Essays in Macroeconomics: Monetary Policy, Interest Rate Spreads, and Financial Markets

Dissertation zur Erlangung des akademischen Grades
Doktor rerum politicarum
der Technischen Universität Dortmund

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Dortmund, März 2017

Contents

1 Introduction **1**

2 Collateralized Lending, Direct Lending, and Interest Rate Spreads **10**

2.1 Introduction 10

2.2 The Model 18

2.2.1 Households 19

2.2.2 Financial Intermediaries 22

2.2.3 Firms 33

2.2.4 Public Sector 39

2.2.5 Equilibrium and Interest Rate Relations 43

2.3 Model Analysis 47

2.3.1 Calibration 49

2.3.2 Crisis Experiment 50

2.3.3 Policy Simulations 53

2.4 Conclusion 57

2.A Appendix 61

2.A.1 Value Function 61

2.A.2 Equilibrium 62

2.A.3 Interest Rate Relations 64

3 Evaluating the Interplay of Term Premia, Monetary Policy, and the Economy in the Euro Area 67

3.1 Introduction . . . 67

3.2 Literature Review . . . 73

3.3 The Model . . . 76

3.3.1 The Macro Part . . . 77

3.3.2 The Term Structure Model . . . 83

3.3.3 Term Structure Premia and the Expectation Hypothesis . . . 89

3.4 Estimation . . . 93

3.4.1 Data . . . 94

3.4.2 The State-Space System . . . 96

3.4.3 Estimation Method . . . 99

3.4.4 Parameter Restrictions and Prior Distributions . . . 104

3.5 Results . . . 108

3.5.1 Policy Rule Coefficients . . . 109

3.5.2 The Model's Dynamic . . . 112

3.6 Conclusion . . . 124

3.A Appendix . . . 127

3.A.1 Parameter Vectors and Matrices . . . 127

3.A.2 Recursive Bond Prices . . . 129

3.A.3 Yield Premia and Return Premia . . . 130

3.A.4 Computation of the $i+1$-period Return Premium . . . 132

3.A.5 The Likelihood Function . . . 134

4 Risk Aversion, Macro Factors, and non-fundamental Components in Euro Area Yield Spreads: A Macro-Financial Analysis 135

4.1 Introduction . . . 135
4.2 Literature Review . . . 141
4.3 The Model . . . 145
4.3.1 The State Equation . . . 146
4.3.2 The Term Structure Model . . . 153
4.3.3 Term Premia . . . 158
4.4 Estimation . . . 160
4.4.1 Data . . . 160
4.4.2 The State-Space System . . . 163
4.4.3 Estimation Method . . . 165
4.4.4 Parameter Restrictions and Prior Distributions . . . 168
4.5 Results . . . 172
4.5.1 Impulse Response Functions . . . 178
4.5.2 Variance Decomposition . . . 180
4.5.3 Historical Shock Decomposition . . . 184
4.6 Conclusion . . . 188
4.A Appendix . . . 191
4.A.1 Parameter Matrices . . . 191
4.A.2 Pricing of Defaultable Bonds . . . 197
4.A.3 Pricing Matrices . . . 199
4.A.4 Computation of the one-period Return Premium . . . 203
4.A.5 Zero-Coupon Yield Data . . . 204

5 Conclusion **208**

Bibliography **210**

List of Tables

2.1 Calibration: Parameters and targeted Moments 48

3.1 Descriptive Data Statistics 96

3.2 Summary of the Prior Distributions 105

3.3 Results: Posterior Distributions (Part I) 110

3.4 Results: Posterior Distributions (Part II) 111

3.5 FEVD of Macroeconomic Variables 121

3.6 FEVD of Bond Yields 123

4.1 Summary of the Prior Distributions 170

4.2 Results: Posterior Distributions (Part I) 173

4.3 Results: Posterior Distributions (Part II) 174

4.4 Results: Posterior Distributions (Part III) 175

4.5 Variance Decomposition 183

List of Figures

2.1 Assets of the Federal Reserve 12

2.2 Composition of the liquidity facilities 13

2.3 IRFs to a capital quality shock for different calibrations of the managerial cost parameter 51

2.4 IRFs to a collateralized lending and a direct lending shock for different values of the managerial cost parameter 53

3.1 IRFs to a risk shock 113

3.2 IRFs to an interest rate shock 115

3.3 IRFs to an output gap shock 116

3.4 IRFs to an inflation shock 118

3.5 IRFS to an inflation target shock 119

4.1 Five-year sovereign bond yield spreads of France, Italy and Spain . . 136

4.2 One-year bond yields of Euro area sovereigns 161

4.3 Five-year bond yields of Euro area sovereigns 162

4.4 Estimated time path of the common risk factor 176

4.5 IRFs to a one-standard-deviation risk aversion shock 178

4.6 IRFs to a one-standard-deviation shock to d_t^{fr} 179

4.7 IRFs to a one-standard-deviation shock to d_t^{es} 180

4.8 IRFs to a one-standard-deviation shock to d_t^{it} 181
4.9 IRFs to a one-standard-deviation common non-fundamental risk factor shock 182
4.10 Historical Shock Decomposition of the five-year yield spread of France 185
4.11 Historical Shock Decomposition of the five-year yield spread of Spain 186
4.12 Historical Shock Decomposition of the five-year yield spread of Italy . 187

Acknowledgment

This thesis was written during my time as a doctoral student at the TU Dortmund University and the Ruhr Graduate School in Economics (RGS Econ). My appreciation goes to a number of people that provided their support or contributed to this thesis with discussions and helpful comments. Without them, this thesis would not exist in the current form.

First, I would like to thank Roland Winkler and Ludger Linnemann for their advice, their valuable support, and their time effort. They provided me with an amazing guidance and have always been interested in the development and success of my work. Their comments and critique helped me to improve my work. I also want to express my thank to Peter Posch for refereeing this thesis and to Andreas Schabert for helpful discussions and comments.

I benefitted a lot from discussions at different international conferences in Cardiff, Istanbul, Kyoto, Mailand, and Taipeh. I want to thank the TU Dortmund University for the financial assistance that allowed me to attend these conferences. Moreover, financial support of the RGS Econ and the SFB 823 of the German Research Foundation (DFG) is gratefully acknowledged.

I am thankful that I had the opportunity to study at the RGS Econ. The well-organized doctoral program taught me valuable skills and guided me through the first year of my doctoral studies. My thanks are also extended to my classmates at

the RGS Econ and my colleagues at TU Dortmund University who have been helpful in a lot of discussions and with their suggestions and comments.

Last but not least, I want to express my gratitude to my parents, Gisela and Reiner, as well as my brother Tim, for their support and their patience during this time. Their encouragement helped me a lot during the past years.

Chapter 1

Introduction

Starting with the collapse of the U.S. subprime mortgage market in the second half of 2007, the global financial crisis led to a disruption of global financial markets and an economic downturn in advanced economies all over the world. To mitigate the crisis and its aftermath, central banks did set up a variety of new unconventional programs and lending facilities. However, since then, turbulent years for the global economy have been passed. The global financial crisis was followed by the Great Recession, a phase marked by a sharp decline in economic activity of economies around the world. At the end of 2009, amplified by the global financial crisis, the European debt crisis hit European economies, inducing periods of low economic growth, instabilities of financial markets and institutions, and rising sovereign bond yield spreads in Euro Area.

The past years raised important and exciting questions for researchers and practitioners alike. How did the unconventional monetary policy programs affect the economy? Assessing the effectiveness of those programs is non-trivial since the counterfactual - how the economy would have evolved in the absence of these programs – is not directly observable, as noted by Bernanke (2012). In addition to the provision

of liquidity, some of these programs also aimed to reduce long-term rates in order to ease financial conditions. The term premium part is one important component in long-term bond rates. Therefore, movements in the term premium should affect aggregate spending and to the extent that movements in term premia do affect the economy, monetary policy should take these movements into account. However, so far, the evidence of how movements in term premia affect the economy is less clear. Moreover, during the European debt crisis, yield spreads rose dramatically. What were the drivers of the observed surge in yield spreads? For the transmission of monetary policy to financial markets, sovereign bond yields play an important role (see ECB, 2012, p. 67). Understanding the drivers of yields and yield spreads is thus important for the conduction of monetary policy.

This thesis consists of three self-contained chapters contributing to different research strands in monetary and financial economics. Chapter 2 analyzes the effects of different unconventional monetary policy actions on interest rate spreads in a stylized macroeconomic model. Chapter 3 evaluates the interplay of term premia movements, monetary policy, and the economy in the Euro area. Chapter 4 investigates the effects of changes in economic fundamentals, risk aversion, and a common non-fundamental risk factor on selected spreads of Euro area sovereign bond yields. The final chapter concludes.

The chapters of the thesis also differ methodologically. In Chapter 2, I apply a numerical simulation of a Dynamic Stochastic General Equilibrium (DSGE) model in order to evaluate the effectiveness of different unconventional monetary policy actions on the economy. DSGE models allow to analyze the transmission mechanisms of policy interventions in a fully dynamic micro-founded framework and help to analyze the effectiveness of these policy interventions by providing the counterfactual policy simulations. Chapter 3 and 4 use different types of macro-finance models of the term

structure of interest rates. Both models belong to the class of no-arbitrage affine term structure models. The canonical no-arbitrage affine term structure model, as proposed by Duffie and Kan (1996), uses a set of latent variables that span the yield curve. By the assumption of the absence of arbitrage opportunities in bond markets, cross-equation restrictions are derived that tie the dynamic of yields over the yield curve closely together. Exploiting all information available over the entire yield curve helps to identify potentially unobservable and observable drivers of movements in yields and yield spreads and to separate the expectation part from the term premium part in long-term bond yields. Based on the class of no-arbitrage affine term structure models, macro-finance models of the term structure of interest rates include, in addition to latent variables, also observable macro variables. Thus, both, observable and unobservable variables span together the yield curve. Therefore, these models offer insight of the economic drivers of movements in yields and help to analyze the evolution of macroeconomic variables, the yield curve, and term premia jointly.

Before the financial crisis, monetary policy was mainly considered as interest rate policy and monetary policy decisionmaking usually consisted of setting an adequate operating target for an overnight interest rate. The target for the short-term interest rate is implemented by adjusting the supply of reserves in open market operations (see e.g. Woodford, 2003, pp. 24). Until the financial crisis, the Fed adjusted reserve balances by purchasing or selling almost exclusively treasury bonds in open market operations (a policy referred to as "Treasuries only"; see Goodfriend, 2011).

Since the onset of the financial crisis in late 2007, central banks of different advanced economies introduced a variety of unconventional policy measures to fight the financial crisis and its aftermath. These unconventional policy measures were implemented to support the liquidity of different kinds of asset markets and of financial

intermediaries, to reduce long-term interest rates, and to improve the stability of financial markets. The programs and their theoretical mechanism through which they might have affected the economy have become quickly into the focus of researchers (see e.g. Cúrdia and Woodford, 2011, Del Negro et al., 2016, Gertler and Karadi, 2011, Gertler and Karadi, 2013, Gertler, et al., 2016, Schabert, 2014, or Schabert, 2015). Gertler and Karadi (2011, 2013) show in a model where financial intermediaries are limited in their ability to arbitrage by an agency problem that the benefits of the provision of public financial intermediation (e.g. direct lending) can be positive. Under this kind of policy the central bank channels funds from households directly to non-financial corporations.

However, under numerous programs which were implemented during the financial crisis (e.g. extended discount window lending, extended open market operations, and more specific programs like the Term Auction Facility), the Fed did not provide public financial intermediation but supplied liquidity by exchanging high-powered money against eligible collaterals with financial institutions like banks and other depository institution. Under these programs, the Fed did not only accepted treasuries as collateral in open market operations but also different types of private assets like e.g. securities of non-financial firms.

Chapter 2 contributes to the literature by studying the effects of changes in the central bank's collateral policy in a model where private financial intermediation is subjected to an agency problem as in Gertler and Karadi (2011). In particular, under this kind of policy, the central bank supplies reserves, i.e. high powered money, against private securities under repurchase agreements (also referred to as collateralized lending, see Schabert, 2015). Thus, in contrast to direct lending, under collateralized lending, the central bank does not intermediate funds directly but exchanges liquidity providing reserves against eligible collaterals under repurchase

agreements. Moreover, I compare the effects of collateralized lending to the effects of direct lending.

For the analysis, I use a macroeconomic model with sticky prices, liquidity providing reserves, a central bank that supplies reserves against eligible collaterals, two different types of financial intermediaries, and financial frictions. Specifically, the agency problem induces a leverage constraint on one type of financial intermediaries. The leverage constraint amplifies the effects of financial disturbances. I evaluate the effectiveness of collateral lending under different settings in which the leverage constraint, induced by the agency problem, affects the dynamic of the economy differently strong.

Chapter 2 shows that both policies can work to reduce interest rate spreads, however, they work through different channels, and the effectiveness of both policies is in general not identical. Thus, in order to evaluate the effects of central bank's asset purchases, it is important to take into account how the asset purchases are conducted. Under direct lending, the central bank sidesteps the agency problem by channeling funds directly to non-financial firms. In contrast, under collateralized lending, the central bank manipulates the liquidity premium incorporated in private assets.

The effectiveness of both policies depends on how strong the leverage constraint affects the dynamic of the model. Specifically, in a setting where the leverage constraint affects the dynamic of the model considerably, the effects of collateralized lending on interest rate spreads are only moderate, while direct lending works well to reduce these spreads. In contrast, if the leverage constraint has only mild effects on the dynamic of the model, collateralized lending is more suitable to reduce interest rate spreads. In other words, if the leverage constraint does strongly affect the dynamic of the model, the effects of a purchase of private assets in open market

operations under repos (i.e. a temporary central bank holding of private assets) on interest rate spreads are at its best only moderate.

Although central banks use the interest rate policy to conduct monetary policy, aggregate demand does not only depend on short-term interest rates but also on long-term interest rates. Indeed, as argued by e.g. Woodford (2005), spending decisions do rather depend on long-term interest rates than on short-term interest rates. The rate of a long-term bond can be decomposed into an expectation and a term premium part. The expectation part consists of the average of the expected sum of current and future short-term interest rates until the bond matures. By shaping the market expectations of the future path of short-term interest rates, central banks are able to influence long-term bond yields. The term premium part compensates risk-averse investors for the risk of holding longer-dated instruments. Thus, if aggregate spending depends, among other macroeconomic factors, on long-term rates, changes in the term premium component of these rates affect economic activity. Therefore, the central bank should take changes in term premia into account for the conduction of monetary policy in order to balance output and inflation. This view is most prominently labeled by the term "practitioner" view (see Rudebusch et al., 2007). Under the practitioner view, a rise in term premia slows down economic activity. Thus, in response to a rise in term premia, the central bank should counterbalance the change in term premia by reducing the short-term interest rate. However, using a variety of different empirical models, the findings of the literature on the effects of term premia movements on the economy are less clear. Often these models are only able to estimate a reduced form relationship (see e.g. Hamilton and Kim, 2002, Favero et al., 2005, or Wright, 2006), restrictions in the models are imposed that prevent effects running from changes of term premia to the economy (see e.g. Ang et al., 2006, or Dewachter et al., 2012) or they specify a particular channel through

which changes in term premia potentially affect the economy (see e.g. Chen et al., 2012). Recently, Ireland (2015) presents a macro-finance model of the term structure that explicitly allows term premia movements to affect output and inflation.

The contribution of Chapter 3 is to explore the effects of movements of term premia in Euro area sovereign bond yields on the economy of the Euro area and to study whether the European Central Bank (ECB) does respond to term premia movements. In order to do so, I use a macro-finance model of the term structure in the lines of Ireland (2015). The model uses restrictions on the contemporaneous relation between the state variables to identify structural macroeconomic and monetary policy shocks.

The model is estimated by Bayesian estimation techniques. Using a Bayesian approach helps to rule out economically non-reasonable regions of the parameter space by employing prior information. This is in particular helpful since the non-linearity of the affine term structure model can produce a multimodal likelihood function, as discussed by Chib and Ergashev (2009). Chapter 3 provides evidence for effects running from term premia movements to the economy and back. Indeed, an exogenous rise in term premia does dampen economic activity. Moreover, the model reveals that during the sample period, the ECB lowered the nominal short-term interest rate in response to a rise in term premia. Thus, in line with the practitioner view, by adjusting the policy rate the ECB counteracted changes in term premia.

At the end of 2009, after the global financial crisis, the economies of Euro area countries were hit by the European debt crisis. Accompanied by a sharp decrease in sovereign debt of several Euro area sovereigns, European sovereign debt markets experienced a dramatic surge in spreads between bond yields of several Euro area sovereigns and the yields on German sovereign bonds. However, not did only the yields of bonds of highly indebted countries rise, but also of those with solid fiscal

fundamentals, suggesting that not only credit risk did account for the surge in yield spreads. Although credit risk seems to be an important determinant of yield spreads (see among others Barrios et al., 2009, or Attinasi et al., 2010), the recent literature stresses the relevance of different common factors driving Euro area yield spreads: risk aversion and common non-fundamental factors. The non-fundamental factors are the part of Euro area yield spreads that cannot be explained by changes in economic fundamentals and country-specific factors (see e.g. Dewachter et al., 2015). They are frequently interpreted as redenomination risk or systemic risk. Risk aversion and redenomination risk are not directly observable. In order to quantify the importance of these components of yield spreads, the way these factors are captured becomes important. In the literature, risk aversion is typically proxied by the interest rate spread of U.S. corporate bonds over U.S. treasury bonds or a volatility index of U.S. stock markets (see e.g. Bernoth et al., 2012, Codogno et al., 2003, or Favero et al., 2010).

