# Abstract


This thesis deals with a new approach to tackle binary combinatorial optimization problems. Generally speaking, the idea is to reduce the number of quadratic terms in the objective function to one single, and to analyze the polyhedron which is obtained after a linearization of the quadratic term.

This approach is motivated by several reasons. The original problem with arbitrarily many quadratic terms is NP-hard in general, but although a good polyhedral description with fast separation routines could significantly speed up the optimization when using branch-and-cut algorithms, there is only few information about polyhedral structures so far. Considering the reduced problem with one quadratic term, an efficient optimization is possible if the underlying linear version is tractable. Thus, an efficient separation of the facet defining inequalities is possible in theory. Furthermore, all inequalities which are valid for the reduced problem remain valid for the original problem. In combination, the investigation of the facetial structure of the reduced problem with one quadratic term can yield a better polyhedral description of the original problem.

For a practical usage of such a theoretical approach we consider several specific optimization problems with one quadratic term and analyze their polyhedral structure after linearization. In particular, we consider the minimum spanning forest and the minimum spanning tree, the minimum branching and the minimum arborescence problem, and the minimum assignment and the maximum matching problem. For each of these problems we determine several classes of facet defining inequalities. Furthermore, for the minimum spanning forest and the minimum spanning tree problem, we present a complete description of the corresponding polytopes. For the strongly related minimum branching and the minimum arborescence problem we show on the one hand several similarities, but on the other hand we also have to state that the polyhedral structure becomes much more complicated due to directedness of the edges requiring the degree constraints. When considering the minimum assignment problem with one quadratic term we not only make a conjecture about the complete description but also discover that one single quadratic term can suffice to increase the number of facets from polynomial to exponential. For the polyhedron of the maximum matching problem with one linearized quadratic term we determine the greatest variety of facet classes but however show that they still do not suffice for a complete description. Since most of the derived facet classes are of exponential size, we propose different routines for a polynomial time separation. Our exemplary computational results on the quadratic minimum spanning forest and the quadratic minimum spanning tree problem show the practical relevance of our approach.