi_{2i} + \Pi_{2j}}{2} \\ &= \frac{x_{2i}}{x_{2i} + y_{2j}} V - x_{2i} + \alpha \frac{V - x_{2i} - y_{2j}}{2}. \end{aligned} \tag{4.46}$$

In this stage the individual i is concerned with his payoff relative to the payoff of his active rival. Other members of the groups have lost the first stage competition and therefore dropped out of the contest. Individual i does not include dropped out players in his utility function. Therefore, the utility function just includes two individuals.

Assuming a symmetric Nash equilibrium and a regular interior solution, the first-order condition for player i is

$$\frac{\partial F_{2i}^{rel}}{\partial x_{2i}} = \frac{y_{2j}}{(x_{2i} + y_{2j})^2} V - 1 - \alpha \frac{1}{2} \stackrel{!}{=} 0. \tag{4.47}$$

Since we assume identical agents and search for symmetric equilibria, we set $x_{2i} = y_{2j} = x_2$. This gives

$$x_2^{rel} = \frac{1}{2(2+\alpha)} V. \tag{4.48}$$

To calculate the benefit of entering the second round we have to take a broader view on the whole game. In the second stage the individuals with relative preferences want to maximize their payoff in comparison to those of the players, who are still active in the contest. Under this assumption the optimal effort is given as $\frac{1}{2(2+\alpha)} V$ in the second stage. Now, analyzing the first stage, individuals additionally compete with all $2n$ active players. Therefore, the players want to maximize their material payoff compared to those of all other players. The benefit of entering the second round is now the difference in the material payoff between individual i and the average over all other players. Regarding all players we calculate the utility of entering the second stage.

$$\begin{aligned} \tilde{V}_{SF}^{rel} &= \frac{1}{2} V - \frac{1}{2(2+\alpha)} V + \alpha \left(\frac{V - 2\frac{1}{2(2+\alpha)} V}{2n} \right) \\ &= \left(\frac{(n+\alpha)(1+\alpha)}{2n(2+\alpha)} \right) V \end{aligned} \tag{4.49}$$

Hence, in the first round each player solves the rent-seeking maximization for a reduced rent. For the parametrization $\alpha = -1$, extreme spitefulness, this reduced rent is zero. The players dissipate the complete rent in the competition on the second stage. They do not have an incentive to spent any effort in the first stage.

Given the reduced rent the utility of a relative payoff maximizing individual i in group X in the first stage is

$$F_{1i}^{rel} = \frac{x_{1i}}{\sum x_{1l}} \tilde{V}_{SF}^{rel} - x_{1i} + \alpha \left(\frac{2\tilde{V}_{SF}^{rel} - \sum x_{1l} - \sum y_{1l}}{2n} \right). \tag{4.50}$$

The first-order condition is given by

$$\frac{\partial F_{1i}^{rel}}{\partial x_{1i}} = \frac{\sum x_{1l} - x_{1i}}{(\sum x_{1l})^2} \tilde{V}_{SF}^{rel} - 1 - \frac{\alpha}{2n}. \tag{4.51}$$

Following our assumption that group members are identical within each group, we have $\sum_{l=1}^{n} x_{1l} = nx_1$. Substituting this in the first-order condition, we yield the equilibrium effort level

$$\begin{aligned} x_1^{rel} &= \frac{1}{1+\frac{\alpha}{2n}} \frac{n-1}{n^2} \tilde{V}_{SF}^{rel} \\ &= \frac{1}{1+\frac{\alpha}{2n}} \frac{n-1}{n^2} \left(\frac{(n+\alpha)(1+\alpha)}{2n(2+\alpha)} \right) V. \end{aligned} \tag{4.52}$$

If we take a look at the contest from a welfare perspective, we focus on the rent dissipation RD_{SF}^{rel} in the whole contest to determine the social waste. The aggregate effort of the contest with relative payoff maximizers in proportion to the rent V can be seen in Figure 4.5 and is given as

$$\begin{aligned} RD_{SF}^{rel} &= \frac{2nx_1^{rel} + 2x_2^{rel}}{V} \\ &= \frac{1}{(1+\frac{\alpha}{2n})} \frac{(n-1)}{n^2} \frac{(n+\alpha)(1+\alpha)}{(2+\alpha)} + \frac{1}{2+\alpha}. \end{aligned} \tag{4.53}$$

Relationship between Preference Type and Rent-Seeking in the Semi-Finals Contest

We have seen that the second stage effort in the equilibrium with relative payoff maximizers is given by $x_2^{rel} = \frac{1}{2(2+\alpha)} V$. In comparison, the unique

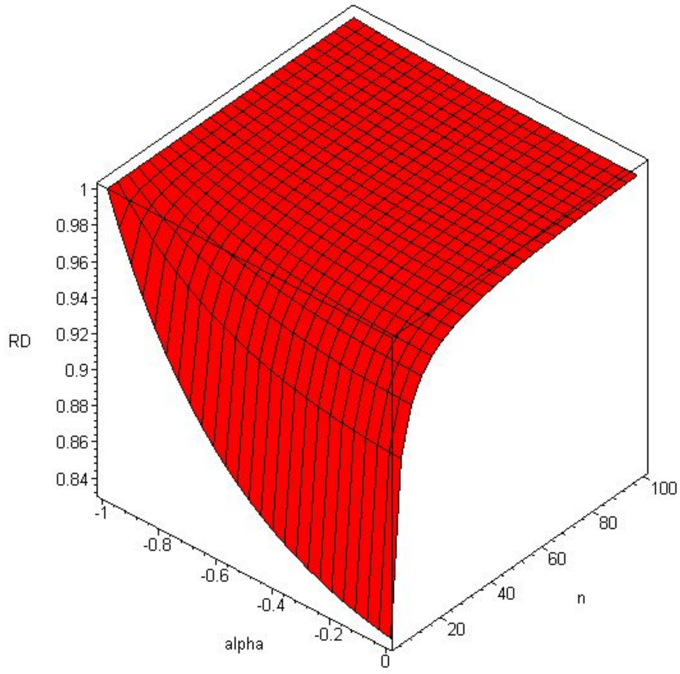

Figure 4.5: Rent dissipation of the Semi-Finals contest.

pure strategy Nash equilibrium with absolute payoff maximizers is given by $x_2^{abs} = \frac{1}{4}V$. When comparing the effort levels of the second stage of the two Nash equilibria, we find that the player interested in relative payoffs invests more effort but gaining lower payoffs than the agent maximizing absolute payoffs. The competition on the second stage between the relative payoff maximizers is stronger. Therefore, they waste more effort. The higher effort reduces the expected payoff. Hence, the incentive of the relative maximizers to reach the second round and therewith the valuation of the rent in the first stage is smaller. These different valuations of the rent in the first stage creates a asymmetry in the player's behavior. In the equilibrium of the first stage with relative preferences the aggregate expenditure is always lower than those of the Nash equilibrium of players with absolute independent preferences. The effects of both stages counteract each other, but since the effect of the second stage is stronger, it dominates the first-stage effect.

Result 4.6 *The total expenditure and therewith the rent dissipation in equilibrium of the overall Semi-Finals contest with negatively interdependent preferences is higher than the overall expenditure in the Semi-Finals con-*

test with absolute payoff maximizers.