Using a multi-country macro-finance model of the term structure, Chapter 4 investigates the drivers of Euro area yield spreads using a measure for risk aversion that is directly derived from Euro area bond markets within in the macro-finance model. In particular, it disentangles the effects of risk aversion and a common non-fundamental factor while accounting for country-specific fiscal variables, the European business cycle, monetary policy and their dynamics and interactions. Sovereign bond yields play an important role for asset pricing and are used as reference rate for key interest rates. Understanding the determinants of yields and yield spreads is important to understand the transmission of monetary policy in a currency union and to detect impairments in the transmission channel, as discussed by the ECB (see ECB, 2014).

The model is applied to yield data of French, German, Italian, and Spanish gov-

ernment bonds. As in Chapter 3, the model is estimated using Bayesian estimation techniques. The results show that although economic fundamentals are the most dominant driver of Euro area yield spreads, the common risk factor accounts for a non-negligible part in Euro area yield spreads. Notably, the contribution of common risk factor shocks to the yield spreads increased from 2012 onwards. Among the economic factors, risk aversion shocks were the most important source for variations in yield spreads. In contrast to the findings of the recent literature (see Dewachter et al., 2015), the contribution of common risk factor shocks to yield spreads is comparably smaller. The results highlight the importance of measuring risk aversion in Euro area bonds markets adequately. Indeed, changes in risk aversion are able to explain most of the spread between 2010 and the Beginning of 2012.

In summary, this thesis contributes to different important topics of the recent literature of financial and monetary economics. Chapter 2 demonstrates that the effects of collateralized lending and direct lending are in general differ. Moreover, under a setting where direct lending is highly effective, collateralized lending does only work modestly to reduce interest rate spreads at its best. Chapter 3 provides evidence that a rise in term premia works to dampen activity and that the ECB does, in turn, respond to movements in term premia. Chapter 4 shows that although a common non-fundamental factor has been important for the dynamic of Euro area yield spreads during the European debt crisis, economic shocks, in particular, risk aversion shocks, were the most dominant drivers of yield spreads.

Chapter 2

Collateralized Lending, Direct Lending, and Interest Rate Spreads

2.1 Introduction

For decades, the main monetary policy tool of the Federal Reserve (Fed) to influence spending, production, employment, and inflation has been the policy rate. However, since the onset of the financial crisis in late 2007, the Fed used a variety of new policy instruments to mitigate the crisis and its aftermath. The Fed's policy actions during the financial crisis can broadly be categorized into three groups: the provision of short-term liquidity to financial institutions like banks and other depository institutions, the provision of liquidity to actors in key financial markets, and the expansion of open market operations to reduce long-term interest rate and the support of the function of credit markets (see Fed, 2015).

In order to support the liquidity of depository institutions, the Fed expanded

discount window lending and introduced the Term Auction Facility (TAF) where depository institutions were able to exchange a broader range of assets for reserves as at the discount window. Moreover, the Fed provided liquidity to primary dealers under the Primary Dealer Credit Facility (PDCF) and the Term Security Lending Facility (TSLF) under which primary dealers were able to receive (fully collateralized) overnight loans as an additional source of liquidity and to trade less liquid assets against Treasuries, respectively. In addition, the Fed did set up liquidity swap lines with foreign central banks to address strains in global dollar funding markets. The second group of policy actions contains programs like the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF), the Commercial Paper Funding Facility (CPFF), the Money Market Investor Funding Facility (MMIFF), and the Term Asset-Backed Securities Loan Facility (TALF) that were aimed to facilitate the provision of liquidity directly to actors in financial markets. Specifically, under the CPFF the Fed provided liquidity to issuers of commercial papers by purchasing commercial papers directly from eligible issuers. The third set of policy actions were implemented by purchasing long-term Treasuries and Mortgage-Backed Securities (MBS) in open market operations (see Fed, 2015).

Figure (2.1) and figure (2.2) depict for illustration purposes the evolution of the asset side of the Fed's balance sheet and the evolution of the composition of the different liquidity facilities set up by the Fed during the financial crisis, respectively. As shown in Figure (2.1), the Fed's balance sheet has grown substantially since the fall of 2008. The newly introduced liquidity facilities did account for a large fraction of the Fed's balance sheet from the fall of 2008 until the fall of 2009. While the size of the different liquidity programs over time did change, the TAF, the CPFF, and the Liquidity Swaps have been the largest facilities among the different liquidity programs, as highlighted in figure (2.2).

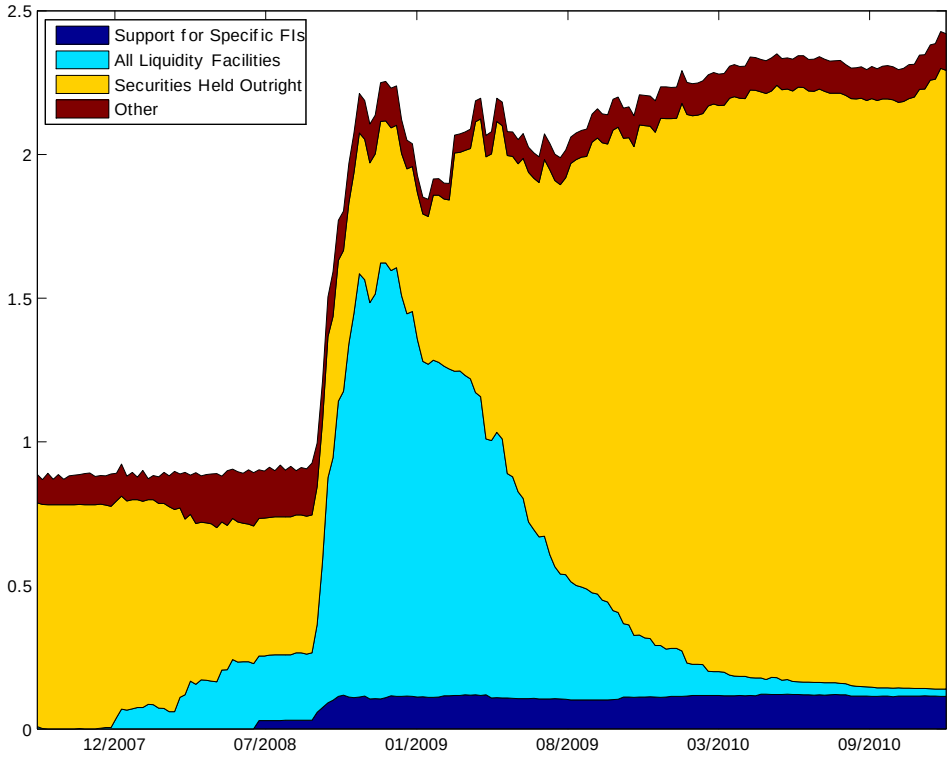


Figure 2.1: Assets of the Federal Reserve (in trillions of dollar). Support for Specific Institutions includes: Maiden Lane LLC; Maiden Lane II LLC; Maiden Lane III LLC; and support to AIG. (Source: Federal Reserve Board.)

The different programs of the Fed and their effects on the economy have gained attention from many researchers since their introduction. Specifically, Gertler and Karadi (2011, 2013) show that if financial intermediaries are limited in their ability to arbitrage, the benefits of the provision of public financial intermediation (e.g. direct lending) can be positive. Under this policy, the central bank substitutes private financial intermediation by central bank financial intermediation. To do so, similar to a private financial intermediary, the central bank issues interest-rate bearing debt in order to fund the purchase of private securities.

However, in open market operations, at the discount window, and under similar

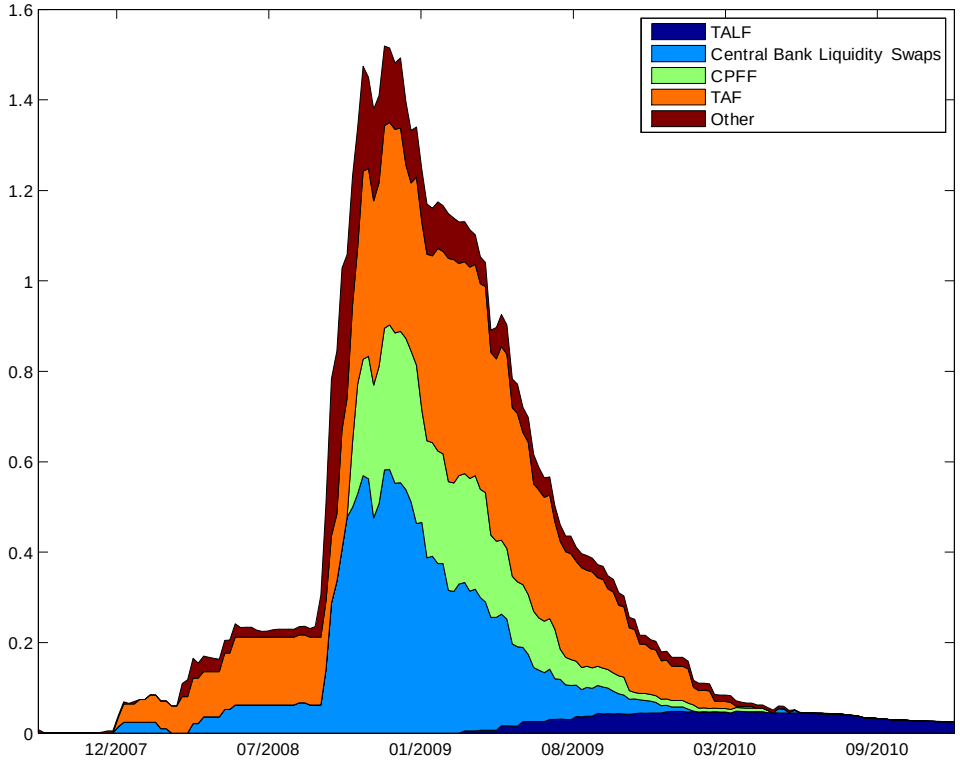


Figure 2.2: Composition of the different liquidity facilities (in trillions of dollar). Other includes: primary credit; secondary credit; seasonal credit; Primary Dealer Credit Facility and Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility. (Source: Federal Reserve.)

but more specific and temporary programs like the TAF, the central bank supplies reserves to financial institutions like banks and other depository institutions against eligible collaterals under repos (e.g. collateralized loans), where reserves are high-powered money. Under this policy, the central bank does not channel funds directly to non-financial firms but provides liquidity to financial intermediaries. In fact, while the effects of direct lending in a model where private financial intermediaries are limited in their ability to arbitrage in financial markets are well studied, the effects of a change in central bank's collateral standards under this kind of friction are less clear.

Contributing to the literature, in this work, I study the effects of the purchase of private securities, i.e. the central bank's exchange of reserves against private assets under repurchase agreements (also referred to as collateralized lending, see Schabert, 2015), where reserves are high-powered money that provide liquidity, and compare these effects to the effects of direct lending in a model where private financial intermediation is subjected to an agency problem.

In order to analyze the effects of collateralized lending, I use a macroeconomic model with sticky prices, reserves that provide a liquidity service, financial intermediaries, and financial frictions. In particular, I extend the Gertler and Karadi (2011) model by a retail banking sector with a reserve demand and a central bank that supplies reserves against eligible collaterals as in Schabert (2014, 2015). Precisely, private financial intermediation consists of two different kinds of banks: retail banks and wholesale banks. Both types of banks are specialized and have different business models. Retail banks collect funds by supplying deposits to households. They use these funds either for lending in the wholesale market or for lending to non-financial firms. Retail banks are not specialized in funding non-financial firms: They lack specific knowledge and may face regulatory constraints. Therefore, they are supposed to be at disadvantage to wholesale banks in funding non-financial firms. This disadvantage is captured by managerial costs. Due to the managerial costs, arbitrage by retail banks does not have to eliminate interest rate spreads. Retail banks also participate in open market operations with the central bank. Similar to Gertler et al. (2016), wholesale banks are modeled as highly leveraged financial institutions that are specialized in the funding of non-financial firms.[1] They rely on short-term interbank

[1] Gertler et al. (2016) extend the Gertler and Karadi (2011) model by a retail and wholesale banking sector in order to study the role of retail and wholesale banks during the financial crisis. The wholesale banking sector in this model is similar to the one in Gertler et al. (2016). In contrast to the here presented work, the retail banks in Gertler et al. (2016) neither have a reserve demand nor participate in open market operations.

market loans and their own net worth to finance the purchase of non-financial firm securities. Based on Gertler and Karadi (2011), a financial friction is introduced by an agency problem between wholesale banks and its creditors. This agency problem leads to an endogenously determined maximum leverage ratio for wholesale banks. In the presence of a financial turmoil, it is this balance sheet constraint that amplifies the economic downturn. Indeed, while retail markets have been relatively stable during the financial crisis, wholesale markets, where banks lend to each other, have been strongly disrupted.[2] By supposing that households rely on demand deposits in order to purchase consumption goods, as in Bredemeier et al. (2015), a demand for reserves is introduced. Specifically, due to a withdrawal of demand deposits before asset markets open, retail banks are required to hold reserves to fulfill their obligations. As in Schabert (2014, 2015), reserves are not supplied in an unbounded way but only against eligible collaterals.

I evaluate the effects of both policy actions on the economy. For this purpose, I consider different scenarios in which the leverage constraint, induced by the agency problem, affects the dynamic of the model differently strong. First, the dynamic of the model in response to a financial disturbance is evaluated. As demonstrated, the amplification of financial disturbances by the leverage constraint does depend, among other factors, on the degree of the elasticity of retail banks' demand for private securities. Next, the effectiveness of collateralized lending and direct lending with respect to different calibrations for the managerial costs are discussed.

The results are as follows: Purchases of private securities either conducted under collateralized lending or under direct central bank lending both potentially work to stimulate output and to reduce the excess return on capital by stimulating the

[2] See Gertler et al. (2016) for a discussion of the effects of the financial crisis on retail and wholesale banking.

demand for non-financial firm securities. However, both policies work through different channels. By direct lending to non-financial firms, the central bank provides central bank financial intermediation. Due to the presence of the agency problem and the managerial costs, central bank financial intermediation does not substitute private intermediation one-to-one. Thus, the increase in central bank's demand for non-financial firm securities due adds up partly to the aggregate demand for non-financial firm securities which raises the price of private securities and reduces credit spreads in capital markets. While central bank financial intermediation directly affects the total demand for non-financial firm securities, under collateralized lending, the central bank manipulates the liquidity premium incorporated in private firm securities. Specifically, by influencing the retail banks' valuation of non-financial firm securities, the central bank is able to stimulate retail banks to increase their demand for non-financial firm securities.

Among other factors, the magnitude of the change in the retails banks' demand for non-financial firm securities in response to a change in financial conditions determines how strong the leverage constraint, induced by the agency problem, affects the dynamic of the economy. The less elastic the retail banks' demand for non-financial firm securities is, the more effective is direct lending and the less effective is collateralized lending. Thus, if retail banks' demand for non-financial firm securities is less elastic, the provision of liquidity has at its best only moderate effects on the economy. In contrast, if retail banks are able to absorb a substantial fraction of capital when financial conditions change, direct central bank lending is less impactful, while the effectiveness of collateralized lending increases. However, in this case, the leverage constraint, induced by the agency problem, is less relevant for the dynamic of the economy. Hence, also the amplification of financial disturbances is less strong and the effects of financial disturbances on the economy become more similar to the

effects of the same disturbance in a more standard New-Keynesian model without the agency problem.

In other words, to the extent that the Fed's liquidity programs worked to eased financial conditions and reduced interest rate spreads,[3] the model is not able to explain these effects of these programs if the agency problem works to affect the dynamic of the economy substantially.

This paper relates to the literature on unconventional monetary policy, in particular, to those papers studying central bank's purchases of private securities as e.g. in Cúrdia and Woodford (2011), Del Negro et al. (2016), Gertler and Karadi (2011, 2013), Gertler, et al. (2016), and Schabert (2014, 2015). While all of these papers consider the purchase of private securities, the mechanism and the conduction of assets purchases differ across these papers. My work shares the closest focus with Gertler and Karadi (2011, 2013) and Schabert (2014, 2015). In particular, Gertler and Karadi (2011, 2013) find in a model where private financial intermediaries ability to arbitrage is restricted that the provision of public financial intermediation has strong effects on the economy if financial markets are disrupted. Schabert (2014) analyzes the effects of asset purchases in open market operations under repos in a model where money serves as a mean of payment and borrowing between households is constrained by a collateral constraint. Schabert (2015) examines optimal monetary policy in a stylized macro model where money is a mean of payment and only supplied in open market operations against eligible collaterals. He shows that under a money supply rationing policy, the central bank is able to improve welfare in the short-run and in the long-run by purchasing private securities when the economy is hit by cost-push shocks. Moreover, the introduction of the reserve demand is based

[3]Indeed evidence by e.g. Campbell et al. (2011), Carpenter et al. (2014), Christensen et al. (2014), Duygan-Bump et al. (2013), Fleming (2012), Mc Andrews et al. (2015), and Wu (2011) suggests that the Fed's liquidity programs worked to ease financial conditions and to reduce interest rate spreads.

on Bredemeier et al. (2015).

