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## Using Microsoft Excel to solve and illustrate mathematical problems

### Abstract

At the University of Kaposvár in BSc and BA education we introduced a new course for the first year students. This course has two objectives.

The first year students have significant differences in mathematics. My colleague Klingné Takács Anna speaks about this problem in his lecture. We use Microsoft Excel to solve and illustrate mathematical problems in Calculus and Excel Solver in Optimization. I will speak about why we use Excel and I would like to demonstrate how we harmonise the traditional education of the mathematics and the opportunities provided by the Excel.

### Introduction

Firstly I would like to talk about mathematics education at Kaposvár University: actually how the number of classes have changed in the past few years, what is the structure of course like and how we are trying to substitute the lessons which have been taken away.

In the second part of my presentation I am emphasising two two examples of Computer in Mathematics where we use Excel and Excel Solver to solve and illustrate problems. The objective is in both examples is to use the opportunities given by Excel combined with so called traditional maths methods. We compare and analyse the advantages and disadvantages of both methods.

### Change in numbers of mathematics classes

Let's have a look how the number of classes/periods have changed compared to the one before the introduction of the Bologna process. Now there is Finance and Agricultural rural development to offer for the students at Kaposvár University at the Faculty of Economic Science. I compare the number of maths classes to the number of maths classes at 5 year long university courses with similar content.

Before 2006 undergraduate students studied for 5 year at Business and management course and Agribusiness course. Subjects and number of subjects per week taught by Mathematics Department. It can be seen in the first two lines of my table/chart. Since the beginning of Bologna process Finance and Agricultural rural development courses are taught/run. The latter two can be seen in lines 3 and 4.

	1. semester	2. semester	3. semester	4. semester	Sum of numbers of lessons
Business and management 5 years university level programme	Calculus (Analysis) Number of lessons: 3+4 Lecture+practice	Probability and statistics Number of lessons: 3+3	Linear algebra Number of lessons: 3+2	Optimization Number of lessons: 2+2	22
Agribusiness 5 years university level programme	Mathematics Number of lessons: 2+2	Economic mathematics Number of lessons: 3+2	Introduction to the optimization Number of lessons: 2+2	—	13
Finance BA	Calculus I. (Analysis) Number of lessons: 2+2	Calculus II. (Probability) Number of lessons: 2+2	Optimization Number of lessons: 2+2	—	12
Agricultural rural development BSc	Calculus Number of lessons: 2+2				4

Significant decline can be seen.

In addition to Agribusiness 5 years university level programme the student could choose agriinformatics specialisation, where our department trained the following course-unit:

Optimization II. 2+2 lessons

Biometrics 1+1 lessons

Multimedia systems 2+2 lessons (A colleague with engineer degree - who teaches at the department – is a specialist of this theme and so we taught this unit too.)

More variables statistics 2+2 lessons

### The new optional course-units

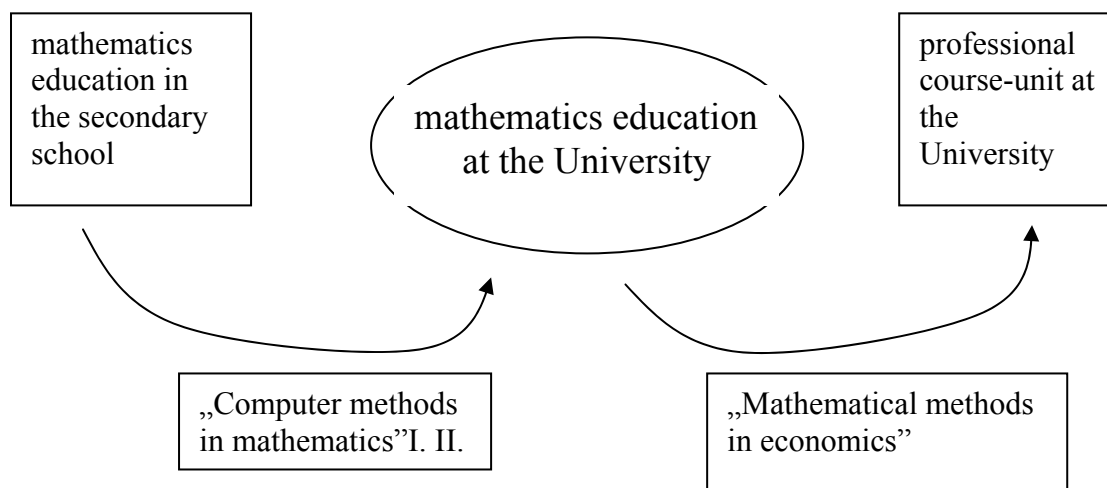
Along with the decline of numbers we can sense the following problems too.

- There is a big difference in the knowledge of incoming students.
- They can't use their acquired knowledge later while studying professional-unit.
- Their ability to use a computer appropriately is not enough (with some exceptions).

To overcome this difficulty we are beginning to develop some optional course-units that can be fitted in the direction of university to solve this temporary problem.

We offer Computer methods in Mathematics to fill in the gap existing between the secondary school and the university education.

We also offer Mathematical methods in economics to have a closer link between mathematics education at university and professional course-unit at the university.



Beside the above mentioned subjects we teach mathematics and physics at the Animal Sciences Faculty, which is even less than the quantity taught at Economics Sciences Faculty. Our department gives lectures in basic information technology at Teacher Training College, Science and Arts and Mathematics and Arts.

Let's have a closer look at the aim and syllabus of the above mentioned subjects.

#### *Computer methods in mathematics I.*

The aim of this training is to revise some units of the secondary education curriculum to ensure sufficient practice in it, to use Microsoft Excel, where it's possible in order to solve and illustrate maths problems.