Proof. The idea of this proof is to show that the minimum of the rent dissipation with respect to α under the assumption of relative preferences lies at $\alpha = 0$ and this coincides with the rent dissipation under the assumption of absolute preferences. The rent dissipation function is differentiable. The partial derivative of the rent dissipation of the Semi Finals contest with respect to the preference parameter is given as

$$\begin{aligned}\frac{\partial RD_{SF}^{rel}}{\partial \alpha} &= \frac{2(n-1)(1+\alpha)}{(1+\frac{\alpha}{2n})n^2(2+\alpha)} - \frac{(n-1)(n+\alpha)(1+\alpha)}{2n^3(1+\frac{\alpha}{2n})^2(2+\alpha)} \\ &\quad - \frac{(n-1)(n+\alpha)(1+\alpha)}{(1+\frac{\alpha}{2n})n^2(2+\alpha)^2} - \frac{1}{(2+\alpha)^2}.\end{aligned}$$

First, we show $\frac{\partial RD_{SF}^{rel}}{\partial \alpha} < 0$. Leaving the negative fourth term out makes the derivative even bigger. Therefore, we can analyze the first three terms:

$$\frac{2(n-1)(1+\alpha)}{(1+\frac{\alpha}{2n})n^2(2+\alpha)} - \frac{(n-1)(n+\alpha)(1+\alpha)}{2n^3(1+\frac{\alpha}{2n})^2(2+\alpha)} - \frac{(n-1)(n+\alpha)(1+\alpha)}{(1+\frac{\alpha}{2n})n^2(2+\alpha)^2} < 0$$

$$\begin{aligned}&\Leftrightarrow \quad 2 - \frac{n+\alpha}{2n+\alpha} - \frac{n+\alpha}{2+\alpha} < 0 \\ &\Leftrightarrow \quad -n^2 + 3n + \alpha < 0.\end{aligned}$$

This is obviously fulfilled for $n \geq 4$ for all values of α and for $n = 3$ with $\alpha \in [-1, 0)$. The case $n = 3$ with $\alpha = 0$ can be shown by simply plugging in the values into $\frac{\partial RD_{SF}^{rel}}{\partial \alpha}$. The derivative is still negative. Now, it is left to prove that $\frac{\partial RD_{SF}^{rel}}{\partial \alpha} < 0$ for $n = 2$:

$$\frac{2(1+\alpha)}{(1+\frac{\alpha}{4})4(2+\alpha)} - \frac{(2+\alpha)(1+\alpha)}{16(1+\frac{\alpha}{4})^2(2+\alpha)} - \frac{((2+\alpha)(1+\alpha)}{(1+\frac{\alpha}{4})4(2+\alpha)^2} - \frac{1}{(2+\alpha)^2} < 0$$

$$\begin{aligned}&\Leftrightarrow \quad \frac{2(1+\alpha)}{(4+\alpha)(2+\alpha)} - \frac{(1+\alpha)}{(4+\alpha)^2} - \frac{(1+\alpha)}{(4+\alpha)(2+\alpha)} - \frac{1}{(2+\alpha)^2} < 0 \\ &\Leftrightarrow \quad \alpha^2 - 2\alpha - 12 < 0.\end{aligned}$$

The last inequality is always fulfilled since $\alpha \in [-1, 0]$.

We have proved that the partial derivative of RD_{SF}^{rel} with respect to α is always smaller than zero ($\frac{\partial RD_{SF}^{rel}}{\partial \alpha} < 0$). The rent dissipation function is strictly monotonic decreasing in α. A decrease of the preference parameter α increases the rent dissipation in the Semi-Finals contest. The minimum of the rent-dissipation function is situated at $\alpha = 0$, because α is restricted to the interval $[-1, 0]$. If and only if $\alpha = 0$ the rent-dissipations are equal $RD_{SF}^{rel} = RD_{SF}^{abs}$. ■

4.4.3 Comparison of Between-Group Contest and Semi-Finals Contest

An equilibrium x_i displays overdissipation (full dissipation, underdissipation) if and only if $\sum_{i=1}^{n} x_i > (=,<)V$, respectively. In equilibrium of the Between-Group contest as well as of the Semi-Finals contest we find for absolute payoff maximizer always underdissipation and for relative payoff maximizers underdissipation as long as $\alpha > -1$. Comparing the rent dissipations of the Between-Group contest with those of the Semi-Finals contest, we find the following:

Result 4.7

i.) *Total expenditure and rent dissipation under the assumption of absolute payoff maximizers in a Semi-Finals contest is always higher or equal than in a Between-Group contest.*

ii.) *Total expenditure and rent dissipation of relative payoff maximizers in a Semi-Finals contest is for a sufficiently small α and $n \geq 3$ lower or equal than in a Between-Group contest.*

Proof. ad i.) To show: $RD_{BG}^{abs} \leq RD_{SF}^{abs}$

$$\frac{1}{2n^2} + \frac{n-1}{n} \leq \frac{2n-1}{2n} \Leftrightarrow \frac{1}{n} \leq 1$$

The last inequality is always fulfilled because there are always at least two players in the contest, $n \geq 2$.

ad ii.) To show: $RD_{BG}^{rel} \geq RD_{SF}^{rel}$

$$\left(\frac{(1+\frac{\alpha}{2})(1+\alpha)}{(2n+\alpha)(n+\alpha)} + \frac{n-1}{n+\alpha}\right) \geq \frac{2(n-1)(n+\alpha)(1+\alpha)}{n(2n+\alpha)(2+\alpha)} + \frac{1}{2+\alpha}$$

The preference parameter $\alpha \in (-1,0)$ has to fulfill the following inequalities to satisfy the above condition:

$$-1 \leq \alpha \leq -\frac{3n^2 - 4n - \sqrt{n(9n^3 - 36n^2 + 44n - 16)}}{3n-4}.$$

This dependency is shown in the right panel of Figure 4.6. For a given number of players n the sufficient preference parameter α for the relation $RD_{BG}^{rel} \geq RD_{SF}^{rel}$ is calculated. ■

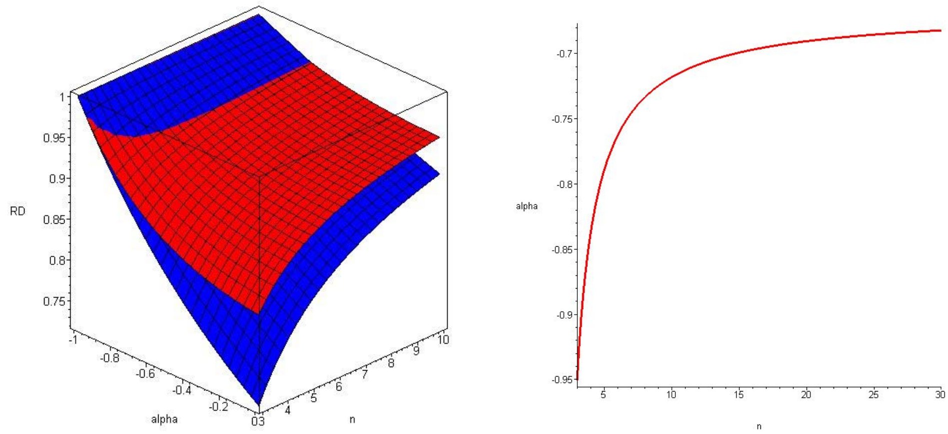

Figure 4.6: Left panel: Rent dissipation for the BG contest (blue) and SF contest (red). Right panel: Dependency of α to n. For preference parameter smaller than this α the rent dissipation of the Semi-Finals contest under the assumption of relative payoff maximizers is lower than the rent dissipation in the Between-Group contest.