The remainder of the paper is structured as follows. In section (2.2) the model is presented and the equilibrium properties are discussed. Section (2.3) discusses the calibration of the model, examines the dynamic of the model in response to a financial disturbance, and evaluates the effects of collateralized central bank lending and direct central bank lending on the economy. Section (2.4) concludes.

2.2 The Model

This section discusses the model. The presented framework is a monetary DSGE model with nominal rigidities based on Calvo (1983) and financial frictions as in Gertler and Karadi (2011). Financial intermediaries consist of two types of banks: retail banks and wholesale banks. Both types of banks are specialized and have different business models. Retail banks collect deposits and use these funds either to lend to wholesale banks or to purchases non-financial firm securities. Wholesale banks are specialized in investing in non-financial firm securities. For funding their investments, they rely on interbank market credit and own net worth. In addition, following Bredemeier et al. (2015), a reserve demand of retail banks is introduced by a premature withdrawal of a fraction of demand deposits by households. As in Schabert (2014, 2015), the central bank does supply reserves only against eligible collaterals.

The timing in the period is as follows. At the beginning of the period, after aggregate shocks are realized but before asset markets open, open market operations are conducted, where the central bank supplies money either outright or under repurchase agreements (repos) against eligible collateral. After open market operations are conducted, production takes place and final good markets open. Households

withdraw a fraction of their demand deposits to finance consumption and repurchase agreements are settled. In order to be able to satisfy the premature withdrawal of demand deposits, retail banks need to hold reserves. Then, repos are settled. Finally, asset markets open. Wholesale banks and retail banks receive payoffs from their investments. Retail banks issue deposits and use their funds to lend to wholesale banks in the interbank market and to purchase non-financial firm securities. Wholesale banks use their funds to invest in non-financial firm securities.

2.2.1 Households

The economy is populated by a continuum of infinitely-lived and identical households of mass one with identical endowments. As in Gertler and Karadi (2011, 2013), it is supposed that each household consists of workers and wholesale bankers. Both types of household members share a perfect consumption insurance. In the following, the fraction of household members being workers is denoted by $1-\tau$ and the fraction of household members being wholesale bankers is denoted by τ. Workers supply labor to non-financial firms. Bankers run wholesale banks. Both types of household members switch occupations over time. Each period, a random fraction $1-\sigma$ of bankers retire and become workers. Hence, the average survival time of a wholesale banker is given by $1/(1-\sigma)$. In order to keep the ratio of bankers to workers constant over time, each period the same number of workers change their occupation and become wholesale bankers. Due to an agency problem that limits wholesale bankers ability to raise funds, wholesale banks retain all earnings until the period they retire. When a wholesale banker retires, retained earnings are paid to its household. In turn, each new banker starts operating its wholesale bank with a start up transfer from the household.

Households' preferences are defined over consumption c_t and labor l_t. The ex-

pected discounted lifetime utility of the representative household is given by

$$u_t = E_t \sum_{i=0}^{\infty} \beta^t \left[\ln \left(c_{t+i} - h\bar{c}_{t+i-1} \right) - \frac{\chi}{1+\eta} l_{t+i}^{1+\eta} \right], \tag{2.1}$$

where $\bar{c}_t$ indicates external habit formation (as in Bredemeier et al., 2015), $\beta \in (0,1)$ is the discount factor and $h \in [0,1)$ is the habit formation parameter.

The representative household enters the period t with holdings of demand deposits and term deposits. It receives additional income from labor. Moreover, any profits from retail banks, retiring wholesale bankers, and non-financial firms are transferred to the representative household. It uses its income for consumption or saving. Households can store wealth by holding demand deposits and term deposits at retail banks. Term deposits are one-period contracts that deliver a safe nominal return. Demand deposits also pay a safe interest rate, but in addition to saving services, they supply a liquidity function. In contrast to term deposits, demand deposits can be withdrawn before maturity. The budget constraint of the household is given by

$$P_t c_t + D_t / \left(1 + r_t^d\right) + B_t^h / \left(1 + r_t\right) + P_t \tau_t = W_t l_t + P_t \Pi_t^h + D_{t-1} + B_{t-1}^h, \tag{2.2}$$

where P_t is a price index, D_t are demand deposits, B_t^h are term deposits, τ_t are lump sum transfers from the government, W_t is the nominal wage, and Π_t^h contains profits from non-financial firms and retail banks and the cumulated earnings from retiring wholesale bankers.

A liquidity demand of households is introduced as in Bredemeier et al. (2015). Following Bredemeier et al. (2015), the household needs to pay cash in order to purchase consumption goods. While the household could hold money to satisfy its liquidity needs, demand deposits offer the same function. Thus, in order to purchase

consumption goods, the following goods market constraint has to hold:

$$P_t c_t \leq \mu D_{t-1}, \tag{2.3}$$

where the parameter $\mu \in [0, 1]$ is the fraction of withdrawn demand deposits. For $\mu < 1$, the goods market constraint takes into account that, on average, household's period deposit holdings may be higher than its period consumption. As in Bredemeier et al. (2015), the specific form of this type of cash-in-advance constraint can be motivated as follows: Consider an idiosyncratic shock with a bounded support that hit each household before the goods market opens and shift their valuation of consumption. In an efficient allocation, each household holds exactly the amount of money that it needs in order to purchase the desired amount of the consumption good after the valuation shock realized. However, since households need to decide about cash holding before they know the realization of the idiosyncratic valuation shock, their cash holding would lead to an ex-post inefficient allocation. Instead, households can rely on interest-bearing demand deposits to satisfy their liquidity needs. Bredemeier et al. (2015) show that a constrained efficient allocation can be implemented if all households hold an amount of deposits that accords to the maximum of the valuation shock and then, after the valuation shock hits and goods market open, withdraw the required amount. Therefore, the parameter μ refers to the mean fraction of withdrawn deposits.

Household's maximization problem is to choose its holding of demand deposits, consumption, labor supply, and term deposits in order to maximize its expected discounted life-time utility subject to the budget constraint (2.2) and the goods

market constraint (2.3). The first order conditions are given by

$$d_t \;:\; \frac{1}{1+r_t^d} = \beta E_t \frac{\lambda_{t+1} + \mu \nu_{t+1}^{hh}}{\lambda_t \pi_{t+1}}, \tag{2.4}$$

$$c_t \;:\; u_{c,t} = \lambda_t + \nu_t^{hh}, \tag{2.5}$$

$$l_t \;:\; \chi l_t^{\sigma_n} = \lambda_t w_t, \tag{2.6}$$

$$b_t^h \;:\; \frac{1}{1+r_t} = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{-1}, \tag{2.7}$$

where $\pi_t = P_t/P_{t-1}$ is the inflation rate, $d_t = D_t/P_t$, $c_t = C_t/P_t$, $w_t = W_t/P_t$, $b_t^h = B_t^h/P_t$,

$$u_{c,t} = \frac{1}{c_t - h c_{t-1}},$$

is the marginal utility, and ν_t^{hh} is the multiplier on the goods market constraint (2.3). Moreover, a transversality condition and the following complementary slackness conditions have to hold:

$$c_t \leq \mu d_{t-1} \pi_t^{-1},\; 0 \leq \nu_t^{hh},\; 0 \leq \nu_t^{hh} \left(\mu d_{t-1} \pi_t^{-1} - c_t \right). \tag{2.8}$$

2.2.2 Financial Intermediaries

The banking system consists of two different types of financial intermediaries: retail banks and wholesale banks. Both types of financial intermediaries have different business models. Retail banks offer demand deposits and term deposits to households, hold government bonds and reserves, invest in non-financial firm securities, and lend in the interbank market. They hold reserves to satisfy their need for liquidity which is induced by households' withdrawal of demand deposits before assets markets open. For simplification purposes, reserves are supplied by the central bank only in open market operations (and not e.g. at the discount window against a higher

penalty rate). The wholesale banks use funds from retail banks and their own equity - or net worth - in order to acquire non-financial firm securities. Wholesale banks are specialized in non-financial firm security management. Therefore, following Gertler et al. (2016), I suppose that wholesale banks have a cost advantage over retail firms in the management of non-financial firm securities. Thus, the rate of return on non-financial firm securities is larger for wholesale banks than for retail banks for any positive amount of non-financial firm securities hold by retail banks. If financial markets work frictionless, retail banks do not invest in non-financial firm securities. However, if the ability of wholesale banks to raise funds in the interbank market is constrained, then also retail banks will acquire non-financial firm securities.

Retail banks

Set-up Retail banks invest in government bonds, reserves, and capital (via the purchase of non-financial firm securities) and provide funds to wholesale banks. In order to raise funds, the retail banks rely on household saving. There exists a continuum of perfectly competitive retail banks $i \in [0, 1]$. The structure of the retail bank is based on Bredemeier et al. (2015). The introduction of managerial costs follows Gertler et al. (2016).

Consider a retail bank i in period t. It enters the period with holdings of treasuries $B_{i,t-1}$, interbank market loans $B^{ib}_{i,t-1}$, money $M_{i,t-1}$, and non-financial firm securities $S_{i,t-1}$. Non-financial firms use the funds acquired by issuing non-financial firm securities to purchase new capital. As in Gertler and Karadi (2011, 2013), each unit of the security is a state-contingent claim to all future returns of the financed unit of capital. Let z_t denote the income flow to the wholesale bank from a security that is financing one unit of capital, δ the depreciation rate of one unit of capital, Q_t the market price of physical capital in terms of the final good, and ξ_t a random

capital quality disturbance, then the value of retail bank i's holding of non-financial firm securities at the beginning of period t equals $(z_t + (1-\delta)\, Q_t)\, \xi_t S_{i,t-1}$.

At the beginning of period t, the retail bank can adjust its stock of money by receiving money injections from the central bank. Following Schabert (2014, 2015), money is supplied by the central bank in open market operations. Open market operations are conducted either outright or under repurchase agreements against eligible collateral at the monetary policy rate r_t^m. Specifically, in "normal" times, the central bank only accepts treasuries as a collateral in open market operations. Hence, in order to be able to participate in open market operations, the retail bank needs to hold treasuries. However, in the event of a disruption of financial markets, the central bank may also decide to accept non-financial firm securities as a collateral in open market operations. Thus, the retail bank may also receive reserves by exchanging non-financial firm securities for reserves. The money injections $I_{i,t}$ of the retail bank are given by

$$I_{i,t} = \frac{\Delta B_{i,t}^C}{1 + r_t^m} + \frac{\Delta S_{i,t}^C}{1 + r_t^m}, \tag{2.9}$$

where $\Delta B_{i,t}^C \leq \kappa_t^b B_{i,t-1}$ and $\Delta S_{i,t}^C \leq \kappa_t^s \left(z_t + (1-\delta)\, Q_t\right) \xi_t S_{i,t-1}$. The policy parameters κ_t^b and κ_t^s are set by the central bank. In the steady state, $\kappa_t^s = 0$ holds.

As in Bredemeier et al. (2015), the retail bank's money demand is induced by households withdrawing demand deposits before maturity. Specifically, after open market operations are conducted, the goods market opens and households withdraw a fraction of deposits. Thus, the retail bank faces the following liquidity constraint:

$$M_{i,t-1} + I_{i,t} \geq \mu D_{i,t-1}, \tag{2.10}$$

where μ denotes the fraction of deposits that is withdrawn.

Before asset markets open, repurchase agreements are settled. The retail bank

repurchases those government bonds that were traded under repurchase agreements, $B_{i,t}^R = (1 + r_t^m) M_{i,t}^R$. Then, its holding of government bonds in period t, after repurchase agreements are settled, reads $B_{i,t-1} - \Delta B_{i,t}^C + (1 + r_t^m) M_{i,t}^R$. In the case of a financial crisis, the central bank may choose to also accept firm claims in open market operations ($\kappa_t^s > 0$). The central bank only exchanges reserves against non-financial firm securities under repos (e.g. collateralized lending). In this case, the financial intermediary repurchases firm claims $\Delta S_{i,t}^C = (1 + r_t^m) M_{i,t}^S$ and its holding of firm claims is given by $Q_t S_{i,t-1}$. Thus, its money holding is given by $M_{i,t-1} - (1 + r_t^m) \left(M_{i,t}^R + M_{i,t}^S\right) + I_{i,t}$. When asset markets open, the retail bank receives demand deposits $D_{i,t}$ at the price $1/\left(1 + r_t^d\right)$ and term deposits $B_{i,t}^h$ at the price $1/(1 + r_t)$. In the asset market, the retail bank uses its funds either to invest in government bonds $B_{i,t}$ at the price $1/\left(1 + r_t^b\right)$, for lending in the interbank market $B_{i,t}^{ib}$, or to directly acquire capital by purchasing state-contingent non-financial firm securities $S_{i,t}$ at the market price of capital Q_t. Loans to the wholesale banks are one-period risk-free debt contracts that pay the market interest rate r_t^{ib}. In contrast to wholesale banks, retail banks are not specialized in capital management and therefore not as efficient as wholesale banks in the screening and monitoring of investment projects. Moreover, they also may face regulatory constraints. As in Gertler et al. (2016), these ideas are captured by the introduction of managerial costs. Specifically, the real management cost function is given by $\kappa(s_{i,t}) = c_m \eta^{-1} s_{i,t}^{\eta}$, where $c_m > 0$, $\eta \geq 0$, and $s_{i,t} = S_{i,t}/P_t$. Due to the managerial costs, retail banks' arbitrage does not eliminate interest rate spreads in the non-financial firm security market. If $\eta = 0$, the retail banks do not face management costs. In contrast, if $\eta \to \infty$, the retail banks do not invest in non-financial firm securities. The nominal

profits of the retail bank i are then given by

$$\begin{aligned} P_t\Pi^{rb}_{i,t} &= D_{i,t}/\left(1+r^d_t\right) - D_{i,t-1} + B^h_{i,t}/\left(1+r_t\right) - B^h_{i,t-1} + \left(1+r^{ib}_{t-1}\right)B^{ib}_{i,t-1} - B^{ib}_{i,t} \\ &\quad + B_{i,t-1} - B_{i,t}/\left(1+r^b_t\right) + \left(z_t + (1-\delta)\,Q_t\right)\xi_t S_{t-1} - Q_t S_{i,t} - P_t\kappa\left(s_{i,t}\right) \\ &\quad - M_{i,t} + M_{i,t-1} - r^m_t I_{i,t}. \end{aligned}$$

Maximization Problem The retail bank's maximization problem is to choose demand deposits, non-financial firm securities, loans to wholesale banks, term deposits, treasuries, money holding, and money injections to maximizes the sum of expected discounted profits,

$$E_t\beta\sum_{l=0}^{\infty}\frac{\lambda_{t+l+1}}{\lambda_{t+l}}\Pi^{rb}_{i,t+l},$$

subject to the money supply constraints (2.9) and the liquidity constraint (2.10). The first order conditions with respect to demand deposits, non-financial firm securities, loans to wholesale banks, term deposits, treasuries, money holdings, and money injections read:

$$d_{i,t} \;:\; \frac{1}{1+r^d_t} = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1+\mu\nu^{dc}_{i,t+1}}{\pi_{t+1}}, \tag{2.11}$$

$$s_{i,t} \;:\; 1 = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\left[\left(1+r^k_{rb,t+1}\right)\left(1+\kappa^s_{t+1}\nu^{mc}_{i,t+1}\right)\right]\pi^{-1}_{t+1}, \tag{2.12}$$

$$b^{ib}_{i,t} \;:\; \frac{1}{1+r^{ib}_t} = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\pi^{-1}_{t+1}, \tag{2.13}$$

$$b^h_{i,t} \;:\; \frac{1}{1+r_t} = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\pi^{-1}_{t+1}, \tag{2.14}$$

$$b_{i,t} \;:\; \frac{1}{1+r^b_t} = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1+\kappa^b_{t+1}\nu^{mc}_{i,t+1}}{\pi_{t+1}}, \tag{2.15}$$

$$m_{i,t} \;:\; 1 = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1+\nu^{dc}_{i,t+1}}{\pi_{t+1}}, \tag{2.16}$$

$$i_{i,t} \;:\; 1+\nu^{dc}_{i,t} = \left(1+r^m_t\right)\left(1+\nu^{mc}_{i,t}\right), \tag{2.17}$$

where

$$\frac{1+r^k_{rb,t+1}}{\pi_{t+1}} = \frac{z_{t+1}+(1-\delta)\,Q_{t+1}}{Q_t+\kappa'\left(s_{i,t}\right)}\xi_{t+1}, \tag{2.18}$$

$d_{i,t} = D_{i,t}/P_t$, $b^{ib}_{i,t} = B^{ib}_{i,t}/P_t$, $b_{i,t} = B_{i,t}/P_t$, $m_{i,t} = M_{i,t}/P_t$, $i_{i,t} = I_{i,t}/P_t$, and $\nu^{mc}_{i,t}$ and $\nu^{dc}_{i,t}$ are the multipliers on the money supply constraints (2.9) and on the liquidity constraint (2.10), respectively. Moreover, the following complementary slackness conditions have to hold:

$$\left(1+r^m_t\right) i_{i,t} \leq \kappa^b_t b_{i,t-1}\pi^{-1}_t + \kappa^s_t\left(z_t+(1-\delta)\,Q_t\right)\xi_t s_{i,t-1}\pi^{-1}_t,\ \nu^{mc}_{i,t} \geq 0,$$
$$\nu^{mc}_{i,t}\left(\kappa^s_t\left(z_t+(1-\delta)\,Q_t\right)\xi_t s_{i,t-1}\pi^{-1}_t + \kappa^b_t b_{i,t-1}\pi^{-1}_t - \left(1+r^m_t\right) i_{i,t}\right) = 0,$$

and

$$\mu d_{i,t-1}\pi^{-1}_t \leq m_{i,t-1}\pi^{-1}_t + i_{i,t},\ \nu^{dc}_{i,t} \geq 0,\ \nu^{dc}_{i,t}\left(m_{i,t-1}\pi^{-1}_t + i_{i,t} - \mu d_{i,t-1}\pi^{-1}_t\right) = 0.$$

Aggregation The aggregate holding of non-financial firm securities by retail banks $s_{rb,t}$, the aggregate retail banks' demand for government bonds $b_{rb,t}$, and the aggregate supply of interbank market lending $b^{ib}_{rb,t}$ are given by $s_{rb,t} = \int_0^1 s_{i,t}di$, $b_{rb,t} = \int_0^1 b_{i,t}di$, and $b^{ib}_{rb,t} = \int_0^1 b^{ib}_{i,t}di$, respectively. In the following, the aggregation of the remaining retail banks' variables is facilitated by recognizing that in equilibrium all retail banks will behave identical. Thus, the index i can be dropped. Combing eq. (2.4) from the households and eq. (2.11), establishes an equilibrium relation between ν^{hh}_t and ν^{dc}_t,

$$E_t\beta\frac{\lambda_{t+1}+\mu\nu^{hh}_{t+1}}{\lambda_t}\pi^{-1}_{t+1} = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1+\mu\nu^{dc}_{t+1}}{\pi_{t+1}}.$$

If $\nu^{hh}_t = \lambda_t\nu^{dc}_t$ holds, this condition is satisfied (see Bredemeier et al., 2015). Finally, since retail banks do behave identical and do not face idiosyncratic risks, in equi-

librium each retail bank will hold the same amount of non-financial financial firm securities. Hence, the aggregate management costs are simply given by $\kappa\left(s_{rb,t}\right)$.