### Themes:

1. Revision of some parts of secondary education curriculum  
Algebraic identity, absolute value, solve of equations and inequality  
Raising to a power, extraction of a root, definition of logarithm, identity of logarithm  
Definition of function, function attributes, draw and elementary functions graphs  
Function transformations, operations  
Definition of sequences, arithmetic, geometric sequences
2. Solving tasks which are closely connected to the syllabus of basic mathematics.  
Compared to the practice of Calculus it helps the student achieve the necessary level by solving task rows built on top of each other in a smaller and easier steps.
3. Calculation and draw of elements of the sequences, draw functions graphs, geometrical illustration of Newton quotient, derivative in Excel

### *Computer methods in mathematics II.*

The aim of teaching the subject to present computer methods which help acquire Calculus, Analysis, Optimization. It includes solving tasks by Excel or LINV during the course. It also includes comparing and analyzing traditional mathematics and computer methods.

### Themes:

Linear approximation and approximation by Taylor series of the functions.

Riemann sum of function (integral)

Calculation of factorial and binomial coefficient. Permutations with and without repetitions, combinations with and without repetitions

Modeling the random effects, frequency, relative frequency. Probability distribution (discrete and continuous)

Matrix operations: multiplication, inverse. Determinant, solution to a system of linear equations with computation inverse matrix and with Cramer's rule

System of linear equations, linear and nonlinear programming (LP, NLP) problems, transport optimization, sensibility analysis in Excel Solver and LINV program („home (self) made program”)

### *Mathematical methods in economics*

The aim of teaching this subject is to apply previously learnt mathematical methods via economics tasks and to enlighten the close relation between mathematics and the professional course-units.

We find out how mathematics is related to the material covered in maths classes and when maths is used in professional course-units.

We realized that we have already covered the same thing under a difference name.

### Themes:

Sequences and series in the financial and economic calculations (compound interest, taking inflation into account, present value, comparison of different, interest methods, calculation of annuity, etc.)

Account and degree of changing in the economy, differentiable and empirical functions. The derivative and the elasticity

Exponential and logarithmic function, exponential and logarithmic applications, semi-log graph, elasticity and logarithmic derivative

One-variable optimization, global and local extreme value, solve extreme value problems in economics

Applications of integration in the economics (store of exchange, distribution of income, etc.)

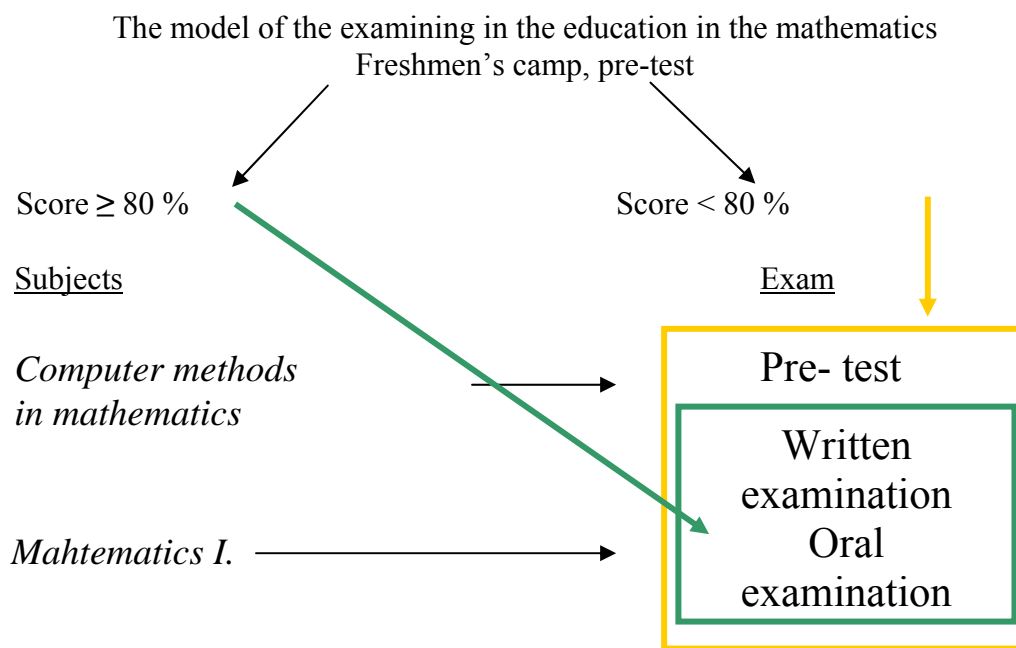
Functions of several variables, role of partial derivative in economics, multivariable optimization (demonstration with a computer program)  
 Formulating and solving of conditional extreme value problem (elimination and contour line method) using Microsoft Excel

**The system of the examining in the education in the mathematics Freshmen’s camp, pre-test**

The reasons are as follows: Most problems come from the fact that the students cannot apply their knowledge gained in secondary school in tasks not from the fact that the haven’t covered enough material on the first course.

They acquired the necessary material in the secondary school, met the requirements however they are not able to apply the theory in new tasks. That’s why we make our students sit for a test based on secondary school material before the end of the first semester an especially pointed one on the pasts which university built mathematics on. In case one cannot get over 80% they are offered Computer methods in maths where they have the opportunity to re-learn material needed. This process is mentioned in a lecture given by my colleague Klingné Takács Anna.

The model of the examination and education structure run during the preparatory camp for freshmen. Having passed pre-test they can do their university course test. In case they failed the pre-test they are offered Computer methods in mathematics and do the pre-test again in order to get ready for the university course test.



**Using Microsoft Excel to solve and illustrate mathematical problems**

The other aim of Computer methods in mathematics is to solve tasks in Excel and to illustrate certain phenomena. I’ d like to talk about this now.

Why do we use Excel?

It is available for everybody. Even though a lot of excellent mathematical software exist students cannot access them having finished their studies at university. Teachers of professional units (finances, economics) also use Excel. In the future during MSc (Master of sciences) training We would like to use a specific maths program (Maple or Mathematica).

I will show two examples in details. It is important in both examples that they use traditional mathematical methods.

### 1. The graph representation of the function in Excel

We put a lot of effort to draw the graph of a function with Excel because we experience that the students can solve function analysis problems well, except drawing the graph of a function. They determine the 1<sup>st</sup> and 2<sup>nd</sup> derivative, their roots, signs, but the drawing of the graph of a function still causes trouble.