Let's now endogenize the contest structure. One might imagine a contest organizer who is interested in maximizing the efforts expended by the players.[13] The designer can choose between the Between-Group contest and the Semi-Finals contest. He picks the contest structure with the highest rent-dissipation. The preferred contest structure depends on the assumed type of players preferences. If the contest organizer believes that the players have absolute preferences, he should prefer a Semi-Finals model to elicit higher total aggregate effort. However, if he thinks they have strong negatively interdependent preferences, he should favor in the most cases the Between-Group model.

4.4.4 Application: Electoral Rules

An election is a decision making process where a population chooses an individual to hold official offices. This is the usual mechanism by which modern democracy fills offices in the legislature, sometimes for regional and local government. There is a high universal acceptance of elections as a tool for selecting representatives in modern democracies. In most democratic

[13] See Gradstein and Konrad (1999) for endogenizing contest structures.

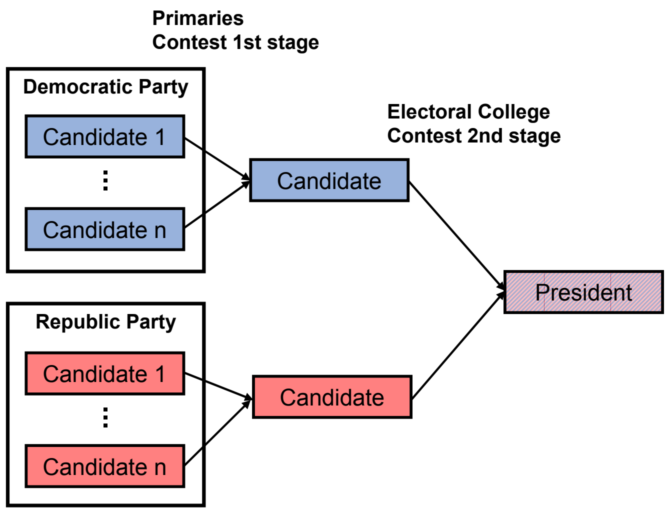

Figure 4.7: United States presidential election.

political systems, there is a range of different types of election. Here, we compare the United States presidential election with the election of a federal minister of the German cabinet concerning the arising rent dissipation. To give a benchmark for the aggregated spendings during a presidential election in the USA we refer to a table provided by Center for Responsive Politics on the CNN web page[14]: total spending by presidential candidates 240 million US-$ in 1996, 343 million US-$ in 2000, 718 million US-$ in 2004 and 586 million US-$ in 2008.

The election process of the President of the United States is split into two stages as shown in Figure 4.7. Primary elections, the first stage of the contest, serve to narrow down a field of candidates. Once a candidate has been elected for each party, the two candidates need to compete for office in the general elections. The presidential primary election is the first stage in the process of electing the President of the United States of America. A primary election (nominating primary) is an election in which voters in a jurisdiction select candidates for a subsequent election. In other words, primary elections are one means by which a political party nominates candidates for the following general election. A political party is a political organization that seeks to attain political power within a government. There are two major political parties in the United States, the Democratic Party and the Repub-

[14]See CNN Politics web page (17/03/09):
http://edition.cnn.com/ELECTION/2008/money/index.html.

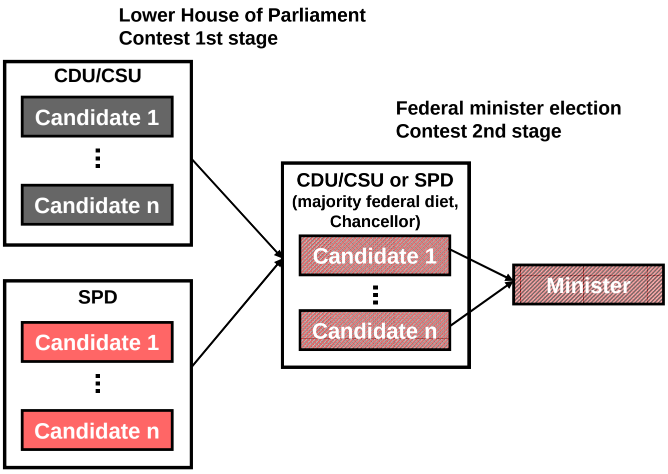

Figure 4.8: Elections of a federal minister of the German cabinet.

lican Party. The general election following the primaries is the second and final step. This election system coincides with the above explained Semi-Finals model. There are two groups, the two political parties, competing on the first stage in primaries trough expanding effort in election campaigns. The winning member is nominated for the second stage, the general election, where he again spends effort in campaigning against the winner of the other party.

By contrast the election of federal minister of the German cabinet coincides with the Between-Group model. The election occurs in two stages as shown in Figure 4.8. The first stage is the election of the Federal Diet. The Federal Diet is the lower house of the German Parliament. Germany has a multi-party system with two strong parties, the Christian Democratic Union (CDU) and the Social Democratic Party (SPD). According to experience, one of these major parties supplies the Chancellor of Germany and therewith nominates the federal ministers. Therefore, we model two groups, the parties, which compete through expanding effort in a election campaign for the majority in the Federal Diet. Simplified seen, on a second stage the winning group, the party which holds the majority, has the right to elect the Chancellor and the federal minister out of their party. For it they expand again effort in the election process.

In view of the above analysis of the Between-Group model and the Semi-Finals model we can say that under the assumption of absolute players-preferences the rent dissipation of the Between-Group model is lower. This means that under this assumption the resources in the German election system are spent more efficiently than in the USA system. If we act on the assumption that politicians are status seeker, which means they have negatively interdependent preferences, the resources are spend more efficiently in the United States presidential election than in the German election. Summing up, it depends on the assumption of preferences whether resources are spent more efficiently in a USA election system with primaries or a German election structure.

4.4.5 Application: Awarding Prestige Sport Events

This section compares the awarding systems of the Olympic Games and the FIFA World Cup. To host a match of the FIFA World Cup cities have to go through a two-stage contest. On a first stage the country where the city belongs to has to apply for the FIFA World Cup. Once the country is assigned to the World Cup the individual cities in the country can bid for being home for a match. This awarding system of the World Cup coincides with the above explained Between-Group model.

In a stylized way the awarding system of the Olympic games coincides with the Semi-Finals model. Countries around the world select cities within their national territory to put forward bids for hosting the Olympic Games. On the first stage cities in one country fight for the nomination as a candidate. On the second stage the elected cities of each country bidding to host the Summer Olympic Games or the Winter Olympic Games compete aggressively to have their bid accepted by the International Olympic Committee (IOC). The IOC members, representing most of the member countries, vote to decide where the Games will take place. Typically, the decision is made approximately seven years prior to the games. The bidding process for the 2012 Summer Olympics was considered one of the most hotly contested in the history of the IOC. London and Paris made it to the final round of voting. Paris was seen as the front-runner for most of the campaign, but last-minute lobbying by London's supporters was one factor that led to the success of its bid. Partly this lobbying can be characterized as spiteful. For example, the London bid consultants stated that the Stade de France, the national stadium of France, was not adequate for athletics, an action that goes against the IOC rules which forbid to make statements about a rival bid. In return the French President Jacques Chirac made comments stating that the only food worse than British food is Finnish and that the only thing the British have done for Europe's agriculture is mad cow disease. In recent years, the contest for the right to host the games has grown increasingly fierce and there is a serious rivalry between the candidates.