Wholesale banks

Set-up Wholesale banks are specialized in capital management. As in Gertler et al. (2016), wholesale banks do not face management costs. In contrast to retail banks, they solely rely on collecting funds in the interbank market to finance the purchase of non-financial firm securities. However, their ability to raise funds in the interbank market is limited by an agency problem. The wholesale banking sector is based on Gertler and Karadi (2011) and Gertler et al. (2016).

There exists a continuum of wholesale banks. Consider an wholesale bank j that uses net worth $N_{j,t}$ and funds obtained from the interbank market $B_{j,t}^{ib}$ to acquire state-contingent non-financial firm securities $S_{j,t}$ at the price Q_t. Its balance sheet is given by

$$B_{j,t}^{ib} + N_{j,t} = Q_t S_{j,t}. \tag{2.19}$$

Non-financial firms use their funds to purchase new capital (by purchasing non-financial firm securities). The wholesale banks' real rate of return on non-financial firm securities is given by

$$\frac{1 + r_{t+1}^k}{\pi_{t+1}} = \frac{z_{t+1} + (1-\delta) Q_{t+1}}{Q_t} \xi_{t+1}. \tag{2.20}$$

Net worth is accumulated through retained profits. It is given by the differences between wholesale bank's gross return on non-financial firm securities and the costs of borrowing in the interbank market,

$$N_{j,t} = \left(1 + r_t^k\right) Q_{t-1} S_{j,t-1} - \left(1 + r_{t-1}^{ib}\right) B_{j,t-1}^{ib}. \tag{2.21}$$

If the wholesale bank is constrained in its ability to raise funds in the interbank market, it is optimal for the bank to retain all earnings to overcome the financing constraint. In order to prevent wholesale banks from being able to fund all of its investments by their own net worth, it is assumed that wholesale banks have a finite expected lifetime. Specifically, the wholesale bank j in period t will survive with probability σ until the next period and retires with probability $1-\sigma$. The probability of surviving does not depend on the history of the wholesale bank. Hence, each period the fraction $1-\sigma$ of wholesale banks exit the banking sector. A fraction of the similar size of household members become wholesale bankers, keeping the overall number of wholesale banks constant. A newly founded bank receives a one-time endowment from its household.

A wholesale bank retains all earnings until the point in time when it exits the banking sector and pays out all retained earnings to its household. Its objective is thus to maximize the expected value of discounted terminal wealth,

$$V_{j,t} = E_t \sum_{\tau=t+1}^{\infty} (1-\sigma)\, \sigma^{\tau-t-1} \Lambda_{t,\tau} n_{j,\tau},$$

where $n_{j,t} = N_{j,t}/P_t$, $\Lambda_{t,t+1} = \beta\lambda_{t+1}/\lambda_t$ is a stochastic discount factor, σ is the probability to survive until the next period, and $1-\sigma$ is the probability that a banker exits the sector and becomes a worker.

As in Gertler and Karadi (2011), am agency problem that limits the wholesale bank's ability to finance lending is introduced. After the wholesale bank received all funds, it may choose to divert a fraction of its assets and transfer this fraction to its household. If a wholesale bank decides to fraud, its lender will force it into bankruptcy. However, wholesale bank's creditors are only able to reclaim the remaining fraction, but not the total quantity of funds. Specifically, let θ denote the fraction

of non-financial firm securities the bank can divert, then the divertable amount is given by $\theta Q_t s_{j,t}$, where $s_{j,t} = S_{j,t}/P_t$. It is supposed that diverting assets takes time. A wholesale bank is not able to divert assets immediately. Therefore, the wholesale bank must decide at the end of period t, before knowing the state in period $t+1$, whether it wants to fraud. Retail banks are only willing to supply funds to a wholesale bank, as long as they can be sure that the wholesale bank will not divert. This is the case if the gain from fraud does not exceed the franchise value $V_{j,t}$ of the wholesale bank. Hence, in order to be able to raise funds in the interbank market, the following incentive constraint has to be satisfied

$$V_{j,t} \geq \theta Q_t s_{j,t}. \tag{2.22}$$

Using the balance sheet (2.19) in the net worth accumulation equation (2.21) and dividing by the price level P_t gives the law of motion of wholesale bank j's real net worth,

$$n_{j,t} = \left(r_t^k - r_{t-1}^{ib}\right) Q_{t-1} s_{j,t-1} \pi_t^{-1} + \left(1 + r_{t-1}^{ib}\right) n_{j,t-1} \pi_t^{-1}. \tag{2.23}$$

Maximization Problem Let $V_t\left(s_{j,t}, n_{j,t}\right)$ denote the maximized franchise value of a wholesale bank j given $s_{j,t}$ and $n_{j,t}$ at the end of period t. Then, the franchise value of a bank at the end of period $t-1$ can be expressed by the following Bellman equation:

$$V_{t-1}\left(s_{j,t-1}, n_{j,t-1}\right) = E_{t-1}\Lambda_{t-1,t}\left[\left(1-\sigma\right) n_{j,t} + \sigma \max_{s_{j,t}} V_t\left(s_{j,t}, n_{j,t}\right)\right]. \tag{2.24}$$

The term in the square brackets in eq. (2.24) reflects that the wholesale bank may retire with probability $1-\sigma$ and survives with probability σ. The wholesale bank's problem in period t is to choose $s_{j,t}$ to maximize the franchise value $V_t\left(s_{j,t}, n_{j,t}\right)$

subject to the incentive constraint eq. (2.22) and the law of motion of real net worth eq. (2.23). I follow Gertler and Karadi (2011) and conjecture that the value function $V_{j,t}$ is a linear function of the balance sheet components,

$$V_t\left(s_{j,t}, n_{j,t}\right) = \mu_t^k Q_t s_{j,t} + \mu_t^n n_{j,t}. \tag{2.25}$$

Appendix (2.A.1) shows that the coefficients of the value function are given by

$$\mu_t^k = E_t \Lambda_{t,t+1} \Xi_{t+1} \left(r_{t+1}^k - r_t^{ib}\right) \pi_{t+1}^{-1}, \tag{2.26}$$

$$\mu_t^n = E_t \Lambda_{t,t+1} \Xi_{t+1} \left(1 + r_t^{ib}\right) \pi_{t+1}^{-1}, \tag{2.27}$$

where μ_t^k is the excess returns on non-financial firm securities, μ_t^n is the saving in costs for funds by holding another unit of net, and Ξ_{t+1} is the shadow value of one more unit of net worth at $t+1$ averaged across existing and continuing states,

$$\Xi_t = (1-\sigma) + \sigma \left[\mu_t^k \phi_t + \mu_t^n\right], \tag{2.28}$$

where

$$\phi_t = \frac{Q s_{j,t}}{n_{j,t}} \tag{2.29}$$

is the leverage ratio. Due to the symmetricity of the structure of wholesale banks' maximization problem, the leverage ratio is independent of bank-specific characteristics.

Insert the conjectured solution (2.25) into the Bellman equation (2.24), then the wholesale bank's maximization problem is to chose $s_{j,t}$ to maximize this objective subject to the incentive constraint (2.22). The first order condition of the wholesale

bank's maximization problem is given by

$$\mu_t^k = \frac{\nu_t^k}{1+\nu_t^k}\theta, \tag{2.30}$$

where ν_t^k is the Lagrange multiplier on the incentive constraint (2.22). Moreover, the complementary slackness conditions,

$$V_{j,t} \geq \theta Q_t s_{j,t}, \nu_t^k \geq 0, \nu_t^k \left(V_{j,t} - \theta Q_t s_{j,t}\right) = 0,$$

have to be satisfied. Eq. (2.30) shows that the marginal benefit of holding non-financial firm securities μ_t^k equals its marginal costs, that is, the tightening in the incentive constraint (2.22) from holding one more unit of non-financial firm securities. If the incentive constraint does not bind $\left(\nu_t^k = 0\right)$, then, from eq. (2.26), the excess returns on non-financial firm securities are zero. Thus, if the incentive constraint is not binding, wholesale banks extend lending to non-financial firms until the rate of return on non-financial firm securities is equal to the marginal costs of borrowing in the interbank market.

Under a binding incentive constraint (2.22), $\nu_t^k > 0$, a restriction on the ratio of bank's net worth to assets is imposed. The maximum asset to net worth ratio is given by

$$\phi_t = \frac{\mu_t^n}{\theta - \mu_t^k}. \tag{2.31}$$

Aggregation Since the maximum leverage ratio is independent of firm-specific characteristics, the relation between the aggregate net worth and the aggregate asset portfolio of wholesale banks is obtained by simply summing up over eq. (2.29):

$$Q_t s_{w,t} = \phi_t n_t. \tag{2.32}$$

where $s_{w,t} = \int_0^1 s_{j,t} dj$. Eq. (2.32) illustrates that a deterioration in aggregate net worth generates fluctuations in the aggregate asset portfolio of the wholesale banks. The wholesale banks' demand for interbank market lending is given by $b^{ib}_{w,t} = \int_0^1 b^{ib}_{j,t} dj$. Moreover, the aggregate net worth consists of net worth of existing wholesale banks n^e_t and the net worth of entering wholesale banks n^n_t:

$$n_t = n^e_t + n^n_t.$$

Summing up across the wholesale banks' net worth (2.23) yields the aggregate net worth of existing financial intermediaries,

$$n^e_t = \sigma \left[\left(r^k_t - r^{ib}_{t-1} \right) Q_{t-1} s_{w,t-1} + \left(1 + r^{ib}_{t-1} \right) n_{t-1} \right] \pi_t^{-1},$$

where σ is the fraction of bankers that survived from $t-1$ to t. New wholesale banks receive a start-up transfer from their household. It is assumed, that this transfer is a fraction $\omega / (1-\sigma)$ of the value of total assets that the exiting wholesale banks held in period $t-1$, $(1-\sigma) Q_t s_{w,t-1} \pi_t^{-1}$. The net worth of newly founded wholesale banks is thus given by

$$n^n_t = \omega Q_t s_{w,t-1}.$$

Hence, aggregate net worth evolves according to

$$n_t = \sigma \left[\left(r^k_t - r^{ib}_{t-1} \right) Q_{t-1} s_{w,t-1} + \left(1 + r^{ib}_{t-1} \right) n_{t-1} \right] \pi_t^{-1} + \omega Q_{t-1} s_{w,t-1} \pi_t^{-1}. \quad (2.33)$$

2.2.3 Firms

The economy's production sector consists of four types of firms: intermediate goods producing firms, monopolistically competitive retailers, final good producers, and

capital good producers. Competitive intermediate firms produce intermediate goods using capital and labor as inputs. Retail firms repack these intermediate goods and sell them in a monopolistically competitive market to a representative final goods producing firm. Capital producers produce new capital which is sold to intermediate firms. They use solely the final good as input and are subjected to adjustment costs.

Intermediate firms

A continuum of perfectly competitive intermediate firms produce differentiated goods that are sold to retailers. Each intermediate firm j, $j \in [0,1]$, operates under a constant returns to scale production technology,

$$y_{j,t} = (\xi_t k_{j,t-1})^{\alpha} l_{j,t}^{1-\alpha}, \tag{2.34}$$

where $k_{j,t-1}$ and $l_{j,t}$ are firm j's capital and labor input, respectively. In each period t, a firm j hires labor from households taking as given the market wage rate, uses existing capital $k_{j,t-1}$ as input, and purchases new capital $x_{j,t}$ for production in period $t+1$. The variable ξ_t denotes a capital quality shock. As in Gertler and Karadi (2011, 2013), this shock is meant to capture economic obsolescence and is a simple source of exogenous variation in the capital stock. Thus, the effective quantity of capital used for production in period t is given by $\xi_t k_{j,t-1}$. At the end of the period, firm j is left with the depreciated capital stock $(1-\delta)\,\xi_t k_{j,t-1}$. It purchases new capital $x_{j,t}$ from capital producers for production in period $t+1$. Hence, intermediate firm $j's$ capital stock evolves according to

$$k_{j,t} = (1-\delta)\,\xi_t k_{j,t-1} + x_{j,t}, \tag{2.35}$$

Firms are required to collect funds to finance the purchase of new capital. In

order to finance new capital, an intermediate firm issues state-contingent securities $s_{j,t}$ to financial intermediaries. As in Gertler and Karadi (2011, 2013), each unit of new capital is financed by issuing a claim to all future returns of this unit. One unit of capital pays the state-contingent net nominal interest rate r^k_{t+1} in period $t+1$. In equilibrium, the price of one claim equals the price of one unit of capital Q_t. The supply of non-financial firm securities of firm j at the end of period t is given by

$$s_{j,t} = k_{j,t}. \tag{2.36}$$

Denote the price of the intermediate goods by P^m_t, then, following Gertler and Karadi (2011, 2013), the intermediate firms' labor demand is given by

$$w_t = (1-a)\frac{P^m_t}{P_t}\frac{y_{j,t}}{l_{j,t}}. \tag{2.37}$$

Since intermediate firms operate under a constant return to scale technology in perfect completive markets, they earn zero profits in equilibrium. The zero profit condition implies that intermediate firms' gross return on capital which is paid out to the holders of non-financial firm securities is given by

$$\frac{1+r^k_t}{\pi_t} = \frac{z_t + Q_t(1-\delta)}{Q_{t-1}}\xi_t, \tag{2.38}$$

where $z_t \equiv a\left(P^m_t/P_t\right)\left(y_{j,t}/\left(\xi_t k_{j,t-1}\right)\right)$ is the real marginal product of capital.