We have to select the interval, on which we draw the graph of a function, so we give here to a subset of the domain of a function. After this we choose a step value, it is important how large the step value is, because it may happen on the case of a big step value, that everywhere differentiable functions have breakpoints on the graph. (We can correct this, when we select "smooth lines" diagram)

As an example we selected a function, which has a break point, extremal value, inflection point also.

How do we choose the right interval?

Which ones are the important, exciting points, which have to contain the selected subset of the domain of a function?

These are the singularity points, roots, extremal values, inflection points of a function.

Let us look at an example. We would like to show the graph of the function  $f(x) = \frac{x^2}{(x+1)^2}$ .

First we determine the domain of  $f$ , that immediately gives the break points also:

$D_f : x \in \mathbb{R} \setminus \{-1\}$   $x$  element of set of all real numbers except the number -1

After this we calculate the root, the result is  $x = 0$

Then we give the roots of the derivative:

$$f'(x) = \frac{2x(x+1)^2 - x^2 \cdot 3(x+1)}{(x+1)^4} = \frac{2x(x+1) \cdot (x+1-x)}{(x+1)^4} = \frac{2x}{(x+1)^3}$$

$$f'(x) = 0, \text{ if } x = 0$$

Finally we define the root of the second derivative.

$$f''(x) = \frac{2(x+1)^3 - 2x \cdot 3(x+1)^2}{(x+1)^6} = \frac{2(x+1)^2 \cdot (x+1-3x)}{(x+1)^6} = \frac{2(1-2x)}{(x+1)^4}$$

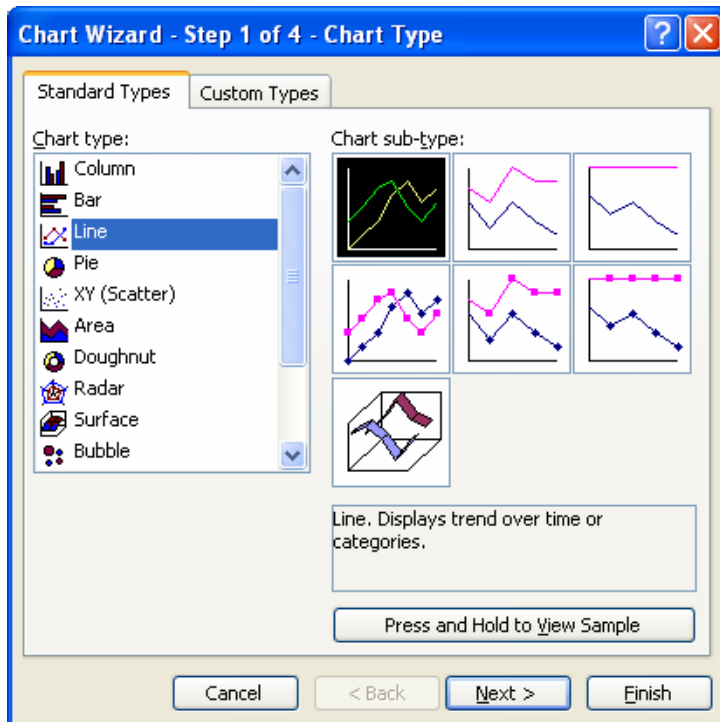
$$f''(x) = 0, \text{ if } x = \frac{1}{2}$$

$$\text{Limits at infinity: } \left( \lim_{x \rightarrow \pm\infty} \frac{x^2}{x^2 + 2x + 1} = 1 \right)$$

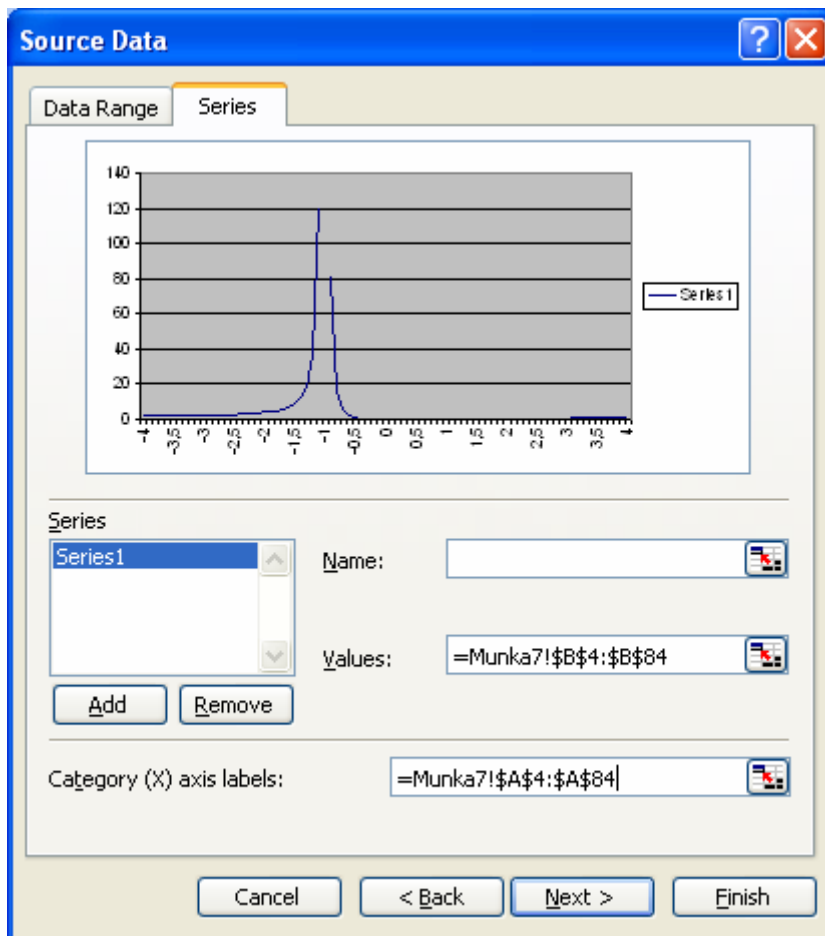
Firstly we determine to illustrate an interval which contains as an inner point and these points

are:  $x = -1$ ,  $x = 0$ ,  $x = \frac{1}{2}$ . Let be the interval  $[-4, 4]$ . In general I add and subtract two extreme points 3 or 4. Consider the step value 0,1.