Comparing the two-stage Between-Group model with the two-stage Semi-Finals model is a strongly simplified comparison of the awarding systems of the World Cup and the Olympic Games. Given the above results we can state that under the assumption of absolute payoff maximizers the rent-dissipation of the Between-Group model is lower and therewith the World Cup awarding system is more efficient. If participating cities are spiteful and have negatively interdependent preferences, the resources are spent more efficiently in the awarding system of the Olympic Games.

4.4.6 Conclusion

This section examines the rent dissipation in two-stage rent-seeking contests. Two different types of contests in which rent-seeking activities take place in two stages are compared. Changing the common assumption of purely self-interested preferences in rent-seeking contests, it is shown that the choice of the contest structure is ambiguous. It depends on the type of preference which the players exhibit. We find that rent dissipation given absolute payoff maximizers in a Between-Group contest is lower than in a Semi-Finals contest and given strongly relative seekers it is the reverse.

On the one hand this result carries over to the case of elections: It depends on the assumption of preferences whether resources are spent more efficiently in a USA election system with primaries or a German election structure. On the other hand it carries over to the case of awarding prestige sport events: There the efficiency of the awarding system of the World Cup or the Olympic Games depends on the assumed preferences.

Chapter 5

Conclusion and Further Research

5.1 Summary

Contexts as seemingly diverse as economic organizations, sport competitions, wars, competition for natural monopoly and patent race can be described as contests whereby players expend resources to win a prize. One special field of the contest literature is political rent-seeking. Rent-seeking generally implies the extraction of uncompensated value from others without making any contribution to productivity. A classical example is the natural monopoly setting. The opportunity of making monopolistic profits invites competitive rent-seeking expenditures on the part of the potential monopolists to ensure the privileged position in the industry for itself by influencing the politicians.

This work contributes to the branch of rent-seeking literature in contest theory. It focus in all chapters on rent-seeking contests with a Tullock contest success function. We extend the branch of literature by altering the assumption on purely self-interested preferences. Precisely, we explore rent-seeking contests, where players have negatively interdependent preferences. The set of players is either homogeneous envious (Chapter 2 and 4) or heterogeneous envious (Chapter 2 and 3).

For a long time the assumption of preferences being independent from other players' payoff has been unquestioned in economics. In the more recent literature, especially the literature about evolutionary game theory, it has become more common to relax this assumption of purely self-interested preferences.

The aim of the Chapter 2 is to show how negatively interdependent preferences can arise beside the evolutionary approach. One source is delegation. Therefore, we analyze strategic delegation by firms' owners to a manager, first in the framework of Cournot competition and afterwards in the case of Tullock contests. Previous literature about delegation in Cournot competition, like for instance,Vickers (1985), Fershtman and Judd (1987), Sklivas (1987) focus exclusively on simultaneous contracting. The first part of Chapter 2 complements their studies by considering a game where firm owners choose incentive contracts sequentially. We show that the leader chooses a contract that leads to more aggressive manager behavior than the follower. More aggressive behavior means that the agents behave as if they have more spiteful preferences. This behavior creates a first-mover advantage for them. The second part of Chapter 2 considers strategic delegation in contests. We extend the standard Tullock contest by introducing in advance to the contest stage an additional delegation stage. Unlike existing models we analyze the impact of relative performance contracts on the delegation decision. First we discuss simultaneous contracting. If there are more than two firms in the market, there exists a unique, subgame perfect and symmetric equilibrium in the simultaneous contracting case. All players react as if they have the same spiteful preferences. Afterwards, we give an outlook on the changes arising through sequential contracting before the contest. There again it ends in heterogeneous players concerning their seemingly negatively interdependent preferences.

The next chapter shows the different consequences these negatively interdependent preferences can have for the outcome of different styles of contests. In Chapter 3 we analyze standard one-stage Tullock contests, in which players have negatively interdependent preferences. Those prefernce functions are of different interdependence. We study asymmetric equilibria in the case of constant marginal efficiency. In the case of increasing marginal efficiency we assume that the interdependent preferences of players are close but still distinguishable. Then we can use a simple but powerful technique of linear approximation. Its basic idea is that efforts of weakly heterogeneous players in equilibrium are close to those of homogeneous players. The equilibrium of identical players with negatively interdependent preferences is well characterized in the part before, and the behavior of weakly heterogeneous players can then be analyzed by Taylor-expanding the corresponding function around the homogeneous equilibrium. In general it is interesting to know how robust the symmetric equilibrium is with respect to weak heterogeneity. In contests the elasticity of effort with respect to a player's own preference is large, which implies that weakly heterogeneous players respond strongly to changes in their relative advantage or disadvantage. In this chapter we stress the importance of the exact preferences in order to understand the impact of the technology of rent-seeking on the structure of the outcome of the game. The arising question is whether a small amount of heterogeneity of players due to their different preferences is capable of causing large changes in the equilibrium structure. Indeed, heterogeneity due to different preferences alters the structure of the equilibrium with respect to the question who drops out first. A second question is whether the chosen technology has an impact on the structure of the outcome. We calculate the threshold values of the discriminatory power for which players sequentially drop out. These threshold values are smaller compared to those identified for players with independent preferences. Therefore, the technology parameter of rent-seeking contests has an impact on the structure of the outcome.

One purpose of Chapter 4 is to compare the rate of rent dissipation in a Between-Group rent-seeking contest in which rent seeking activities take place in two stages with those of a one-stage one-shot individual contest. Especially, the effect of the extension by introducing rent-seekers with negatively interdependent preferences is analyzed. Focusing on the relationship between contest-structure and preference-type we find that rent dissipation given absolute payoff maximizer is lower in a two-stage contest than in a one-stage contest. Given players with relative preferences we find the reverse. Comparing aggregate behavior in hierarchical systems of federalism (two-stage contests) with those in unified systems of a single central government (one-stage contests), these results show that the preferableness depends on the assumption of preferences. This shows that it depends on the assumption of preferences if more or less resources are spent in aggregate on appropriative activities under a hierarchical system of federalism than in a unified jurisdiction with a single central government.

Another purpose of Chapter 4 is to compare the Between-Group contest and the Semi-Finals contest, two variations of a two-stage rent-seeking contest. In the first stage of the Between-Group model two groups compete. Based on group expenditures, one winning group is probabilistically determined. On the second stage, members of the winning group compete with each other for the rent. Whereas, in the Semi-Finals model one player is chosen from each of the two groups and then these two players compete for the rent. We compare the degree of rent dissipation in the Between-Group and the Semi-Final model. The contribution of this chapter is to extend these two models by introducing rent-seekers with negatively interdependent preferences. Focusing on the relationship between contest-structure and preference-type we find that rent dissipation is generally lower in the Between-Group model than in the Semi-Finals model given that the players maximize absolute payoffs. When assuming negatively interdependent preferences, we can find the reverse. To carry this findings over to the framework of elections, we can compare the German election of a Federal minister (Between-Group model) and the United States presidential election (Semi-Finals model). Under the assumption of independent preferences the resources in the German election system are spent more efficiently than in the USA system. If we act on the assumption that politicians are status seeker, which means that they have negatively interdependent preferences, the resources are spend more efficiently in the United States presidential election than in the German election of the Chancellor. In summary, it depends on the assumption of preferences whether resources are spent more efficiently in a USA election system with primaries or a German election structure.