Retail firms

Monopolistically competitive retail firms purchase intermediate goods and simply repack them into retail goods. A retail firm uses one unit of intermediate goods to produce one unit of the differentiated retail good. Thus, the production function of

a retail firm f, $f \in [0,1]$ is given by $y_{f,t} = y_{j,t}$. The real marginal costs of a retail firm are given by $mc_t = P_t^m / P_t$. Nominal frictions are introduced as in Calvo (1983). In each period a retail firm f is only allowed to adjust the nominal price of its good $y_{f,t}$ with exogenous probability $1 - \phi$. With probability ϕ the firm is not allowed to adjust the price. Following Yun (1996), if the firm cannot adjust its price, it indexes the price to the steady state inflation, $P_{f,t} = \pi P_{f,t-1}$. Consider the problem of a retail firm that is allowed to reset its price. The adjusted price is denoted by $\tilde{P}_{f,t}$. The firm seeks to maximize its expected discounted sum of profits by choosing $\tilde{P}_{f,t}$ subject to the demand function for its good (which is given by eq. (2.41), as described below in more detail). The maximization problem reads:

$$\max_{\tilde{P}_{f,t}} E_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t,t+s} \left\{ \pi^s \frac{\tilde{P}_{f,t}}{P_{t+s}} - P_{t+s} mc_{t+s} \right\} y_{f,t+s},$$
$$\text{s.t.} \quad y_{f,t+s} = \left(\frac{\pi^s \tilde{P}_{f,t}}{P_{t+s}} \right)^{-\varepsilon} y_{t+s}.$$

The first order condition for this problem is given by

$$\frac{\tilde{P}_{f,t}}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t,t+s} (\pi^s)^{-\varepsilon} mc_{t+s} (P_{t+s}/P_t)^{\varepsilon} y_{t+s}}{E_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t,t+s} (\pi^s)^{1-\varepsilon} (P_{t+s}/P_t)^{\varepsilon - 1} y_{t+s}}.$$

Since all re-optimizing firms face the same demand function and technology, their problem is symmetric. Thus, each of these firms chooses the same price $\tilde{P}_{f,t}$. Define $\tilde{x}_t \equiv \tilde{P}_{f,t}/P_t$, then the first order condition can be expressed recursively by

$$\tilde{x}_t = \frac{\varepsilon}{\varepsilon - 1} \frac{x_{1,t}}{x_{2,t}}, \tag{2.39}$$

where $x_{1,t} \equiv \lambda_t mc_t y_t + \beta E_t \phi \pi_{t+1}^{\varepsilon} x_{1,t+1}$ and $x_{2,t} \equiv \lambda_t y_t + \beta E_t \pi_{t+1}^{\varepsilon - 1} \phi x_{2,t+1}$. Finally, by using the price index and the law of large numbers, the aggregate price level evolves

according to

$$1 = (1-\phi)\left(\tilde{x}_t\right)^{1-\varepsilon} + \phi\left(\frac{\pi_t}{\pi}\right)^{\varepsilon-1}. \tag{2.40}$$

Final goods producers

The final output is produced by a representative final goods firm using solely retail goods as input. Its production technology is given by the following CES function

$$y_t = \left[\int_0^1 y_{f,t}^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\epsilon-1}}.$$

The final goods firm operates in a competitive market. Hence, it takes the final goods price P_t and the retail goods price as given. Final goods firm's real profits are given by $y_t - P_t^{-1}\int_0^1 P_{f,t}y_{f,t}df$. Its demand for input $y_{f,t}$ is derived from the first order condition of profit maximization:

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\varepsilon} y_t. \tag{2.41}$$

Moreover, since there is perfect competition in the final goods sector and the production technology is linear homogeneous, the final goods firm earns zero profits. Thus, the aggregate price level is given by

$$P_t = \left[\int_0^1 P_{f,t}^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}}.$$

Capital Goods Producers

Capital producing firms operate in a perfectly competitive environment. They produce new capital x_t using the final good as the only input. The new capital is sold

to intermediate firms at the price Q_t. All profits are distributed to the shareholders. The objective of the representative capital goods producing firm is to maximize profits by choosing x_t,

$$\max_{I_t} E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left[Q_\tau x_\tau - \left(1 + f\left(\frac{x_\tau}{x_{\tau-1}} \right) \right) \right],$$

where $f(\cdot)$ are convex adjustment costs. From the first order condition, the price of capital goods is

$$Q_t = 1 + f\left(\frac{x_t}{x_{t-1}} \right) + \frac{x_t}{x_{t-1}} f'\left(\frac{x_t}{x_{t-1}} \right) - E_t \Lambda_{t,t+1} \left(\frac{x_{t+1}}{x_t} \right)^2 f'\left(\frac{x_{t+1}}{x_t} \right). \tag{2.42}$$

Aggregation

The aggregate factor demand of intermediate firms is derived by integrating over eq. (2.37) and (2.38),

$$l_t = (1-a)\, mc_t \frac{y_t}{w_t} \Delta_t, \tag{2.43}$$

$$k_{t-1} = \alpha mc_t \left[\left(1 + r_t^k\right) Q_{t-1} - Q_t \left(1-\delta\right) \xi_t \right]^{-1} y_t, \tag{2.44}$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_{f,t}}{P_t} \right)^{-\varepsilon} df$ is the price dispersion. The price dispersion can be expressed in recursive form as in Yun (1996),

$$\Delta_t = (1-\phi)\, \tilde{x}_t^{-\varepsilon} + \phi \left(\frac{\pi_t}{\pi} \right)^{\varepsilon} \Delta_{t-1}. \tag{2.45}$$

The aggregate output is obtained by integrating over eq. (2.34) and (2.41),

$$\Delta_t y_t = \left(\xi_t k_{t-1} \right)^{\alpha} l_t^{1-\alpha}. \tag{2.46}$$

Finally, by integrating over eq. (2.35) the aggregate capital stock evolves according to

$$k_t = (1-\delta)\,\xi_t k_{t-1} + x_t. \tag{2.47}$$

2.2.4 Public Sector

The public sector consists of the government and the central bank. The government issues one-period nominal risk-free government bonds at a constant growth rate. As in Schabert (2015), it is assumed that the government's bond supply B_t is given by the following rule

$$B_t = \Gamma B_{t-1}, \tag{2.48}$$

where $\Gamma > \beta$ and $B_{-1} > 0$. The government purchases goods G_t, receives seigniorage revenues $P_t\tau_t^{cb}$ from the central bank, and pays lump sum transfers $P_t\tau_t$ to the household to balance it budget,

$$B_{t-1} + P_t\tau_t + G_t = B_t/\left(1+r_t^b\right) + P_t\tau_t^{cb}.$$

The central bank supplies money in open market operations outright, $M_t = \int_0^1 M_{i,t}^B di$, and under repurchase agreements against treasuries, $M_t^R = \int_0^1 M_{i,t}^R di$, and against non-financial firm securities, $M_t^S = \int_0^1 M_{i,t}^S di$. As in Hörmann and Schabert (2015), the central bank transfers its earnings from interest rate payments to the government and reinvests its wealth only in government bonds. Thus, at the beginning of a period, the central bank holds government bonds $B_{cb,t-1}$ and its liabilities, given by the stock of outstanding money, are equal to M_{t-1}. It then receives treasuries ΔB_t^C and non-financial firm securities ΔS_t^C in exchange for newly issued reserves,

$$I_t = M_t - M_{t-1} + M_t^R + M_t^S. \tag{2.49}$$

Repos are settled before asset markets open. After repos are settled, the amount of outstanding reserves is reduced by $M_t^R + M_t^S$, the holding of government bonds is reduced by B_t^R, and the holding of non-financial firm securities by ΔS_t^C. At the end of the period, if the central bank does not provide public financial intermediation (see the discussion of the different central bank policies below), its budget constraint reads

$$P_t \tau_t^s + \frac{B_{cb,t}}{1 + r_t^b} = B_{cb,t-1} + r_t^m \left(M_t - M_{t-1} + M_t^R + M_t^S \right) + M_t - M_{t-1},$$

where I used $\Delta B_t^C + \Delta S_t^C = (1 + r_t^m) I_t$. It is assumed that all central bank profits from interest rate earnings are transferred to the government and that maturing government debt is rolled over, $P_t \tau_t^s = B_{cb,t} - B_{cb,t} / \left(1 + r_t^b\right) + r_t^m \left(M_t - M_{t-1} + M_t^R + M_t^S \right)$. Thus, the central banks holding government bonds evolves according to $M_t - M_{t-1} = B_{cb,t} - B_{cb,t-1}$. Moreover, assuming that the initial values of outstanding money and government bonds are equal, $M_{-1} = B_{cb,-1}$, the central bank's end-of-period holding of government equals

$$B_{cb,t} = M_t.$$

The central bank has different conventional and unconventional instruments for the conduction of monetary policy. First, it sets the policy rate r_t^m. Specifically, I assume that the central bank sets the policy rate according to the following Taylor-type interest rate rule (see Taylor, 1992)

$$1 + r_t^m = \max \left\{ \left(1 + r_{t-1}^m\right)^{\rho_R} \left[(1 + r^m) \left(\frac{\pi_t}{\pi} \right)^{\rho_\pi} \left(\frac{y_t}{y} \right)^{\rho_y} \right]^{1-\rho_R}, 1 \right\}, \tag{2.50}$$

where the parameters $\rho_R \in [0, 1]$, $\rho_\pi \geq 0$, $\rho_y \geq 0$, r^m denotes the steady state value of the policy rate, π the inflation target, and y the steady state value of output.

The policy rule respects the zero lower bound. Moreover, the central bank sets the inflation target π and chooses the ratio of money supplied outright and under repos in exchange for government bonds Ω_t,

$$\Omega_t M_t = M_t^R. \tag{2.51}$$

As in Schabert (2014, 2015), Ω_t is set sufficiently large enough to guarantee the non-negativity of Injections. In addition, the central bank can choose the fraction of randomly selected treasuries in open market operations, $\kappa_t^b \in (0, 1]$.

The central bank in this economy has two unconventional monetary policy instruments. It may either provide liquidity by purchasing non-financial firm securities under repos (e.g. collateralized lending), or it may choose to channel funds directly to non-financial firms (e.g. direct lending). Under collateralized lending, the central bank changes its collateral policy. Precisely, following Schabert (2014), by setting $\kappa_t^s \in [0, 1]$, the central bank decides about the fraction of randomly selected non-financial firm securities that are eligible in open market operations.

Under direct central bank lending, the central bank provides public financial intermediation. Suppose that the central bank intermediates the value $Q_t S_{d,t}$ of non-financial firm securities.

As in Gertler and Karadi (2011, 2013), direct central bank lending is financed by the issuance of one-period risk-free debt $B_{cb,t}^{ib}$. The central bank issues $B_{cb,t}^{ib}$ to retail banks at a risk-free nominal interest rate and lend the acquired funds to non-financial firms.[4] Thus, the equation $Q_t S_{d,t} = B_{cb,t}^{ib}$ always holds. The central bank debt is effectively government debt. Interbank market lending and central bank's short-term debt are both one-period risk-free nominal contracts. Hence,

[4]Since retail banks are not constrained in their ability to collect funds, this is equivalent to an economy where the central bank issues debt to the households.

both assets pay in equilibrium the same market rate. A role for central bank's intermediation arises from the frictions of private financial intermediation. The wholesale banks' ability to collect funds is restricted by the agency. Due to the management costs, retail banks' arbitrage does not eliminate interest rate spreads. Hence, public financial intermediation does not substitute private financial intermediation one-to-one. Following Gertler et al. (2016), public financial intermediation does not come without costs. The central bank faces managerial costs for direct lending to non-financial firms, $\kappa^{cb}(s_{d,t}) = c_{m,cb}(\eta)^{-1} s_{d,t}^{\eta}$, where $c_{m,cb} > c_m$. Thus, it is supposed that the central bank is less efficient in funding non-financial firms than the private financial intermediaries.[5] All profits or losses from the purchase of non-financial firm securities are transferred to the government, $P_t\tau_t^{d,cb} = \left(1 + r_t^k\right) Q_{t-1}S_{d,t-1} + B_{cb,t}^{ib} - P_t\kappa^{cb}(s_{d,t}) - Q_tS_{d,t} - \left(1 + r_{t-1}^{ib}\right) B_{cb,t-1}^{ib}$. Thus, the total transfers from the central bank to the government are given by

$$P_t\tau_t^{cb} = P_t\tau_t^s + P_t\tau_t^{d,cb}.$$

In the following, in order to evaluate and compare solely the effects of the purchase of non-financial firm securities under both policies, the total amount of money supplied by the central bank is left unchanged under the respective policy measure. This implies that if the central bank decides to accept non-financial firm securities in open market operations, the change in κ_t^s is accompanied by a neutralizing adjustment of κ_t^b and an adjustment of the ratio of money supplied outright and under repos in exchange for treasuries. Precisely, I impose that a rise in the amount of reserves against non-financial firm securities supplied under repos M_t^S reduces the

[5] In contrast, the acquisition of non-financial firm securities in open market operations does not involve efficiency costs, since there is no structural difference to the purchase of treasuries in open market operations, as discussed in Schabert (2015).

amount of reserves supplied against treasuries under repos M_t^R proportionally, keeping the ratio of reserves supplied outright M_t and under repos $M_t^R + M_t^S$ untouched from the policy action.

2.2.5 Equilibrium and Interest Rate Relations

This section presents the equilibrium and discusses the relation of the different interest rates.

Equilibrium

In equilibrium all markets are clear. Thus, the aggregate supply of funds in the interbank market equals the aggregate demand,

$$b_{rb,t}^{ib} = b_{w,t}^{ib} + b_{cb,t}^{ib}, \tag{2.52}$$

where I used that interbank market borrowing by wholesale banks and one-period central bank debt the are perfect substitutes for retail banks to simplify the notation. Market clearing in the market for non-financial firm securities implies

$$s_t = k_t, \tag{2.53}$$

where s_t is the aggregate demand for non-financial firm securities given by

$$s_t = s_{rb,t} + s_{w,t} + s_{d,t}. \tag{2.54}$$

In equilibrium, the total supply of government bonds equals aggregate demand. Government bonds can be either held by retail banks or by the central bank,

$$b_t = b_{rb,t} + b_{cb,t}. \tag{2.55}$$

The aggregate resource constraint is given by

$$y_t = c_t + x_t + f\left(\frac{x_t}{x_{t-1}}\right) x_t + g_t + \kappa\left(s_{rb,t}\right) + \kappa^{cb}\left(s_{d,t}\right), \tag{2.56}$$

where $g_t = G_t/P_t$. I restrict the attention to equilibria where the money supply constraint (2.9), the the liquidity constraint (2.10), and the incentive constraint (2.22) always bind (i.e. $\nu_t^{mc} > 0$, $\nu_t^{dc} > 0$, and $\nu_t^{k} > 0$). The formal definition of the rational expectations equilibrium (REE) is given in Appendix (2.A.2).

Interest Rates and Interest Rate Differentials

The derivations for the presented equations in this section are shown in Appendix (2.A.3). Following Bredemeier et al. (2015), the multiplier on the money supply constraint (2.16) can be expressed by

$$\nu_t^{mc} = \frac{1 + r_t^{IS}}{1 + r_t^{m}} - 1, \tag{2.57}$$

where

$$\frac{1}{1 + r_t^{IS}} = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} \pi_{t+1}^{-1}, \tag{2.58}$$

is the marginal rate of intertemporal substitution (see Schabert, 2015). Hence, the money supply constraint (2.9) binds if the central bank sets the policy rate below the marginal rate of intertemporal substitution. If the money supply constraint is

binding, the marginal rate of intertemporal substitution differs from the policy rate by a liquidity premium.

The multiplier of the goods market constraint can be written as

$$\nu_t^{hh} = u_{c,t}\left(1 - \frac{1}{1 + r_t^{IS}}\right). \tag{2.59}$$

Hence, as long as the rate of intertemporal substitution is larger than one, the goods market constraint of households binds. Then, from $\nu_t^{dc} = \nu_t^{hh}/\lambda_t$ also the liquidity constraint of retail banks does bind. Notably, this holds independently of the policy rate.

The equilibrium interest rate on government bonds can be expressed by

$$\frac{1}{1 + r_t^b} = E_t\left(\frac{\left(1 - \kappa_{t+1}^b\right)\left(1 + r_{t+1}^m\right) + \kappa_{t+1}^b\left(1 + r_{t+1}^{IS}\right)}{\left(1 + r_{t+1}^{IS}\right)\left(1 + r_{t+1}^m\right)}\pi_{t+1}^{-1}\right). \tag{2.60}$$

Eq. (2.60) shows that the interest rate on government bonds is a function of the expected policy rate and the expected marginal rate of intertemporal substitution where the policy parameter $\kappa_t^b \in (0, 1]$ is a weighting factor. If all government bonds are eligible in open market operations ($\kappa_t^b = 1$), the interest rate on government bonds equals the expected policy rate up to first order.

In equilibrium, the interest rate on lending in the interbank market r_t^{ib} follows the expected marginal rate of intertemporal substitution up to first order,

$$\frac{1}{1 + r_t^{ib}} = E_t\frac{1}{1 + r_{t+1}^{IS}}. \tag{2.61}$$

Next, I turn to the retail banks' return on capital. The first order condition for retail banks investment in capital (i.e. non-financial firm securities) is given by eq. (2.12). If the central bank does not accept non-financial firm securities as collaterals

in open market operations (i.e. $\kappa_t^s = 0$), then eq. (2.12) can be expressed by

$$1 = E_t \frac{1 + r_{rb,t+1}^k}{1 + r_{t+1}^{IS}}. \tag{2.62}$$

If $\kappa_t^s > 0$, then it follows that $1 > E_t \left(\left(1 + r_{rb,t+1}^k\right) / \left(1 + r_{t+1}^{IS}\right) \right)$. By setting κ_t^s, the central bank is able to manipulate the retail banks' marginal valuation of non-financial firm securities. Moreover, combining eq. (2.12) and eq. (2.13) shows that the following interest rate condition between the rates of return of non-financial firm securities and interbank market loan has to hold in equilibrium:

$$E_t \frac{\left(1 + r_{rb,t+1}^k\right)\left(1 + \kappa_{t+1}^s \nu_{i,t+1}^{mc}\right) - \left(1 + r_t^{ib}\right)}{1 + r_{t+1}^{IS}} = 0. \tag{2.63}$$

Finally, consider the rate of return on capital r_{t+1}^k and the interbank market rate r_t^{ib}. Wholesale banks borrow funds in the interbank market to invest in non-financial firm securities. A binding incentive constraint (2.22), $\nu_t^k > 0$, induces a spread between the expected discounted rate of return on capital r_{t+1}^k and the riskless interbank market rate r_t^{ib}. Specifically, by combining eq. (2.26) and (2.30), the expected discounted spread is given by

$$E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \Xi_{t+1} \frac{r_{t+1}^k - r_t^{ib}}{\pi_{t+1}} = \frac{\nu_t^k}{1 + \nu_t^k} \theta.$$

Under a binding incentive constraint, the spread is positive. Due to the agency problem, the wholesale banks' ability to obtain funds is limited, preventing wholesale banks effectively from expanding their purchase of non-financial firm securities until the marginal cost of borrowing hits the marginal return from lending. As a result, the capital costs for non-financial firms are larger and investment and output are lower compared to the case where the incentive constraint is not binding. Thus, the

spread between the rate of return on capital r^k_{t+1} and the riskless interbank market rate r^{ib}_t is an important measure of the inefficiency of financial markets caused by the agency problem. Moreover, due to the retail banks' management costs, the rate of return on capital r^k_t exceeds the retail banks' rate of return on non-financial firm securities $r^k_{rb,t}$. If the retail banks do not face managerial costs ($\kappa(s_{rb,t}) = 0 \ \forall s_{rb,t}$), then all capital is funded by retail banks (given that the wholesale banks' incentive constraint is binding). Equivalently, if wholesale banks' incentive constraint (2.22) is not binding, all capital is funded by wholesale banks (given that the retail banks face management costs).