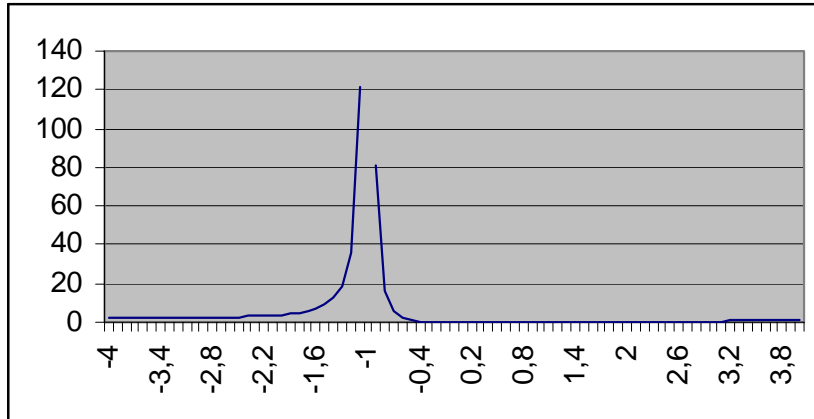
The values of  $[-4, 4]$  with step value 0,1 come to the A column of sheet. We write the values of function to the B column. We mark the values of function, insert a diagram and choose first chart sub-type of line chart type.



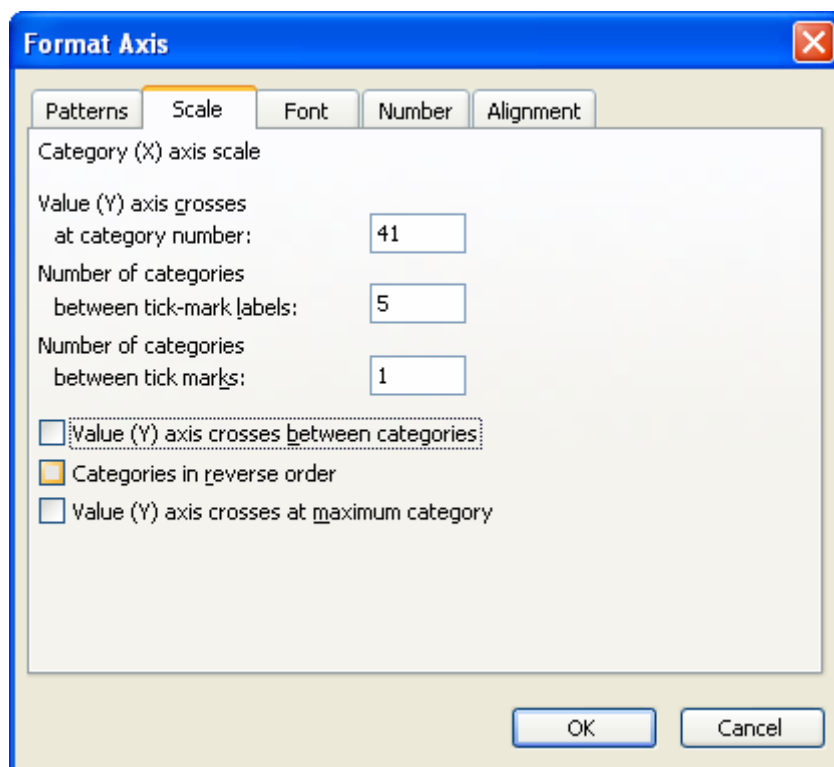
If we see the view sample (minta) it can be seen, that the category (x) axis labels are the values: 1, 2, ... and there are not the value of domain by column A. We can solve this problem by using of the chart wizard in the step 2.



We write the corresponding values of column A to the window of category (X) axis labels. I am not concerned with the chart title, category (x) axis, value (Y) axis and „cosmetics” of the diagram. The aim that the graph has to be similar to the mathematics class common graph representation of the function and role of all important point reflect well in the figure. After we click finish, we receive graph, the biggest problem to be value (Y) axis crosses at the category number in a false point.



After we click with the right mouse button to the (X) axis, we choose first format axis and then scale.

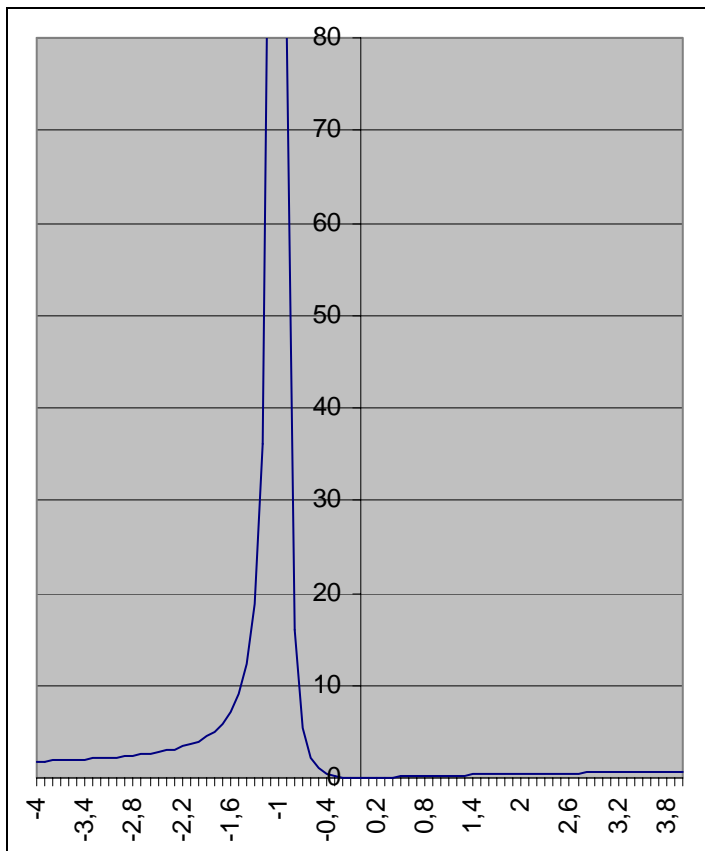


We search for which row is that, where 0 is at the column A? For the 44<sup>th</sup> row in our problem we put the 41<sup>st</sup> value in to the x axis because we wrote the first number in the A4 cell.