All in all we have seen that the assumption of (negatively) interdependent preferences results in totally different equilibria in contests and therewith in other preferable contest structures from a social welfare point of view. For this reason interdependent preferences are not negligible in analytical considerations.

5.2 Further Research

For a long time the assumption of preferences being independent from other players' payoff has been unquestioned in economics. In the more recent literature especially the literature about evolutionary game theory it became more common to relax this assumption of purely self-interested pref-

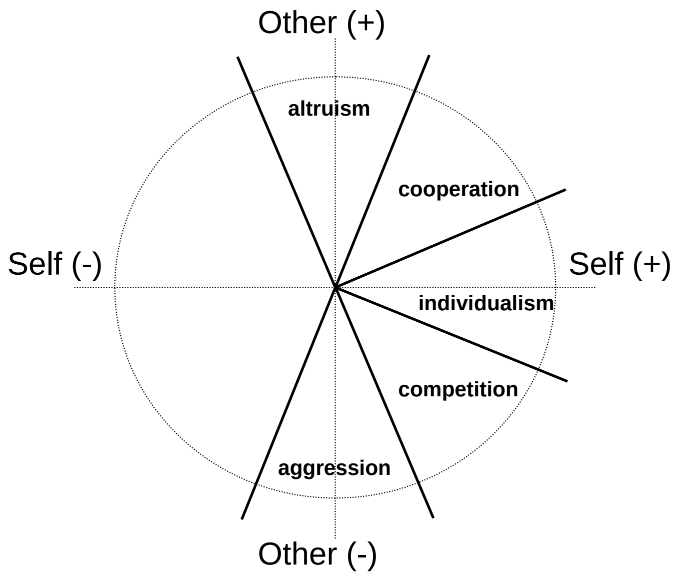

Figure 5.1: Decomposed Game: the value orientation circle.

erences. There have been very few attempts to explain the influence of other-regarding preferences in contest theory. This is particularly true for the study of negatively interdependent preferences. This work is only another step towards a proper understanding of the role of not purely payoff maximizing utility functions and their potential within contests.

Furthermore, an empirical verification, particularly of negatively interdependent preferences would be of great interest. This would help to indicate how envious people are and how huge the spread of this spiteful preferences in the population is. It has been long recognized that there is considerable heterogeneity in individuals preferences but little is known about the distribution of preference types.

Another step would be to verify by a proper experiment the implications of negatively interdependent preference in contests. A first attempt is done by Herrmann and Orzen (2008), who investigate the importance of spite in experimental rent-seeking contests. The patterns they observe are not consistent with inequality aversion but with spite and excessive rivalry between players. The problem in their work is that the pretest, a prisoner's dilemma game, cannot be used to distinguish between selfish types and spiteful types. Therefore, we would suggest for further studies to repeat the lottery contest experiment with another pretest, e.g. a decomposed game. Then, the hypothesis that individuals differ according to their social value orientations in the effort chosen in a lottery contest is testable. We shortly sketch the idea of such a pretest here: In order to test other-regarding behavior of individuals, the decomposed game technique is employed in the pre-test. The decomposed game technique is developed by social psychologists (Wyer (1969), Griesinger and Livingstone (1973) and Liebrand (1984)) and attempts to assess an individual's social motivation. Using this technique, subjects play 24 decomposed games. In each of these games, they are asked to choose between two own/other payoff combinations. Each of these payoff combinations assigns a certain amount of money to the subjects themselves, the own payoff x, and a certain amount to the other subject, the other payoff y. A typical choice may involve, e.g., a combination $(75, -130)$ vs. $(39, -145)$, where one must choose between gaining either 75 or 39, with related losses on the other's part of either 130 or 145. The monetary values of these payoffs were determined such that when plotted as ordered pairs (x, y), in a two-dimensional own-other payoff space (Figure 5.1), they would be located at 24 equally spaced points around a circle, centered at the origin $(0, 0)$ and with an arbitrary radius of 150 monetary units. The pairs of subjects playing these games remained unchanged throughout the whole classification procedure. Hence, the payoffs received by subjects are determined by the 24 decisions that they make themselves and by the 24 decisions that are made by their partners. Subjects are informed of this, but in order to avoid strategic considerations, they do not get any information concerning the identities of their partners nor do they receive any feedback concerning the decisions made by them. Adding up the amounts of money chosen by subjects for themselves and for their partners separately, an estimate of the importance given by the subject to the own payoff and to the partner's payoff is obtained. These estimates are used to approximate a vector in the own-other payoff space, representing the individual's type. Using a standard classification procedure developed for this technique, subjects with a vector lying between 67.5 and 112.5 are classified as altruistic (i.e. maximization of other's payoff); with a vector between 22.5 and 67.5 as cooperative (i.e. maximization of the sum of own and other's payoff); with a vector between 0 and 22.5 or between 337.5 and 360 as individualistic (i.e. maximization of own payoff); and with a vector between 292.5 and 337.5 as competitive (i.e. maximization of the difference between own and other's payoff). The length of this vector serves as an index for a subject's consistency (i.e. it indicates whether the chosen own-other payoff combinations are the closest ones to the subject's motivational vector). Making 24 consistent choices yields a vector length of 300 (twice the radius of the circle). Random choices, on the other hand, result in a vector length of zero.

Once the preferences of the players are identified, one can compare the social value with the effort choice in a rent-seeking contest. This could be pursued in further experimental studies.

Bibliography

ABBINK, K., J. BRANDTS, B. HERRMANN, AND H. ORZEN (forthcoming): "Inter-Group Conflict and Intra-Group Punishment in an Experimental Contest Game," *American Economic Review.*

ALEXOPOLOS, M., AND S. SAPP (2006): "Exploring the Behavior of Economic Agents: The Role of Relative Preferences," *Economics Bulletin*, 12(2), 1–7.

AMEGASHIE, J. A. (1999): "The Design of Rent-Seeking Competitions: Committees, Preliminary and Final Contests," *Public Choice*, 99(1–2), 63–76.

——— (2000): "Some Results on Rent-Seeking Contests with Shortlisting," *Public Choice*, 105(3–4), 245–253.

ANDREONI, J., M. CASTILLO, AND R. PETRIE (2003): "What Do Bargainers' Preferences Look Like? Experiments with a Convex Ultimatum Game," *American Economic Review*, 93(3), 672–685.

ANDREONI, J., AND J. H. MILLER (2002): "Giving According to GARP: An Experimental Test of the Consistency of Preferences for Altruism," *Econometrica*, 70, 737–753.

ANDREONI, J., AND J. H. MILLER (2008): "Analyzing Choice with Revealed Preference: Is Altruism Rational?," vol. 1 of *Handbook of Experimental Economics Results*, chap. 54, pp. 481–487. Elsevier.

APPELBAUM, E., AND E. KATZ (1986): "Rent Seeking and Entry," *Economic Letters*, 20(3), 207–212.