2.3 Model Analysis

This section presents the calibration of the model's parameters and evaluates the dynamic of the model. The first part discusses the choice for the parameter values. Then, the dynamic of the model is evaluated. First, the response of the model to a financial disturbance is considered. The financial turmoil is caused by an unanticipated decline in capital quality. The decline in capital quality leads to a drop in the value of non-financial firm securities, inducing a tightening of the leverage constraint of wholesale banks. The amplification of the financial disturbance on the economy is discussed for different calibrations of the managerial cost parameter η. Then, the effects of collateralized central bank lending and direct central bank lending on the economy are evaluated. Moreover, the impact of different calibrations of the managerial cost parameter η on the effectiveness of collateralized lending and direct lending on the economy are discussed.

Table 2.1: Calibration: Parameters and targeted Moments

Households		
β	0.990	Discount Rate
σ_n	0.276	Inverse Frisch Elasticity of Labor Supply
n	0.333	Steady State Working Time
Intermediate Good Firms		
α	0.330	Capital Share
δ	0.025	Depreciation Rate
Capital Producing Firms		
γ	1.728	Inv. Elasticity of Invest. to the Price of Capital
Retail Firms		
ε	4.167	Elasticity of Substitution between Goods
ρ	0.779	Probability of keeping Prices constant
Retail Banks		
$S_{rb}/\left(S_{rb}+S_w\right)$	0.500	Share of Firm Securities
Wholesale Banks		
σ	0.900	Survival Rate of Bankers
ϕ	20.00	Steady State Leverage Ratio
$\left(r^k - r^{ib}\right)\pi^{-1}$	0.016	Real Steady State Excess Return p.a.
Government		
ρ^π	1.500	Inflation Coefficient of Interest Rate Rule
ρ^y	0.050	Output Gap Coefficient of Interest Rate Rule
ρ^r	0.900	Smoothing Coefficient of Interest Rate Rule
r^m	0.015	Steady State Policy Rate
Ω	1.000	Fraction of Money supplied under Repos
π	1.011	Steady State Inflation Rate
Γ	1.011	Growth Rate of Treasuries
G/Y	0.200	Proportion of Government Expenditures
κ^b	1.000	Fraction of Treasuries eligible in OMOs
κ^s	0.000	Fraction of Firm Securities eligible in OMOs

2.3.1 Calibration

Overall there are 22 parameters to calibrate. Table (2.1) lists the calibrated parameter values and the targeted steady state values of some variables. Following Gertler and Karadi (2013), households' discount rate is set to $\beta = 0.99$, the inverse Frisch elasticity of labor supply is set to $\sigma^n = 0.276$, the relative utility weight of labor supply is picked to hit a steady state working time of $n = 1/3$, the capital share is set equal to $\alpha = 0.33$, the depreciation rate is set to $\delta = 0.025$, the inverse elasticity of investment to the price of capital is set to $\gamma = 1.728$, the elasticity of substitution is set to $\varepsilon = 4.167$, and the probability of keeping prices constant is set to $\rho = 0.779$. For the calibration of the parameters belonging to financial intermediaries, I follow closely Gertler et al. (2016). The scaling parameter c_m in the management cost function is set to target the steady-state share of non-financial firm securities hold by retail banks. This share is set equal to 0.5 which corresponds to the pre-crisis ratio of capital held by retail banks and wholesale banks in 2007 (see Gertler et al., 2016). The managerial cost parameter of retail banks η is important for the amplification of financial disturbances in the economy. It determines the degree of retail banks' ability to adjust capital holding if financial conditions change. As a result, as demonstrated below, it controls how strong the leverage constraint, induced by the agency problem, affects the dynamic of the model.

Lower values of η imply that retail banks are able to absorb larger amounts of capital during a financial turmoil, mitigating the effects of the tightening of wholesale banks' leverage constraint on the total demand for non-financial firm securities. For the crisis experiment and the evaluation of the effectiveness of collateralized lending and direct lending, different values of this parameter are considered and discussed. As in Gertler et al. (2016), the survival rate of wholesale banks is set to $\sigma = 0.9$.

The proportional transfers to new entering wholesale banks ω and the fraction of divertable capital θ are picked to lead to a steady state leverage ratio of 20 and an annual spread between the real return on capital and the real interbank rates equal to 1.6 percentage points. The picked values are based on the targets of Gertler et al. (2016). The policy rate and inflation rate are set to $r^m = 1.0635^{1/4} - 1$ and $\pi = 1.045^{1/4}$ which accords to their respective average of the sample 1964.Q2 - 2008.Q2 (see Bredemeier et al., 2015). Moreover, following Bredemeier et al. (2015), the growth rate of treasury bonds is set equal to the inflation target $\Gamma = \pi$.[6] The fraction of money supplied against treasuries under repos is set, for simplicity, to $\Omega = 1$. In the steady state, the central bank only accepts treasuries in open market operations, thus $\kappa^s = 0$. This policy corresponds to the "treasury only" doctrine of the Federal Reserve (Fed) before the financial crisis (see Goodfriend, 2011). Further, treasuries are fully eligible in the steady state, $\kappa^b = 1$. Finally, similar to Bredemeier et al. (2015), the ratio of government expenditure to output is set to 0.2 and the interest rate rule coefficients are set to conventional values $\rho^r = 0.8$, $\rho^y = 1.5$ and $\rho^y = 0.05$.[7]

2.3.2 Crisis Experiment

In this subsection, I evaluate the response of the model to a capital quality shock for different calibrations of the managerial cost parameter. This parameter determines the degree of retail banks' ability to adjust capital holding if financial conditions change. Indeed, the managerial cost parameter η is an important parameter for the amplification of the capital quality shock, as shown in this subsection. As in Gertler and Karadi (2011, 2013), the crisis is triggered by a capital quality shock. The shock

[6] Schabert (2015) shows that the central bank can still implement its inflation target if $\Gamma \neq \pi$.

[7] Moreover, following Bredemerier et al. (2015), the parameter μ is set equal to 1. While this choice is somewhat arbitrary, the parameter does not affect the model's dynamic.

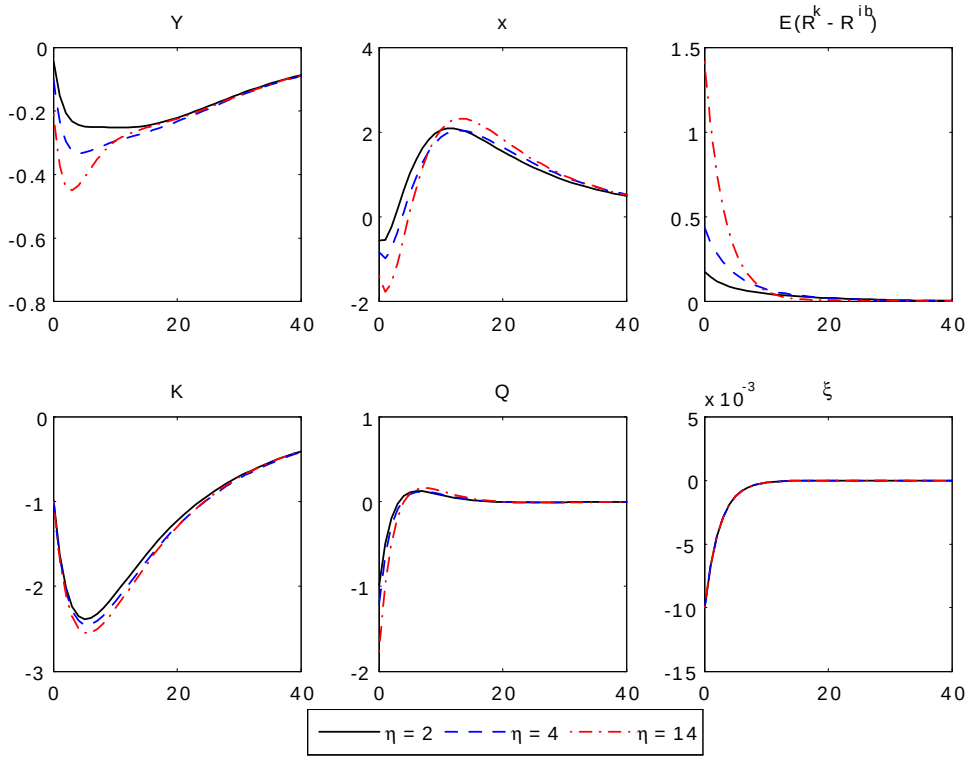


Figure 2.3: Impulse Responses of output, investment, interest rate spread, capital, and asset prices to a capital quality shock for different calibrations of the managerial cost parameter η.

is calibrated to lead to a drop in the capital quality of 1 percent on impact. The shock process is modeled by a stationary AR(1) process with an autoregressive factor of 0.66. The corresponding impulse response functions (IRFs) are displayed in figure (2.3). The IRFs displayed in figure (2.3) describe the response of the model to a capital quality shock for different calibrations of the managerial cost parameter η. Most of the variables are depicted in percentual deviations from their steady state. The real interest rate spread $E_t\left(R^k_{t+1} - R^{ib}_t\right)$ is depicted in annual percentage points. The variables ξ_t is presented in absolute variations from its steady state.

The fall in capital quality leads to a drop in capital, output, and investment. Moreover, the unanticipated decline in capital reduces the value of non-financial firm securities, resulting in a drop in wholesale banks' net worth. From the decline in

wholesale banks' net worth, the leverage constraint is tightened, forcing wholesale banks to sell assets. The decline in their demand for non-financial firm securities puts downward pressure on their price. The drop in the price of non-financial firm securities further enhances the downturn by tightening wholesale banks' leverage constraint and forcing them to fire sale non-financial firm securities in order to meet the leverage constraint. Retail banks, who do not face an agency problem, increase their holding of capital by purchasing non-financial firm securities. However, due to the capital management costs their ability to fund capital is limited. Thus, they are not able to fully compensate the drop in wholesale banks' demand for non-financial firm securities. As a result, both, investment and output drop. Investment declines on impact and needs roughly one year to recover.

As shown in figure (2.3), the amplification of the capital quality shock does depend on the calibration of the managerial cost parameter η. The capital quality shock leads to a tightening in the leverage constraint of wholesale banks, inducing a drop in their demand for non-financial firm securities which results eventually in a rise of the interest rate on non-financial firm securities. Retail banks are not leverage constrained. Thus, the rise in the interest rate spread leads, ceteris paribus, to an increase in the retail banks' demand for non-financial firm securities. Higher values of η imply that a rise in retail banks non-financial firm security holdings is associated with higher management costs. Thus, for high values of η, the magnitude of the change of retail banks' demand for non-financial firm securities in response to a change of financial conditions is comparably small. In contrast, for low values of η, the magnitude of the change in the retail banks' demand for non-financial firm securities in response to a rise in financial conditions is considerably stronger. As a result, the rise in the retail banks' demand for non-financial firm securities during the financial turmoil is stronger for low values of η. The increase in the retail banks'

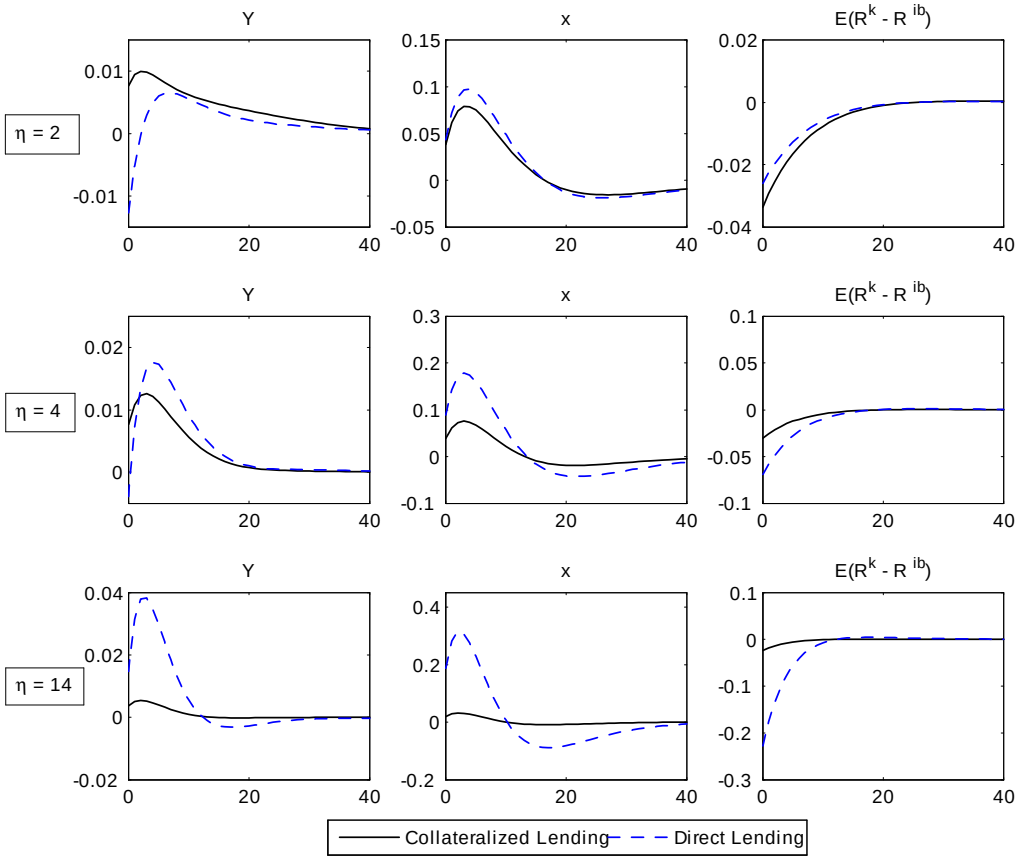


Figure 2.4: Impulse Responses of output, investment, and the interest rate spread to a collateralized lending and a direct lending shock for different values of the managerial cost parameter η.

demand for non-financial firm securities dampens the rise in the interest rate spread and mitigates the amplification of the capital quality shock by the leverage constraint.

2.3.3 Policy Simulations

This section analyses the effects of a small purchase of non-financial firm securities either conducted by collateralized lending or by direct lending on the economy for different values of the managerial cost parameter η. If $\eta > 0$ (and $c_m > 0$), arbitrage by retail banks does not eliminate the spread between the interest rate on capital and

the interbank market rate due to the management costs. Specifically, the parameter η determines the magnitude of the change in the retail banks' demand for non-financial firm securities in response to a change in financial conditions. High values of η reduce the amount of capital the retail bank can absorb if financial conditions change. Figure (2.4) displays the impulse responses to an unanticipated purchase of non-financial firm securities of three key variables (output, investment, and the excess returns on capital) for three different values of η. The central bank either provides funds to non-financial firms directly (direct lending) or purchases non-financial firm securities under repos in open market operations (collateralized lending). Direct lending is modeled as an unanticipated purchase of non-financial firm securities directly from non-financial firms. As described in section (2.4), the central bank issues one-period riskless bonds to finance lending to non-financial firms. The purchase of non-financial firm securities is modeled as an AR(1) process with an autoregressive factor of 0.9. The size of the shock is set to achieve that, on impact, the amount of purchased non-financial firm securities equals 0.05 (which accords to roughly 0.9 percent of the total capital stock). Similar to Gertler et al. (2016), I suppose that the central bank intermediation comprises an efficiency loss. The efficiency cost is interpreted as the cost of publicly channeling funds to non-financial firms. I choose a moderate value of $c_{m,cb} = 0.005$. Collateralized lending is modeled as an unanticipated rise in the fraction of non-financial firm securities accepted in open market κ_t^s. The value of non-financial firm securities purchased in open market operations under repos is denoted by I_t^S.[8] In order to compare the effects of both policy actions, the purchased amounts of non-financial firm securities are of similar magnitude. Specifically, the shock is calibrated so that, on impact, the value of non-financial firm securities purchased in

[8] I_t^S is defined as the value of non-financial firm securities that the central bank purchases under repos, $I_t^s = \kappa_t^s \left(z_t + Q_t \left(1 - \delta\right)\right) s_{rb,t-1}$.

open market operations equals also roughly 0.9 percent of the total capital stock.[9]

Both policies affect the economy through different channels. By direct lending, the central bank substitutes private financial intermediation by public financial intermediation, contributing directly to the aggregate demand for non-financial firm securities. In contrast, collateralized lending does not affect the demand for non-financial firm securities directly. By accepting non-financial firm securities as a collateral in open market operations, the central bank is able to manipulate the liquidity premium incorporated in non-financial firm securities and is eventually able to stimulate the retail banks' demand for non-financial firm securities.