1	x	$x^2$
2		$(x+1)^2$
3		
4	-4	1,777778
5	-3,9	1,808561
6	-3,8	1,841837
7	-3,7	1,877915

42	-0,2	0,0625
43	-0,1	0,012346
44	0	0
45	0,1	0,008264
46	0,2	0,027778
47	0,3	0,053754

Then we get a better view presentation to the figure, the point of discontinuity can be seen better, but the extreme value and inflexion point are not.



We can correct this problem by decreasing the maximum value (y) axis scale, major unit and minor unit change automatically.

Format Axis	Format Axis	Format Axis
Patterns   <b>Scale</b>   Font   Num Value (Y) axis scale Auto <input checked="" type="checkbox"/> Minimum: 0 <input type="checkbox"/> Maximum: 80 <input checked="" type="checkbox"/> Major unit: 10 <input checked="" type="checkbox"/> Minor unit: 2	Patterns   <b>Scale</b>   Font   Num Value (Y) axis scale Auto <input checked="" type="checkbox"/> Minimum: 0 <input type="checkbox"/> Maximum: 10 <input checked="" type="checkbox"/> Major unit: 1 <input checked="" type="checkbox"/> Minor unit: 0,2	Patterns   <b>Scale</b>   Font   Num Value (Y) axis scale Auto <input checked="" type="checkbox"/> Minimum: 0 <input type="checkbox"/> Maximum: 2 <input checked="" type="checkbox"/> Major unit: 0,2 <input checked="" type="checkbox"/> Minor unit: 0,04

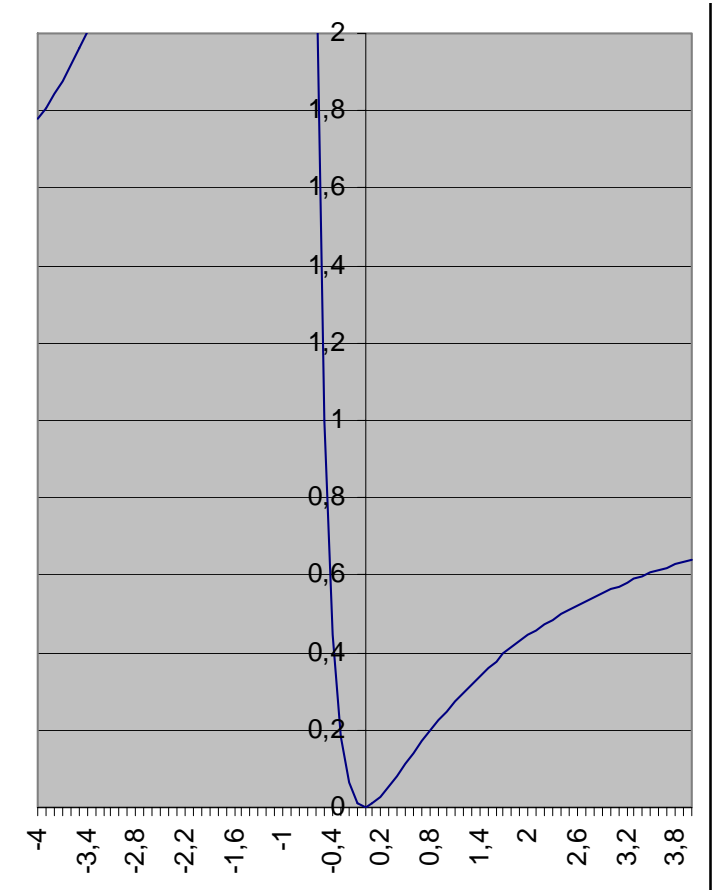
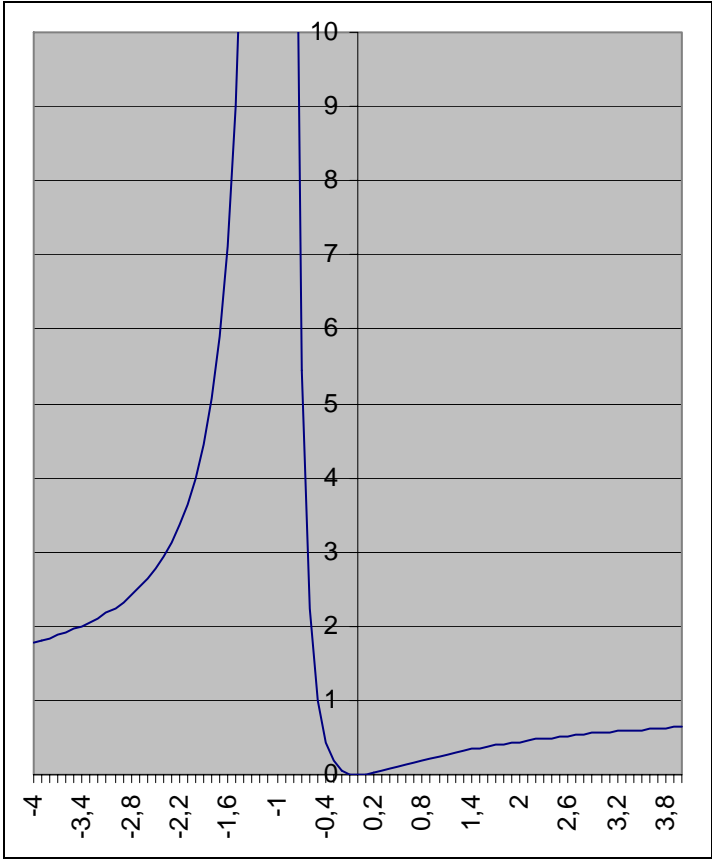
The settings of the original figure