BAIK, K. H. (2007): "Equilibrium Contingent Compensation in Contests with Delegation," *Southern Economic Journal*, 73(4), 986–1002.

Baik, K. H., and I. G. Kim (1997): "Delegation in Contests," *European Journal of Political Economy*, 13, 281–298.

Baik, K. H., and S. Lee (2000): "Two-Stage Rent-Seeking Contests with Carryovers," *Public Choice*, 103, 285–296.

Basu (1995): "Stackelberg Equilibrium in Oligopoly: An Explanation Based on Managerial Incentives," *Economics Letters*, 49(4), 459–464.

Baye, M. R., and H. Hoppe (2003): "The Strategic Equivalence of Rent-Seeking, Innovation and Patent-Race Games," *Games and Economic Behavior*, 44(2), 217–226.

Baye, M. R., D. Kovenock, and C. G. de Vries (1996): "The All-Pay Auction with Complete Information," *Economic Theory*, 8, 291–305.

——— (2005): "Comparative Analysis of Litigation Systems: An Auction-theoretic Approach," *Economic Journal*, 115(505), 583–601.

Beath, J., Y. Katsoulacos, and D. Ulph (1989): "Strategic R&D Policy," *Economic Journal Supplement*, 99(395), 74–83.

Becker, G. S. (1974): "A Theory of Social Interactions," *Journal of Political Economy*, 82(6), 1063–1094.

Bell, D. E., R. L. Keeney, and J. D. C. Little (1975): "A Market Share Theorem," *Journal of Marketing Research*, 12(2), 136–141.

Bergstrom, T. (1995): "On the Evolution of Altruistic Ethical Rules for Siblings," *American Economic Review*, 85(1), 58–81.

Bester, H., and W. Güth (1998): "Is Altruism Evolutionarily Stable?," *Journal of Economic Behavior & Organization*, 34(2), 193–209.

Bewley, T. (1995): "A Depressed Labor Market as explained by Participants," *American Economic Review*, 85, 250–254.

Bolton, G., and A. Ockenfels (2000): "ERC: A Theory of Equity, Reciprocity, and Competition," *American Economic Review*, 90, 166–193.

Bush, C. (1994a): "Interdependent Preferences and Status: A Taxonomy of Demand," Working Paper 385, Rochester Center for Economic Research.

——— (1994b): "Interdependent Preferences and Status in Consumption: Empirical Evidence," Working Paper 387, Rochester Center for Economic Research.

Charness, G., and M. Rabin (2002): "Understanding Social Preferences with Simple Tests," *Quarterly Journal of Economics*, 117, 817–869.

Clark, A. E., and A. J. Oswald (1996): "Satisfaction and Comparison Income," *Journal of Public Economics*, 61(3), 359–381.

Clark, D. J., and C. Riis (1996): "On the Win Probability in Rent-Seeking Games," Memorandum 5, University of Oslo, Department of Economics.

——— (1998): "Contest Success Functions: An Extension," *Economic Theory*, 11(1), 201–204.

Congleton, R., A. Hillman, and K. Konrad (2008): *40 Years of Research on Rent Seeking, Volumes 1 and 2*. Springer Verlag, Hamburg.

Corchon, L. (2000): "On the Allocative Effects on Rent Seeking," *Journal of Public Economic Theory*, 2(4), 483–492.

——— (2007): "The Theory of Contests: A Survey," *Review of Economic Design*, 11(2), 69–100.

Cornes, R., and R. Hartley (2005): "Asymmetric contests with general technologies," *Economic Theory*, 26, 923–946.

Davis, D., and C. Holt (1993): *Experimental Economics*. Harvard University Press.

Dixit, A. (1987): "Strategic Behavior in Contests," *American Economic Review*, 77, 891–898.

Duesenberry, J. (1949): *Income, Savings and the Theory of Consumers Behavior*. Harvard University Press.

Dufwenberg, M., and G. Kirchsteiger (2004): "A Theory of Sequential Reciprocity," *Games and Economic Behavior*, 47(2), 268–298.

Easterlin, R. A. (2009): *Game Theory: Strategies, Equilibria, and Theorems*chap. Excessive Expenditure in wo-Stage COntests: Theory and Experimental Evidence. F. Columbus, Nova Science Publishers, New York.

Eaton, B., and M. Eswaran (2003): "The Evolution of Preferences and Competition: A Rationalization of Veblen's Theory of Invidious Comparisons," *Canadian Journal of Economics*, 36(4), 832–859.

Epstein, G., and S. Nitzan (2002): "Asymmetry and Corrective Public Policy in Contests," *Public Choice*, 113, 231–240.

Etro, F. (2006): "Aggressive Leaders," *Rand Journal of Economics*, 37(1), 146–154.

——— (2007): *Competition, Innovation, and Antitrust: A Theory of Market Leaders and its Policy Implications*. Springer.

Falk, A., and U. Fischbacher (2006): "A Theory of Reciprocity," *Games and Economic Behavior*, 54, 293–316.

Fang, H. (2002): "Lottery versus All-Pay Auction Models of Lobbying," *Public Choice*, 112, 351–371.

Farmer, A., and P. Pecorino (1999): "Legal Expenditure as a Rentseeking Game," *Public Choice*, 100, 271–288.

Fehr, E., and K. M. Schmidt (1999): "A Theory of Fairness, Competition and Cooperation," *Quarterly Journal of Economics*, 114, 817–868.

Fershtman, C., and K. L. Judd (1987): "Equilibrium Incentives in Oligopoly," *American Economic Review*, 77(5), 927–940.

Fishman, R., S. Kariv, and D. Markovits (2007): "Individual Preferences for Giving," *American Economic Review*, 97(5), 1858–1876.

Frank, R. H. (1985): *Choosing the Right Pond: Human Behavior and Quest for Status*. Oxford University Press, New York.

Friedman, L. (1958): "Game-Theory Models in the Allocation of Advertising Expenditures," *Operations Research*, 6(5), 699–709.

Fudenberg, D., and J. Tirole (1984): "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and the Hungry Look," *American Economic Review*, 74(2), 361–366.

Garfinkel, M., and S. Skaperdas (1996): *The Political Economy of Conflict and Appropriation*. New York Cambridge University Press.

Gradstein, M., and K. A. Konrad (1999): "Orchestrating Rent Seeking Contests," *The Economic Journal*, 109(458), 536–545.

Griesinger, D., and J. Livingstone (1973): "Toward a Model of Interpersonal Motivation in Experimental Games," *Behavioral Science*, 18, 173–188.

Groh, C., B. Moldovanu, A. Sela, and U. Sunde (2008): "Optimal Seedings in Elimination Tournaments," *Economic Theory*, online.

Guse, T. (2006): "When are Spiteful Players better off? Different Levels of Interdependent Preferences in Rent-Seeking Contests," Unpublished.

Guse, T., and B. Hehenkamp (2006): "The Strategic Advantage of Interdependent Preferences in Rent-Seeking Contests," *Public Choice*, 129, 323–352.

Güth, W., and M. Yaari (1992): "Explaining Reciprocal Behavior in Simple Strategic Games: An Evolutionary Approach," in *Explaining Forces and Changes: Approaches to Evolutionary Economics*, ed. by U. Witt, pp. 23–34. University of Michigan Press.