The bottom panels in figure (2.4) show the IRFs of the three variables in response to both policy actions for a high value of η. The dotted line in figure (2.4) reports the effects of direct lending. Direct central bank lending raises the total demand for non-financial firm securities. Since wholesale banks are balance sheet constrained and retail banks face management costs, the rise in the demand for non-financial firm securities by the central bank does not substitute the demand for non-financial firm securities perfectly. The increase in the total demand for non-financial firm securities drives up the price of non-financial firm securities Q_t, resulting in a drop in the rate of return of capital r^k_{t+1} and the real interest rate spread $E_t\left(R^k_{t+1} - R^{ib}_t\right)$. Moreover, the rise in the demand for capital pushes up investment and output.

As displayed by the solid line in the bottom panels in figure (2.4), also collateralized lending works to stimulate aggregate demand for non-financial firm securities. However, the impact of collateralized lending is rather small compared to direct lending for high values of η. By allowing retail banks to trade non-financial firm securities against reserves in open market operations, the central bank is able to influence the

[9] As described in section (2.4), the change in κ^s_t is accompanied by a neutralizing adjustment of κ^b_t and an adjustment of the ratio of money supplied outright and under repos in exchange for treasuries, in order to keep the total amount of reserves and the amount of reserves supplied outright unchanged by the policy action.

retail banks' marginal valuation of non-financial firm securities. More precisely, the equilibrium relation between the retail banks' expected marginal valuation of non-financial firm securities and the interbank market rate, given in eq. (2.63), together with eq. (2.18) show how the central bank is able to manipulate the retail banks' demand for these securities. If the central bank sets $\kappa_t^s > 0$, ceteris paribus, the retail banks' marginal valuation of non-financial firm securities rises. Thus, by arbitrage, retail banks' start to purchase additional non-financial firm securities until their expected marginal valuation of these securities equals the expected interbank market rate in equilibrium. Thus, a rise in κ_t^s increases retail banks' demand for non-financial firm securities, resulting in a rise in the total demand for these securities and pushing up the price of these securities. As a result, the the real interest rate spread $E_t\left(R_{t+1}^k - R_t^{ib}\right)$ drops. The rise in the total demand for non-financial firm securities pushes up investment, the total capital stock, and output.

The top panels in figure (2.4) display the IRFs of output, investment, and excess returns on capital in response to both policy actions for a low value of η. Again, both, collateralized lending and direct lending work to stimulate the aggregate demand for non-financial firm securities. In result, the excess returns on capital drop and investment and increases.[10] However, for the low value of η, direct lending is less effective than collateralized lending. The drop in the excess returns on capital is stronger under collateralized lending for the low value of η.

The intuition for the different effects of both policies for different values of the managerial cost parameter η is as follows. Direct lending works by increasing the demand for non-financial firm securities directly. If the central bank purchases non-financial firm securities by direct lending, the demand for these securities increases

[10]Notably, under the direct lending policy, output declines on impact. The initial decline in output results from a drop in labor. For moderate public financial intermediation costs, public intermediation of a small amount of assets reduces the aggregate costs of asset management. As a result, under the chosen calibration, households consume more leisure and consumption goods.

which pushes down the rate of return on capital in equilibrium. For a high value of η, retail banks' demand is less elastic. Thus, if the return on capital decreases, the drop in the retail banks' demand for non-financial firm securities is relatively small. Therefore, the overall impact of direct lending on the aggregate demand for non-financial firm securities increases in η. For high values of η, the effects of collateralized lending on the excess return on capital are comparably small. In contrast to direct lending, collateralized lending does not increase the total demand for non-financial firm securities directly, but by influencing the retail banks' marginal valuation of these securities. The less elastic the retail banks' demand for non-financial firm securities is, the less effective is collateralized lending in stimulating retail banks' demand for firm securities.

The analysis shows that the effects of both policies on the economy are in general not identical. Therefore, in order to analyze the effectiveness of different asset purchase programs, it is important to take into account how central bank's asset purchases are conducted. Moreover, overall, the effects of direct lending on the excess returns on capital are of stronger magnitude for a high value of η than the effects of collateralized lending for a low value of η.

2.4 Conclusion

In this work, I evaluate the effects of central bank's purchase of private securities in a macroeconomic model with money serving as a mean of payment, two types of financial intermediaries (namely retail banks and wholesale banks), and financial frictions. Both types of financial intermediaries have different business models. Retail banks rely only on households' saving for funding the acquisition of non-financial firm securities and lending in the interbank market. Wholesale banks use funds acquired

in the interbank market to lend to non-financial firms. Based on Gertler and Karadi (2011), a financial friction is introduced by an agency problem that entails a leverage constraint on the wholesale banks' balance sheets. In addition, due to management costs, retail banks' arbitrage is not able to eliminate spreads in the non-financial firm security market.

I analyze the effects collateralized central bank lending and compare these effects to those of direct central bank lending. Under collateralized central bank lending, the central bank purchases private securities in open market operations, i.e. exchanges reserves against private securities under repos, where reserves are high-powered money. Alternatively, under direct central bank lending, the central bank provides public financial intermediation by lending funds directly to non-financial firms, where direct lending is financed by issuing interest-rate bearing debt, as in Gertler and Karadi (2011, 2013). Thus, similar to a private financial intermediary, the central bank intermediates funds to non-financial firms directly. In order to evaluate the effectiveness of collateralized central bank lending, I consider different settings in which the agency problem affects the model's dynamic differently strong.

First, the dynamic of the model in response to capital quality shock is analyzed. As demonstrated, the effectiveness of both policies depends on how strong the leverage constraint affects the model's dynamic which, in turn, depends on the elasticity of retail banks' demand for non-financial firm securities. Next, the effectiveness of collateralized central bank lending and direct central bank lending are analyzed.

The results show that both policies, collateralized central bank lending and direct central bank lending, are able to reduce excess returns on capital in this model. Ultimately, both policies work by changing the aggregate demand for non-financial firm securities. However, both policies affect the aggregate demand for non-financial firm securities through different channels.

Due to the presence of the financial friction and the management costs, the change in the aggregate demand for non-financial firm securities is not offset by private financial intermediaries. Specifically, under collateralized central bank lending, the central bank is able to manipulate the liquidity premium incorporated in these securities by adjusting the fraction of non-financial firm securities eligible in open market operations. In response, retail banks adjust their demand for non-financial firm securities accordingly. In contrast, by intermediating funds publicly to non-financial firms directly, the central bank provides public financial intermediation. The increase in the demand for non-financial firm securities raises the price of private securities and reduces excess returns on capital.

I want to stress two main points of my results and their implications. First, the effectiveness of both policies is in general not identical. Thus, in order to evaluate the effects of central bank's asset purchases, it is important to take into account how the asset purchases are conducted. In fact, under a considerable number of programs (e.g. the discount window lending or the Term Auction Facility) the Fed supplied reserves against an eligible collateral to financial institutions in order to support the liquidity of these institutions and did not intermediate funds publicly from lenders to borrowers.

Second, collateralized lending and direct lending are each most effective under the exactly opposite market conditions of the private financial sector. More precisely, if retail banks' demand for non-financial firm securities is completely inelastic, collateralized lending does have no impact on the economy, while direct lending is most effective and vice versa. In contrast, if retail banks' are able to expand their demand for private assets relatively elastically when financial conditions change, the effects of direct lending on the economy are less beneficial compared to the analysis of Gertler and Karadi (2011). The presence of retail banks mitigate the effects of the

wholesale banks' leverage constraint on the economy and reduces the effectiveness of direct lending thereby. Moreover, if retail banks are able to adjust its demand for non-financial firm securities more elastically, the amplification of a financial disturbance (here modeled by a reduction in the capital quality) is less strong and the effects of the financial disturbance in this model become more similar to the effects of this disturbance in a more standard New-Keynesian model without the agency problem. Thus, in this case, the agency problem is less relevant for the dynamic of the economy. In other words, under a calibration where the leverage constraint affects the model's dynamic substantially, the effects of central bank's liquidity programs are, at its best, only moderate.

2.A Appendix

2.A.1 Value Function

Inserting the conjectured solution (2.25) of the value function into the Bellman equation (2.24) yields

$$\begin{aligned}\mu_{t-1}^{k} Q_t s_{j,t-1} + \mu_{t-1}^{n} n_{j,t-1} &= E_{t-1}\Lambda_{t-1,t}\left[(1-\sigma)\, n_{j,t} + \sigma\left(\mu_t^{k} Q_t s_{j,t} + \mu_t^{n} n_{j,t}\right)\right] \\ &= E_{t-1}\Lambda_{t-1,t}\Xi_t n_{j,t},\end{aligned}$$

where

$$\Xi_t \equiv (1-\sigma) + \sigma\left[\mu_t^{k}\phi_t + \mu_t^{n}\right].$$

Next, by using the law-of-motion of wholesale bank j, given by eq. (2.23), the following expression is obtained

$$\begin{aligned}\mu_{t-1}^{k} Q_t s_{j,t-1} + \mu_{t-1}^{n} n_{j,t-1} &= E_{t-1}\Lambda_{t-1,t}\Xi_t\left[\left(r_t^{k} - r_{t-1}^{ib}\right) Q_{t-1} s_{j,t-1}\pi_t^{-1}\right. \\ &\quad \left. + \left(1 + r_{t-1}^{ib}\right) n_{j,t-1}\pi_t^{-1}\right].\end{aligned}$$

Comparing terms yields that the conjectured solution (2.25) holds for any $(s_{j,t}, n_{j,t})$ if

$$\begin{aligned}\mu_t^{k} &= E_t\Lambda_{t,t+1}\Xi_{t+1}\left(r_{t+1}^{k} - r_t^{ib}\right)\pi_{t+1}^{-1}, \\ \mu_t^{n} &= E_t\Lambda_{t,t+1}\Xi_{t+1}\left(1 + r_t^{ib}\right)\pi_{t+1}^{-1}.\end{aligned}$$

2.A.2 Equilibrium

A REE is a set of sequences,

$$\left\{\lambda_t, \pi_t, l_t, w_t, c_t, mc_t, y_t, k_t, \tilde{x}_t, x_{1,t}, x_{2,t}, \Delta_t, Q_t, n_t, \Xi_t, \phi_t, x_t, m_t, m_t^R, m_t^S,\right.$$
$$\left. i_t, b_t, b_{rb,t}, s_t, s_{w,t}, s_{rb,t}, \nu_t^{hh}, \nu_t^{dc}, \nu_t^{mc}, \mu_t^n, \nu_t^k, \mu_t^k, r_t^d, r_t, r_{rb,t}^k, r_t^k, r_t^{ib}, r_t^b\right\}_{t=0}^{\infty},$$

satisfying

$$\frac{1}{1+r_t^d} = \beta E_t \frac{\lambda_{t+1} + \mu\nu_{t+1}^{hh}}{\lambda_t \pi_{t+1}}, \tag{2.64}$$

$$u_{c,t} = \lambda_t + \nu_t^{hh}, \tag{2.65}$$

$$\chi l_t^{\sigma_n} = \lambda_t w_t, \tag{2.66}$$

$$\frac{1}{1+r_t} = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{-1}, \tag{2.67}$$

$$1 = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left[\left(1 + r_{rb,t+1}^k\right)\left(1 + \kappa_{t+1}^s \nu_{t+1}^{mc}\right)\right] \pi_{t+1}^{-1}, \tag{2.68}$$

$$c_t = m_t + m_t^R + m_t^S, \tag{2.69}$$

$$\nu_t^{hh} = \lambda_t \nu_t^{dc}, \tag{2.70}$$

$$\frac{1}{1+r_t^{ib}} = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{-1}, \tag{2.71}$$

$$\frac{1}{1+r_t^b} = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{1 + \kappa_{t+1}^b \nu_{t+1}^{mc}}{\pi_{t+1}}, \tag{2.72}$$

$$1 = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{1 + \nu_{t+1}^{dc}}{\pi_{t+1}}, \tag{2.73}$$

$$1 + v_t^{dc} = \left(1 + r_t^m\right)\left(1 + \nu_t^{mc}\right), \tag{2.74}$$

$$\frac{1 + r_{rb,t}^k}{\pi_t} = \frac{z_t + (1-\delta) Q_t}{Q_{t-1} + \kappa'\left(s_{rb,t-1}\right)} \xi_t, \tag{2.75}$$

$$\left(1 + r_t^m\right) i_t = \left(\kappa_t^b b_{rb,t-1} + \kappa_t^s \left(z_t + (1-\delta) Q_t\right) \xi_t s_{rb,t-1}\right) \pi_t^{-1}, \tag{2.76}$$

$$m_t = m_{t-1} \pi_t^{-1} - m_t^R + i_t, \tag{2.77}$$

$$m_t^S = \kappa_t^s \frac{\left(z_t + (1-\delta) Q_t\right) \xi_t}{\left(1 + r_t^m\right) \pi_t} s_{rb,t-1} \tag{2.78}$$

$$\frac{1+r_t^k}{\pi_t} = \frac{z_t+(1-\delta)\,Q_t}{Q_{t-1}}\xi_t, \tag{2.79}$$

$$\mu_t^k = E_t\Lambda_{t,t+1}\Xi_{t+1}\left(r_{t+1}^k - r_t^{ib}\right)\pi_{t+1}^{-1}, \tag{2.80}$$

$$\mu_t^n = E_t\Lambda_{t,t+1}\Xi_{t+1}\left(1+r_t^{ib}\right)\pi_{t+1}^{-1}, \tag{2.81}$$

$$\Xi_t = (1-\sigma)+\sigma\left[\mu_t^k\phi_t+\mu_t^n\right], \tag{2.82}$$

$$\phi_t = \frac{\mu_t^n}{\theta-\mu_t^k}, \tag{2.83}$$

$$\phi_t = \frac{Q_t s_{w,t}}{n_t}, \tag{2.84}$$

$$\mu_t^k = \frac{\nu_t^k}{1+\nu_t^k}\theta, \tag{2.85}$$

$$n_t = \begin{array}{l}\left[\sigma\left[\left(r_t^k-r_{t-1}^{ib}\right)Q_{t-1}s_{w,t-1}+\left(1+r_{t-1}^{ib}\right)n_{t-1}\right]\pi_t^{-1}\right. \\ \qquad\left.+\omega Q_t s_{w,t-1}\pi_t^{-1}\right],\end{array} \tag{2.86}$$

$$Q_t = \begin{array}{l}\left[1+f\left(\frac{x_t}{x_{t-1}}\right)+\frac{x_t}{x_{t-1}}f'\left(\frac{x_t}{x_{t-1}}\right)\right. \\ \left.-E_t\Lambda_{t,t+1}\left(\frac{x_{t+1}}{x_t}\right)^2 f\left(\frac{x_{t+1}}{x_t}\right)\right],\end{array} \tag{2.87}$$

$$l_t = (1-a)\,mc_t\frac{y_t}{w_t}\Delta_t, \tag{2.88}$$

$$k_t = (1-\delta)\,\xi_t k_{t-1}+x_t, \tag{2.89}$$

$$\Delta_t y_t = (\xi_t k_{t-1})^{\alpha}\, l_t^{1-\alpha}, \tag{2.90}$$

$$\tilde{x}_t = \frac{\varepsilon}{\varepsilon-1}\frac{x_{1,t}}{x_{2,t}}, \tag{2.91}$$

$$x_{1,t} = \lambda_t mc_t y_t+\beta E_t\phi\pi_{t+1}^{\varepsilon}x_{1,t+1}, \tag{2.92}$$

$$x_{2,t} = \lambda_t y_t+\beta E_t\pi_{t+1}^{\varepsilon-1}\phi x_{2,t+1}, \tag{2.93}$$

$$1 = (1-\phi)\,(\tilde{x}_t)^{1-\varepsilon}+\phi\left(\frac{\pi_t}{\pi}\right)^{\varepsilon-1}, \tag{2.94}$$

$$\Delta_t = (1-\phi)\,\tilde{x}_t^{-\varepsilon}+\phi\left(\frac{\pi_t}{\pi}\right)^{\varepsilon}\Delta_{t-1}, \tag{2.95}$$

$$y_t = c_t+x_t+f\left(\frac{x_t}{x_{t-1}}\right)x_t+g_t+\kappa\left(s_{rb,t}\right), \tag{2.96}$$

$$b_t = \Gamma b_{t-1}\pi_t^{-1}, \tag{2.97}$$

$$m_t = \Omega_t m_t^R, \tag{2.98}$$

$$s_t = k_t, \tag{2.99}$$

$$s_t = s_{rb,t} + s_{w,t} + s_{dl,t}, \tag{2.100}$$

$$b_t = b_{rb,t} + m_t, \tag{2.101}$$

where the auxiliary variables are given by

$$\begin{aligned} u_{c,t} &= \frac{1}{c_t - hc_{t-1}}, \\ \kappa\left(s_{rb,t}\right) &= \frac{c_m}{\eta} s_{rb,t}^{\eta}, \\ \Lambda_{t,t+1} &= \beta \frac{\lambda_{t+1}}{\lambda_t}, \\ z_t &= amc_t \frac{y_t \Delta_t}{\xi_t k_{t-1}}, \\ f\left(\frac{x_t}{x_{t-1}}\right) &= \frac{\gamma}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2, \end{aligned}$$

a transversality condition, fiscal and monetary policy setting $\{g_t, r_t^m \geq 0, \kappa_t^s \in [0,1]\,,$ $\kappa_t^b \in (0,1], \Omega_t \geq 0, s_{dl,t} \geq 0\}_{t=0}^{\infty}$, $\beta \geq \pi$ and $\Gamma \geq 1$, for given sequences of shocks $\{\xi_t\}_{t=0}^{\infty}$, and given initial values $m_{-1} > 0$, $b_{-1} > 0$, $b_{rb,-1}$, d_{-1}, and $\Delta_{-1} \geq 1$.