The first modification

The second modification



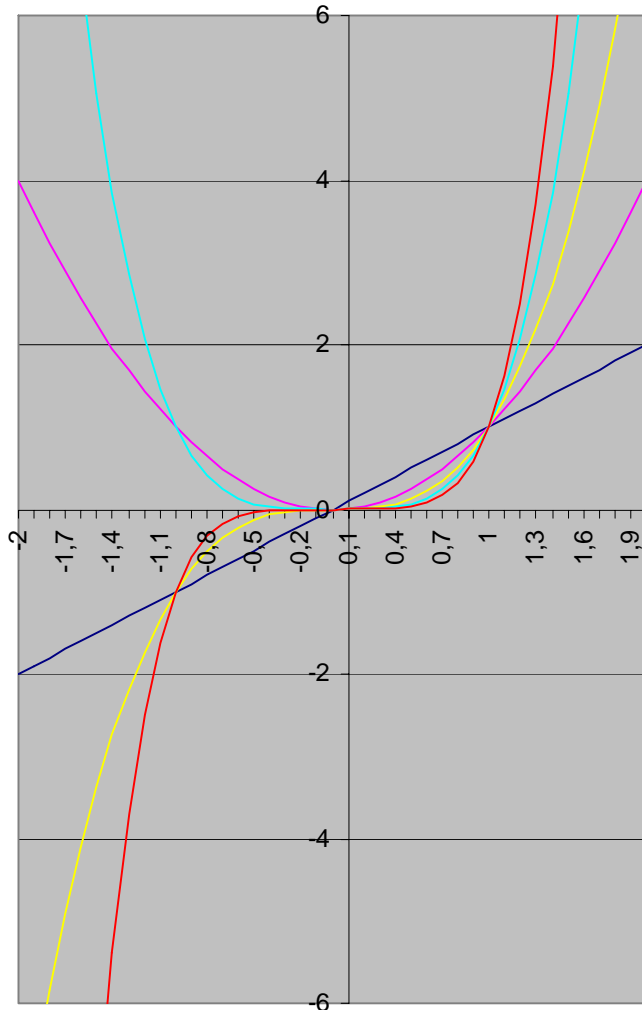
So we see the extreme value well in the following figure, then on the next figure we can see the inflection point.



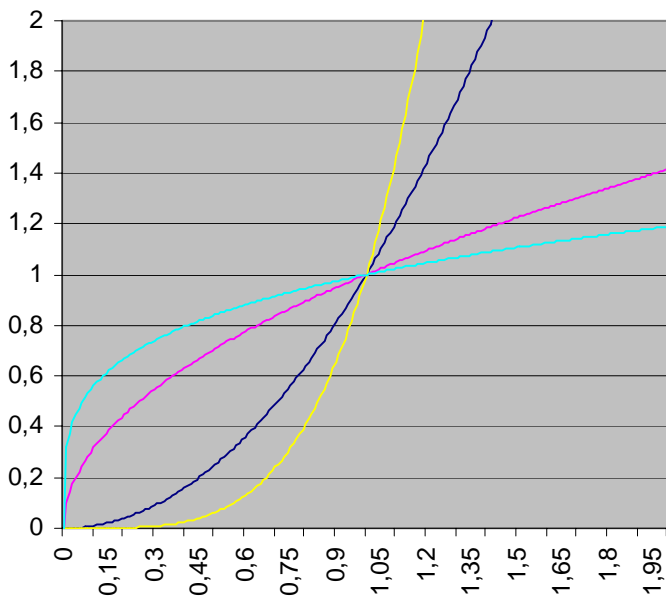
Before the second task we see some examples of the graph representation of the function in Excel:

Power functions

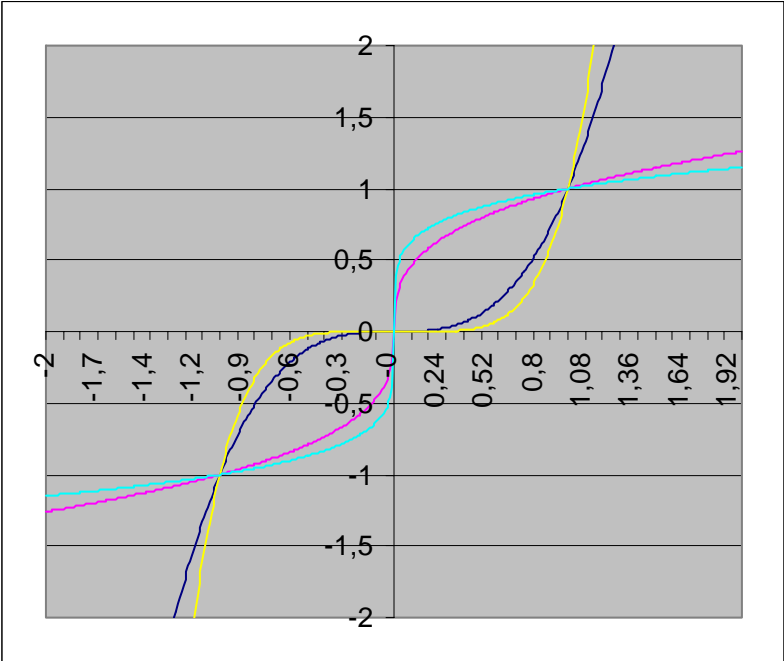
$$f(x) = x \quad g(x) = x^2 \quad h(x) = x^3 \quad i(x) = x^4 \quad j(x) = x^5$$