Hamilton, W. D. (1970): "Selfish and Spiteful Behavior in an Evolutionary Model," *Nature*, 228, 1218–1220.

Harris, C., and J. Vickers (1985): "Perfect Equilibrium in a Model of Race," *Review of Economic Studies*, 52(2), 193–209.

——— (1987): "Racing with Uncertainty," *Review of Economic Studies*, 54(177), 1–21.

Hehenkamp, B., W. Leininger, and A. Possajennikov (2004): "Evolutionary Equilibrium in Tullock Contests: Spite and Overdissipation," *European Journal of Political Economy*, 20(4), 1045–1057.

Herrmann, B., and H. Orzen (2008): "The Appearance of Homo Rivalis: Social Preferences and the Nature of Rent Seeking," Discussion Paper 2008-10, The Centre for Decision Research and Experimental Economics, School of Economics, University of Nottingham.

Herrmann, B., C. Thöni, and S. Gächter (2008): "Antisocial Punishment across Societies," *Science*, 319(5868), 1362–1367.

HIRSHLEIFER, J., AND J. G. RILEY (1992): *The Analytics of Uncertainty and Information*. Cambridge University Press.

HOFFMAN, E., K. MCCABE, AND V. SMITH (1998): "Behavioral Foundations of Reciprocity: Experimental Economics and Evolutionary Psychology," *Economic Inquiry*, 36, 335–352.

HOLMSTROM, B. (1982): "Moral Hazard in Teams," *Bell Journal of Economics*, 13(2), 324–340.

HUCK, S., K. A. KONRAD, AND W. MÜLLER (2002): "Merger and Collusion in Contests," *Journal of Institutional and Theoretical Economics*, 158(4), 563–575.

JANSEN, T., A. VAN LIER, AND A. VAN WITTELOOSTUIJN (2007): "A Note on Strategic Delegation : The Market Share Case," *International Journal of Industrial Organization*, 25(3), 531–539.

KAPTEYN, A., S. VAN DER GEER, H. VAN DE STADT, AND T. WANSBECK (1997): "Interdependent Preferences: An Econometric Analysis," *Journal of Applied Econometrics*, 6, 665–686.

KATZ, E., AND J. TOKATLIDU (1996): "Group Competition for Rents," *European Journal of Political Economy*, 12(4), 599–607.

KLUMPP, T., AND M. K. POLBORN (2006): "Primaries and the New Hampshire Effect," *Journal of Public Economics*, 90(6–7), 1073–1114.

KOCKESEN, L., E. A. OK, AND R. SETHI (2000): "The Strategic Advantage of Negatively Interdependent Preferences," *Journal of Economic Theory*, 92, 274–299.

KONRAD, K. A. (2004): "Altruism and Envy in Contests: An Evolutionarily Stable Symbiosis," *Social Choice and Welfare*, 17, 479–490.

——— (2007): "Strategy in Contests - An Introduction," Discussion Paper SP II 2007 01, Wissenschaftszentrum Berlin.

——— (2009): *Strategy and Dynamics in Contests*. Oxford University Press.

KONRAD, K. A., AND D. KOVENOCK (2005): "Equilibrium and Efficiency in the Tug-of-War," Discussion Paper SP II 2005 14, CEPR.

——— (2009): "Multi-battle Contests," *Games and Economic Behavior*, 66(1), 256–274.

——— (forthcoming): "Contests with Stochastic Ability," *Economic Inquiry*.

Kooreman, P., and L. Schoonbeek (1997): "The Specification of the Probability Functions in Tullocks Rent-Seeking Contest," *Economic Letters*, 56(1), 59–61.

Kräkel, M. (2002): "Delegation and Strategic Incentives for Managers in Contests," *Managerial and Decision Economics*, 23(8), 461–470.

Kräkel, M., and D. Sliwka (2006): "Strategic Delegation and Mergers in Oligopolistic Contests," *Journal of Economics and Business*, 58(2), 119–136.

Krüger, A. (1974): "The Political Economy of the Rent-Seeking Society," *American Economic Review*, 64, 291–303.

Lambertini, L. (2000): "Strategic Delegation and The Shape of Market Competition," *Scottish Journal of Political Economy*, 47(5), 550–570.

Layard, R. (1980): "Human Satisfactions and Public Policy," *The Economic Journal*, 90(360), 737–750.

Lazear, E. P., and S. Rosen (1981): "Rank-Order Tournaments as Optimum Labor Contracts," *Journal of Political Economy*, 89, 841–864.

Lee, S. (2003): "Two-Stage Contests With Additive Carryovers," *International Economic Journal*, 17(1), 83–99.

Leininger, W. (2002): "Contests over Public Goods: Evolutionary Stability and the Free-Rider Problem," Working Paper 713(1), CESifo.

——— (2003): "On Evolutionarily Stable Behavior in Contests," *Economics of Governance*, 4, 177–186.

——— (2009): "Evolutionary Stable Preferences in Contests," *Public Choice*, 140, 341–356.

Levine, D. (1998): "Modeling Altruism and Spitefulness in Experiments," *Review of Economic Dynamics*, 1, 593–622.

LIEBRAND, W. (1984): "The Effect of Social Motives, Communication and Group Size on Behavior in an N-Person Multi-stage Mixed-motive Game," *European Journal of Social Psychology*, 14, 239–264.

LOCKARD, A., AND G. TULLOCK (2001): "Efficient Rent Seeking. Chronicle of an Intellectual Quagmire," *Kluwer Academic Publishers, Boston.*

LOURY, G. (1979): "Market Structure and Innovation," *Quarterly Journal of Economics*, 93, 395–410.

MALUEG, D., AND A. YATES (2004): "Rent Seeking with Private Values," *Public Choice*, 119, 161–178.

MATROS, A. (2006): "Stochastic K-player Blotto games," Working Paper 206, University of Pittsburgh, Department of Economics.

MCAFEE, R. P. (2000): "Continuing Wars of Attrition," Workingpaper.

MCBRIDE, M., AND S. SKAPERDAS (2007): "Explaining Conflict in Low-Income Countries: Incomplete Contracting in the Shadow of the Future," in *Institutions and Norms in Economic Development*, ed. by M. Gradstein, and K. Konrad, pp. 141–161. MIT Press, Cambridge.

MCCABE, K., S. RASSENTI, AND V. SMITH (1998): "Reciprocity, Trust, and Payoff Privacy in Extensive Form Bargaining," *Games and Economic Behavior*, 24, 10–24.

MEHLUM, H., AND K. O. MOENE (2004): "So much Fighting, so little Success," Workingpaper.

MILLER, N. H., AND A. PAZGAL (2001): "The Equivalence of Price and Quantity Competition with Delegation," *Rand Journal of Economics*, 32(2), 284–301.

MILLS, H. D. (1961): "A Study in Promotional Competition," in *Mathematical Models and Methods in Marketing*, ed. by F. M. Bass, R. D. Buzzell, M. R. Greene, W. Lazer, E. A. Pessemier, D. L. Shawver, A. Schuchman, C. A. Theodore, and G. W. Wilson, pp. 245–301. Elsevier, Homewood, Illinois.

MOLDOVANU, B., AND A. SELA (2006): "Contest Architecture," *Journal of Economic Theory*, 126(1), 70–96.