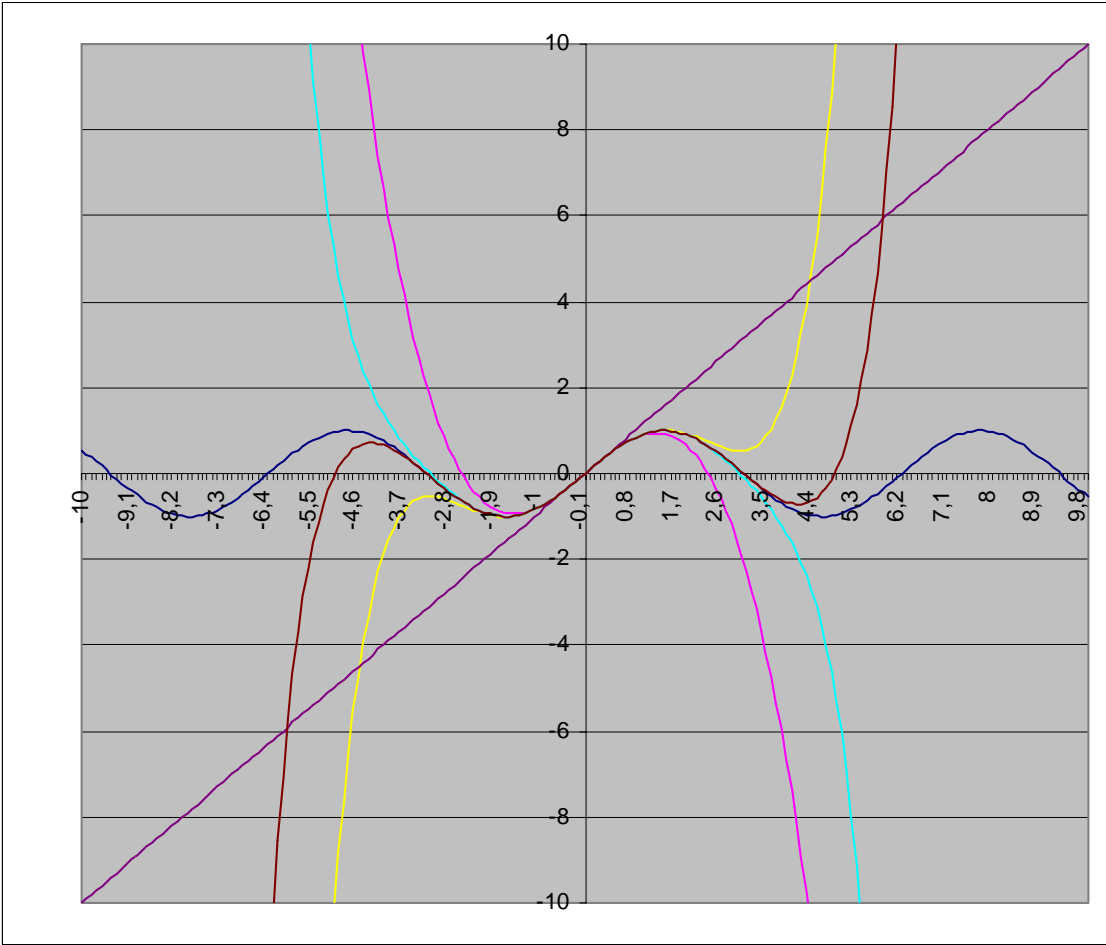
In case of the even power and root function's domain is  $x \geq 0$ , then these functions are inverse functions.



Odd power and root functions



Sin(x) and Taylor approximations, polynomials of degree 1, 3, 5, 7.



## 2. Linear programming and sensitivity analysis

Let's have a look at the second problem.

We generally solve linear and non-linear programming and transportation problem with Excel Solver. The Excel finds the solution for dual problem and sensitivity analysis too. We compare the results to give the simplex method and Excel Solver.

Let's have a look at an example:

The exercise:

$$x_1 + 2x_2 + x_3 \leq 30$$

$$x_1 + x_3 \leq 20$$

$$2x_1 + x_2 + x_3 \leq 35$$

$$z = 2x_1 + x_2 + 4x_3 = \max$$

The first table with simplex method

I.	$x_1$	$x_2$	$x_3$	b
$u_1$	1	2	1	30
$u_2$	1	0	1	20
$u_3$	2	1	1	35
-z	2	1	4	0

With Excel Solver

Microsoft Excel - Book1

File Edit View Insert Format Tools Data Window Help

File Edit View Insert Format Tools Data Window Help

Arial

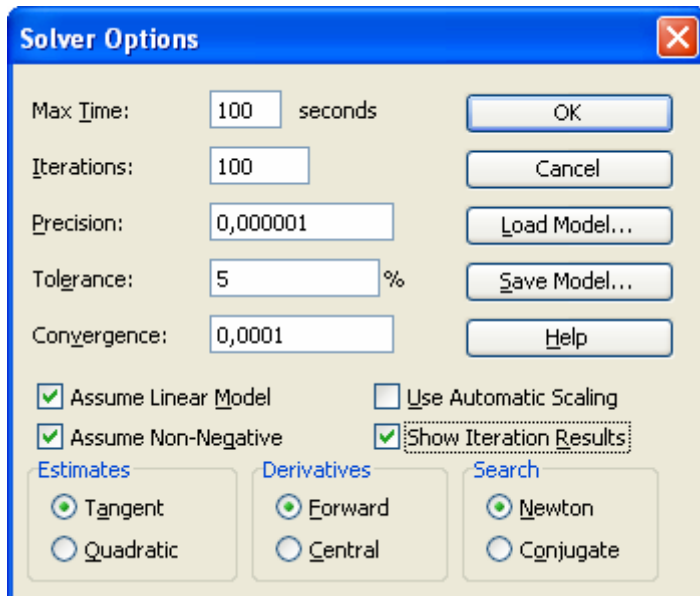
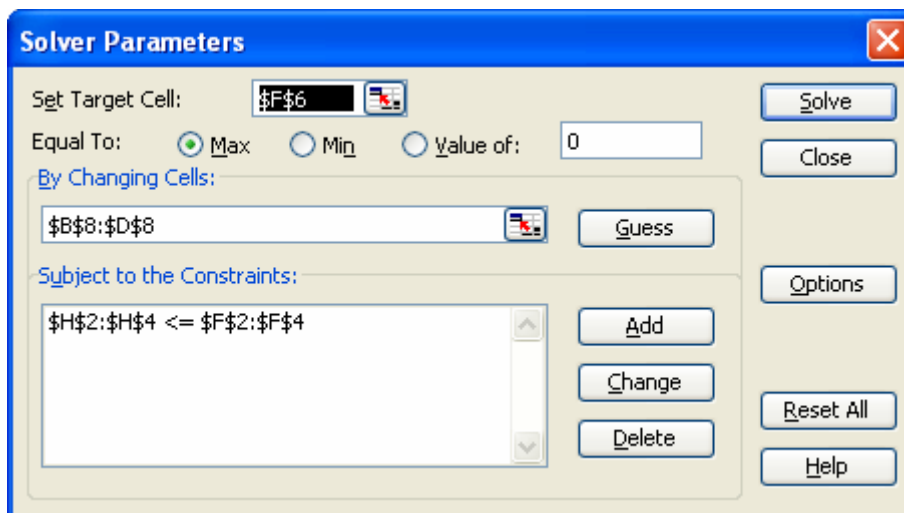
I21