Mujumdar, S., and D. Pal (2007): "Strategic Managerial Incentives in a Two-Period Cournot Duopoly," *Games and Economic Behavior*, 58(2), 338–353.

Münster, J. (2007): "Simultaneous Inter- and Intra-Group Conflicts," *Economic Theory*, 32(2), 333–352.

——— (2009): "Group Contest Success Functions," *Economic Theory*, 41(2), 345–357.

Nalebuff, B., and J. E. Stiglitz (1983): "Prizes and Incentives: Towards a General Theory of Compensation and Competition," *Bell Journal of Economics*, 14, 21–43.

Nitzan, S. (1991): "Collective Rent Dissipation," *Economic Journal*, 101, 1522–1534.

——— (1994): "Modelling Rent-Seeking Contests," *European Journal of Political Economy*, 10(1), 41–60.

Nti, K. O. (1997): "Comparative Statics of Contests and Rent-Seeking Games," *International Economic Review*, 38, 43–59.

Offerman, T., J. Sonnemans, and A. Schram (1996): "Value Orientations, Expectations and Voluntary Contributions in Public Goods," *Economic Journal*, 106, 817–845.

Ok, E. A., and L. Kockesen (2000): "Negatively Interdependent Preferences," *Social Choice and Welfare*, 17, 533–558.

Olson, M. (1965): *The Logic of Collective Action: Public Goods and the Theory of Groups*. Harvard University Press.

Parco, J., A. Rapoport, and W. Amaldoss (2005): "Two-stage Contests with Budget Constraints: An Experimental Study," *Journal of Mathematical Psychology*, 49(4), 320–338.

Perez-Castrillo, J. D., and T. Verdier (1992): "A General Analysis of Rent-Seeking Games," *Public Choice*, 73, 335–350.

Porac, J. F., J. B. Wade, and T. Pollock (1999): "Industry Categories and the Politics of the Comparable Firm in CEO Compensation," *Administrative Science Quarterly*, 44, 112–144.

Possajennikov, A. (2008): "On the Survival of Payoff Maximizing Behavior and Delegation in Contests," Discussion Paper 2008-15, Centre for Decision Reseach and Experimental Economics, University of Nottingham.

——— (2009): "Commitment in Symmetric Contests," *Economics Bulletin*, 29(1), 376–384.

Rabin, M. (1993): "Incorporating Fairness into Game theory and Economics," *American Economic Review*, 83, 1281–1302.

Risse, S. (2010): "Two-stage group rent-seeking with negatively interdependent preferences," *Public Choice*, (DOI: 10.1007/s11127-010-9618-5).

Ritz, R. A. (2008): "Strategic Incentives for Market Share," *International Journal of Industrial Organization*, 26(2), 586–597.

Robson, A. R., and S. Skaperdas (2008): "Costly Enforcement of Property Rights and the Coase Theorem," *Economic Theory*, 36(1), 109–128.

Rosen, S. (1986): "Prizes and Incentives in Elimination Tournaments," *American Economic Review*, 76, 701–715.

Rubin, P., C. Curran, and J. Curran (2001): "Litigation versus Legislation: Forum Shopping by Rent Seekers," *Public Choice*, 107, 295–310.

Ryvkin, D. (2007): "Tullock Contests of Weakly Heterogeneous Players," *Public Choice*, 132(1), 49–64.

Salas Fumas, V. (1992): "Relative Performance Evaluation of Management," *International Journal of Industrial Organization*, 10(3), 473–489.

Schelling, T. C. (1960): *The Strategy of Conflict*. Oxford University Press, New York.

Schmalensee, R. (1976): "A Model of Promotional Competition in Oligopoly," *Review of Economic Studies*, 43(3), 493–507.

Shaffer, S. (2006): "Contests with Interdependent Preferences," *Applied Economics Letters*, 13, 877–880.

Sheremeta, R. M. (2009): "Two-Stage Contests With Additive Carryovers," *Workingpaper*.

Skaperdas, S. (1996): "Contest Success Functions," *Economic Theory*, 7(2), 283–290.

Sklivas, S. D. (1987): "The Strategic Choice of Managerial Incentives," *Rand Journal of Economics*, 18, 452–458.

Sobel, J. (2005): "Interdependent Preferences and Reciprocity," *Journal of Economic Literature*, 43, 392–436.

Solnik, S. J., and D. Hemenway (1998): "Is more always better?: A Survey on Positional Concerns," *Journal of Economic Behavior & Organization*, 37, 373–383.

Stein, W. E. (2002): "Asymmetric Rent-Seeking with more than two Contestants," *Public Choice*, 113, 325–336.

Stein, W. E., and A. Rapoport (2004): "Asymmetric Two-Stage Group Rent-Seeking: Comparison of two Contest Structures," *Public Choice*, 118, 151–167.

Szymanski, S. (2003): "The Economic Design of Sporting Contests," *Journal of Economic Literature*, 41(4), 1137–1187.

Tirole, J. (1988): *The Theory of Industrial Organization*. MIT Press.

Tollison, R. D. (1982): "Rent Seeking: A Survey," *Kyklos*, 35(4), 575–602.

——— (1997): "Rent seeking," in *Perspectives on Public Choice*, ed. by D. Mueller, pp. 506–525. Cambridge University Press, New York.

Tullock, G. (1967): "The Welfare Cost of Tariffs, Monopolies and Theft," *Western Economic Journal*, 5, 224–232.

——— (1980): "Efficient rent seeking," in *Toward a Theory of the Rent-Seeking Society*, ed. by J. Buchanan, R. Tollison, and G. Tullock, pp. 97–112. A&M University, Texas.

——— (2003): "The Origin Rent-Seeking Concept," *International Journal of Business and Economics*, 2(1), 1–8.

Veblen, T. (1899): *The Theory of the Leisure Class*. Macmillan.

VICKERS, J. (1985): "Delegation and the Theory of the Firm," *Economic Journal*, 95, 138–147.

WÄRNERYD, K. (1998): "Distributional Conflict and Jurisdictional Organization," *Journal of Public Economics*, 69, 435–450.

——— (2000): "In Defense of Lawyers: Moral Hazard as an Aid to Cooperation," *Games and Economic Behavior*, 33(1), 145–158.

WEIBULL, J. W., AND M. SALOMONSSON (2006): "Natural Selection and Social Preferences," *Journal of Theoretical Biology*, 239(1), 79–92.

WYER, R. (1969): "Prediction of Behavior in Two-Person Games," *Journal of Personality and Social Psychology*, 13, 222–238.

ZIZZO, D. J. (2003): "Empirical Evidence on Interdependent Preferences: Nature or Nurture?," *Cambridge Journal of Economics*, 27, 867–880.

ZIZZO, D. J., AND A. OSWALD (2000): "Are People Willing to Pay to Reduce Others' Incomes?," The Warwick Economics Research Paper Series (TWERPS) 568, University of Warwick, Department of Economics.

Ich versichere, dass ich diese Dissertation selbständig verfasst habe. Bei der Erstellung der Arbeit habe ich mich ausschliesslich der angegebenen Hilfsmittel bedient. Die Dissertation ist nicht bereits Gegenstand eines erfolgreich abgeschlossenen Promotions- oder sonstigen Prüfungsverfahrens gewesen.

Dortmund, 09. Mai 2010