	A	B	C	D	E	F	G	H
1		$x_1$	$x_2$	$x_3$		b		
2	$u_1$	1	2	1		30		
3	$u_2$	1	0	1		20		
4	$u_3$	2	1	1		35		
5								
6	z	2	1	4				
7								
8		1	1	1				
9								
10								
11								

=SumProduct(B2:D2;B\$8:D\$8)  
 = SumProduct(B3:D3;B\$8:D\$8)  
 = SumProduct(B4:D4;B\$8:D\$8)  
 = SumProduct(B6:D6;B8:D8)

We write initial values of the variable to the purple cell, this cells are changing cells. We calculate the value of the left side of the inequalities with the function of sum of the product to the grey cells and we compare it with the values b, so we receive constraints. We calculate content of target cell with the function of sum of the product and we maximalise it.

The settings:



The result table with simplex method

Opt.	$x_1$	$u_1$	$u_2$	b
$x_2$	0	0,5	-0,5	5
$x_3$	1	0	1	20
$u_3$	1	-0,5	-0,5	10
-z	-2	-0,5	-3,5	-85

With Excel Solver:

Microsoft Excel 11.0 Answer Report

Worksheet: [Book1]Sheet1

Report Created: 2008.03.14. 13:28:38

Target Cell (Max)

Cell Name	Original Value	Final Value
\$H\$6 z	7	85

Adjustable Cells

Cell Name	Original Value	Final Value
\$B\$8 x1	1	0
\$C\$8 x2	1	5
\$D\$8 x3	1	20

Constraints

Cell Name	Cell Value	Formula	Status	Slack
\$H\$2 u1	30	\$H\$2<=\$F\$2	Binding	0
\$H\$3 u2	20	\$H\$3<=\$F\$3	Binding	0
\$H\$4 u3	25	\$H\$4<=\$F\$4	Not Binding	10

$$x_1 + 2x_2 + x_3 \leq 30$$

$$x_1 + x_3 \leq 20$$

$$2x_1 + x_2 + x_3 \leq 35$$

$$0 + 2 \cdot 5 + 20 = 30$$

$$0 + 20 = 20$$

$$2 \cdot 0 + 5 + 20 = 25 < 35$$

### Sensitivity analysis

In most problems in practise we need to extend the constraint, or to change objective coefficient. The sensitivity analysis search for the answer, what bounds are these changes move between, in a way that the signchanges would not change in the last row and column of the optimal table. The value of the constraints stay non-negative and the objective coefficient non-pozitiv. We need to solve inequality systems, which solutions can be seen in the following chart. The minimum and maximum values of objective value ( $z(\min)$ ,  $z(\max)$ ) we recieve it for all  $b_i$ ,  $c_i$  ( $i=1, 2, 3$ ).

With traditional method:

Constraint R.H. side:

	min	max	$z(\min)$	$z(\max)$
$b_1$	20	50	80	95
$b_2$	0	30	15	120
$b_3$	25	$\infty$	85	85

Objective coefficient:

	min	max	$z(\min)$	$z(\max)$
$c_1$	$-\infty$	4	85	85
$c_2$	0	8	80	120
$c_3$	2	$\infty$	25	$\infty$

If we solve linear programming problem with Excel Solver the sensitivity analysis happen always. We get the table below. It is important task to compare the two table (with traditional method and with Excel Solver).

With Excel Solver:

The screenshot shows the 'Microsoft Excel 11.0 Sensitivity Report' window. The report is titled 'Microsoft Excel 11.0 Sensitivity Report' and was created on 2008.03.14 at 13:28:38. It is located on 'Worksheet: [Book1]Sheet1'.

**Adjustable Cells**

Cell Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$8 x1	0	-2	2	2	1E+30
\$C\$8 x2	5	0	1	7	1
\$D\$8 x3	20	0	4	1E+30	2

**Constraints**

Cell Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$H\$2 u1	30	0,5	30	20	10
\$H\$3 u2	20	3,5	20	10	20
\$H\$4 u3	25	0	35	1E+30	10

The solving of dual problem are in this Excel chart, in the columns of reduced cost and shadow price. These datas can be seen in the optimal table with problem solving simplex method.

After this let's have a look at the results of sensitivity analysis in dual way. We receive the minimum of the constraint in a way (the result table with traditional method), that we subtract the allowable decrease from the column of constraint R. H. side (the Excel table of constraints – Sensitivity report). The maximum of the constraint (the result table by traditional method) come with similar calculation, we add the column of constraint R. H. side (the Excel table of constraints – Sensitivity report) to allowable increase.

The minimum and the maximum of the objective coefficient get the result table of the adjustable cells, here we need to subtract the allowable increase from the objective coefficient, and to add the objective coefficient to allowable increase

The traditional method receive more information, the table show that we get with this method the values of the objective function, which belong to minimum and maximum of the objective coefficient. We can not get this information with Excel.