2.A.3 Interest Rate Relations

Starting with the retail banks, using eq. (2.16) and the equilibrium relation $\nu_t^{hh} = \lambda_t \nu_t^{dc}$ yields

$$\lambda_t = E_t \beta \frac{\lambda_{t+1} + \nu_{t+1}^{hh}}{\pi_{t+1}}. \tag{2.102}$$

Furthermore, plugging $\nu_t^{hh} = \lambda_t \nu_t^{dc}$ into eq. (2.17) yields

$$1 + \nu_t^{hh}/\lambda_t = (1 + r_t^m)(1 + \nu_t^{mc}). \tag{2.103}$$

Combining eq. (2.102), (2.103), and (2.5) leads to

$$\begin{aligned} (1 + r_t^m)(1 + \nu_t^{mc}) &= \left[E_t \beta \frac{u_{c,t+1}}{u_{c,t}\pi_{t+1}} \right]^{-1} \\ \Leftrightarrow \nu_t^{mc} &= \frac{1 + r_t^{IS}}{1 + r_t^m} - 1. \end{aligned} \tag{2.104}$$

where

$$\frac{1}{1 + r_t^{IS}} = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} \pi_{t+1}^{-1}.$$

By plugging eq. (2.5) into (2.102), the multiplier of the goods market constraint can be written as

$$\begin{aligned} u_{c,t} - v_t^{hh} &= E_t \beta \frac{u_{c,t+1}}{\pi_{t+1}} \\ \Leftrightarrow \nu_t^{hh} &= u_{c,t} \left(1 - \frac{1}{1 + r_t^{IS}} \right). \end{aligned} \tag{2.105}$$

Moreover, from eq. (2.15) together with (2.5) and (2.102) the equilibrium interest rate on government bonds is given by

$$\begin{aligned} \frac{1}{1 + r_t^b} &= E_t \frac{1}{1 + r_{t+1}^{IS}} \left(1 - \kappa_{t+1}^b + \kappa_{t+1}^b \frac{1 + r_{t+1}^{IS}}{1 + r_{t+1}^m} \right) \pi_{t+1}^{-1} \\ &= E_t \left(\frac{\left(1 - \kappa_{t+1}^b\right)\left(1 + r_{t+1}^m\right) + \kappa_{t+1}^b \left(1 + r_{t+1}^{IS}\right)}{\left(1 + r_{t+1}^{IS}\right)\left(1 + r_{t+1}^m\right)} \pi_{t+1}^{-1} \right). \end{aligned} \tag{2.106}$$

The interest rate on lending in the interbank market r_t^{ib} is obtained by plugging eq. (2.5) and (2.102) into (2.13)

$$\begin{aligned}
\frac{1}{1+r_t^{ib}} &= E_t\beta\left(E_{t+1}\beta\frac{\lambda_{t+2}+v_{t+2}^{hh}}{\pi_{t+2}}\left[E_t\beta\frac{\lambda_{t+1}+v_{t+1}^{hh}}{\pi_{t+1}}\right]^{-1}\right)\pi_{t+1}^{-1} \\
&= E_t\beta\left(E_{t+1}\frac{u_{c,t+2}}{\pi_{t+2}}\left[E_t\frac{u_{c,t+1}}{\pi_{t+1}}\right]^{-1}\right)\pi_{t+1}^{-1} \\
&= E_t\frac{1}{1+r_{t+1}^{IS}}. \qquad (2.107)
\end{aligned}$$

Chapter 3

Evaluating the Interplay of Term Premia, Monetary Policy, and the Economy in the Euro Area

3.1 Introduction

Standard decomposition of yields separates the yield of a long-term bond into an expectation part and a term premium part. The expectation part consists of the average of the expected sum of short-term interest rates until the bond matures while the term premium part compensates risk-averse investors for the risk of holding longer-dated instruments. In order to affect spending, production, employment, and inflation, manipulating the expectations of the future short rates by the forward guidance of future short-term interest rates is one important tool of central banks, as emphasized by Woodford (2005). This routine is known as the expectation channel. However, to the extent that aggregate demand depends, among other macroeconomic factors, not only on the short-term interest rate but also on long-term interest rates,

also changes in the term premium component of these rates affect economic activity. Indeed, recently, Ireland (2015) finds evidence for the U.S. that an increase in term premia dampens economic activity. In turn, by influencing the term premium in long-term bond yields, there is another, less conventional way, how monetary policy might be able to affect economic activity (see e.g. Wu, 2014). This paper analyses the effects of movements in term premia of Euro area government bonds on the economy of the Euro area, the effects of monetary policy on term premia, and whether the ECB responds, in turn, on term premia movements.

The effects of variations in term premia on the economy, and how monetary policy affects these premia, are in the focus of policy makers and researchers, not solely, but especially since the financial crisis. During the crisis, with the short-term nominal interest rate at the zero lower bound in the US, unconventional methods of monetary policy sought to influence the expectation of future short-term rates and to reduce term premia in long-term bond yields in order to ease financial conditions. But also before the onset of the financial crisis, the effects of changes in term premia on the economy and the response of monetary policy to these fluctuations were considered by researchers and policy makers. As explained by then Federal Reserve Chairman Bernanke (2006), "if spending depends on long-term interest rates, special factors that lower the spread between long-term and short-term interest rates will stimulate aggregate demand. Thus, when the term premium declines, a higher short-term rate is required to obtain the long-term rate and the overall mix of financial conditions consistent with maximum sustainable employment and stable prices". This “practitioner” view, as labeled by (Rudebusch et al., 2007), states two assumptions. Firstly, a drop in the term premium and with it in long-term yields, all else is being equal, works to stimulate aggregate demand and output. Secondly, the central bank is required to counteract the drop in the premium by adjusting the short-term interest

rate in order to balance output and inflation. Although this view is prevalent among practitioners (see Rudebusch et al., 2007), less evidence for it has been found so far (as discussed in Ireland, 2015). The empirical findings of the effects of changes in the premium on output are rather mixed, ranging from exactly the opposite relationship of what the practitioner view suggests to the expected inverse relationship between term premia and output. Since a broad literature focuses on the effects of movements of term premia on output, the next section serves a more detailed literature overview of the effects of term premia movements on GDP.

However, not are only the effects of changes in term premia on output unclear, but also how monetary policy should respond to these changes (if it responds at all). The practitioner view advocates that in response to a rise in the term premium, the central bank should lower the policy rate to offset the increase.[1] In contrast, Goodfriend (1993) and McCallum (2005) argue that the central bank should increase the short-term interest rate in response to a rise in the term premium. Both interpret the rise in the term premium as evidence for an increase in inflation scares which the central bank should fight by raising the short-term interest rate. More recently, Ireland (2015) investigates the response of monetary policy to changes in the term premium for the US. He provides evidence that an increase in the premium led the Fed to tighten monetary policy.

This paper seeks to evaluate the interplay of monetary policy, term premia and the economy in the Euro Area. My analysis focuses on the euro area before and during the financial crisis in order to investigate if movements in term premia affect output and inflation, whether the ECB responds to these movements, and if term premia, in turn, respond to conventional monetary policy actions. For this purposes,

[1] Indeed, Carlstrom, Fuerst, and Paustian (2014) demonstrate in a DSGE model with segmented financial markets and imperfect financial intermediation that a negative response coefficient in the monetary policy rule on the term premium increases welfare modestly.

I apply a macro-finance model of the term structure of interest rates based on Ireland (2015) to the Euro area.

The recent period raises questions about the necessity to impose a non-negativity constraint or lower-bound constraint on the short-term interest rate processes of the model, usually known as the "zero lower bound". While the results of Bauer and Rudebusch (2015) stress the relevance of shadow rate models (a particular class of term structure models that respects a lower bound for the short-term interest rate process) for the US, the need for this kind of models for the Euro area is less obvious. As argued by Christensen and Krogstrup (2014, 2016), bond yields in Europe (in their example: German and Swiss bond yields) "have actually been well below zero for intermediate maturities and for extended periods in recent years. Hence, in these cases, a standard Gaussian modeling approach appears to be fully warranted" (Christensen and Krogstrup, 2016, p.2 7). I follow their argumentation and do not enforce a zero-lower bound.

For analyzing the yield curve, and especially term structure premia, macro-finance models bring along several benefits over pure finance term-structure models and structural macro models. In contrast to pure finance models, macro-finance models use a set of macroeconomic variables to span the yield curve and allow the macro fundamentals to evolve jointly over time. The short-end of the yield curve, that is, the short-term risk-free interest rate is under the control of the central bank. Using information of the state of the macroeconomy helps to model the short-term interest rate process. Moreover, since term premia are not only time varying, but are also different across bond maturities, exploiting all information available over the entire yield curve helps to identify the term premium and thus to separate the term premium component from the expectation component of long-term yields. In contrast to structural macro models, macro-finance models do not impose strong

theoretical assumptions on how macroeconomic developments affect term premia and term premia, in turn, affect the economy, but use a more flexible time-series approach, as discussed by Ireland (2015, p. 125). This is in particular appealing because of the conflicting evidence of the effects of movements in term premia on the economy from previous empirical studies.

In order to model the dynamics of yields consistently over the yield curve, macro-finance models of the term structure of interest rates employ cross-equation restrictions. Based on Duffie and Kan (1996), these cross-equation restrictions arise from the assumption of the absence of arbitrage opportunities in bond markets. The precise specification of the term structure part of the model follows Dewachter and Iania (2011), Dewachter et al. (2014), and Ireland (2015): In order to evaluate the interplay of term premia movements, monetary policy, and the economy, a latent risk variable that captures term premia movements is employed. In the spirit of Cochrane and Piazzesi (2005, 2008), the risk variable is constructed to be the only force that drives the one-period expected excess holding return (the one period-return premium) and is integrated into the state-space system. The dynamics of the state variables are modeled as a structural vector autoregressive (VAR) model. The risk variable responds to all state variables and exhibits an autonomous dynamic. Thus, the yield curve is not fully spanned by observable macro factors as suggested by Joslin, Priebsch, and Singleton (2014). Moreover, while Dewachter and Iania (2011) and Dewachter et al. (2014) does not allow term premia to affect the economy, following Ireland (2015), the model allows for feedbacks from term premia movements to the economy. Identification of the structural shocks of the state equations is achieved by imposing restrictions on the contemporaneous relation among the variables of the state equation. The estimation of the model is carried out by Bayesian estimation techniques. The likelihood function is constructed using the Kalman filter. The

posterior is evaluated using an Adaptive Metropolis (AM) algorithm in the lines of Haario et al. (2001).

My results reveal a strong interaction among term premia, monetary policy, and the economy. In line with the practitioner view, I find that a rise in term premia is associated with a drop in the output gap and in inflation. The ECB lowers the short-term interest rate in response to an increase in term premia. Thus, during the sample period, the ECB mitigates the effect of a rise in the term premium on the yield curve by lowering the short-end of the yield curve. However, I find only negligible effects of conventional monetary policy on term premia in turn.

The remainder of the paper is organized as follows. The next section serves a literature overview of the effects of term premia movements on output. Section (3.3) explains the macro-finance model and discusses the decomposition of the yield curve into the expectation part and term premia part. Section (3.4) casts the model into the state-space system, describes the data, and discusses the estimation procedure and the prior distribution. Section (3.5) presents and discusses the results of the estimation. The last section concludes.

3.2 Literature Review

This section covers a literature overview of the empirical results and the theoretical consideration of the effects of term premia movements on output.

In standard linearized New-Keynesian models, term premia do simply not exist. Log-linearization eliminates higher order terms like term premia by construction. In order to analyze term premia in a DSGE framework, limits-to-arbitrage or non-linear setups are required. Rudebusch et al. (2007) show that a non-linear New-Keynesian model with habit formation produces time-varying term premia which respond to the state of the economy. They emphasize that the relationship between the term premium and the output gap depends on the kind of the underlying distortion. However, their model does not offer a feedback from the term premium to the economy. Andrés et al. (2004) use a New-Keynesian model with imperfect substitutability between different financial assets and segmented asset markets to analyze the effect of long-term yields on aggregate demand and supply. They demonstrate that an increase in term premia dampens economic activity. Chen et al. (2012) estimate a linearized DSGE model with segmented financial markets and limits to arbitrage. They evaluate the effects of Large Scale Asset Purchases (LSAP) on the economy where the effects are transmitted by a drop in the term premium of long-term government bonds. Though the decrease in term premia works to stimulate economy activity, their results suggest that the effects are only moderate. Similarly, Kiley (2012) estimates a model with segmented markets and limits-to-arbitrage using not only government long-term bond yields but also private long-term bond yields. His results also suggest that a decline in the term premium has positive but moderate effects on aggregate spending.

Using less structural approaches, either macro-finance models or reduced form

regressions, a broad empirical literature analyzes the effect of changes of term premia on the economy. The following passages summarize their findings.

Hamilton and Kim (2002) use a regression to investigate the effects of the short-long term yield spread on GDP growth. They were the first who decompose the yield spread into an expectation part and a term premium part in order to evaluate the effects of both components of the spread on GDP growth separately. Using ex-post observed short rates as instruments for ex-ante expected rates to isolate the expectation part, they find that a decline in premia is associated with slower future GDP growth, contradicting the practitioner view. Also, Favero, Kaminska, and Söderström (2005) find that a lower term premium predicts slower future GDP growth. They decompose the yield spread similar to Hamilton and Kim (2002) but use an estimated real-time VAR to predict the expectations of future short-term rates. Wright (2006) investigates whether the return forecast factor of Cochrane and Piazzesi (2005) - a linear combination of the spot rate and four forward rates - helps to forecast recessions. He documents that lower term premia raise the odds of a recession.

In contrast to these results, Ang et al. (2006) find that changes in the term premium do not affect output growth. They run a regression of output growth on the term premium and expected future short rates, where the premium and the expected future short rates are computed from the estimates of a VAR with long-term rates, GDP growth, and the short-term interest rate. Also Rosenberg and Maurer (2007) find that the term premium has no predictive power for future GDP growth. They decompose the yield spread as in Hamilton and Kim (2002) and use both components in a recession forecasting model. In their estimation, the term premium is measured by the Kim-Wright (2005) term premium measure - the estimated term premium from a no-arbitrage dynamic latent 3-factor model. Dewachter et al. (2014) use

a macro-finance model of the term structure where a latent variable captures all movement in the one-period expected excess holding return (the return premium). After estimating the macro-finance model, they use the time path of the latent variable in a regression on future GDP growth. They find that the term premium has no predictive power for future output growth.

However, in line with the practitioner view, Rudebusch et al. (2007) find that a decline in the term premium is associated with higher positive GDP growth. Using the Kim-Wright term premium measure, they decompose the term spread in order to perform a regression of GDP growth on changes in the term premium. Also, Jardet, Montfort, and Pegoraro (2013) and Joslin, Priebsch, and Singleton (2014) find both that a rise in the term premium lowers GDP growth in the short run, but has positive effects on GDP growth for longer horizons. While the former use a macro-finance near-cointegrated VAR(p) term structure model, the latter employ a macro-finance model with imperfect correlated macro risk to explore the sources of variations in expected excess returns on bonds and the effects of term premium shocks on GDP growth and inflation. Recently, using a macro-finance model of the term structure, Ireland (2015) find that a rise in the term premium leads to a drop in output.

The aforementioned results show that the findings on the effects of term premia movements on output are rather mixed, depending on the used framework and on how term premia are identified. In view of the unambiguous finding of the literature, the identification of term premia, that is the separation of the term premium from the expectation of future short-term interest rates, and how the expected future becomes even more relevant. Although the estimation of macro-finance models is computationally more challenging than the estimation of regression models, it comes along with benefits. As discussed in the last section, by tying yields together by no-arbitrage assumption, the model uses information over the whole cross-section

of yields. Employing these information helps to separate the term premium part from the expectation part in bond yields. Based on Dewachter and Iania (2011), Dewachter et al. (2014), and Ireland (2015), I employ a single latent variable that captures movements term premia. Specifically, by restrictions on the pricing kernel, this risk variable is identified to be the only source of variation in the prices of risk. Departing from Dewachter and Iania (2011), Dewachter et al. (2014), and following Ireland (2015), the risk variable is explicitly allowed to affect the dynamic of the economy.

In the following, I use a macro-finance model in the lines of Ireland (2015) to evaluate the interplay of term premia, monetary policy, and the economy.

3.3 The Model

In this section, the macro-finance model is presented. It is a joint model of the macroeconomy and the term structure as introduced into the macro-finance literature by Ang and Piazzesi (2003). The structure of the macro part of the model follows closely Ireland (2015). The term structure is modeled by an affine no-arbitrage model of the term structure as developed by Duffie and Kan (1996) and Dai and Singleton (2000). Motivated by the evidence of Cochrane and Piazzesi (2008) that one single factor accounts for most of the movements in expected excess holding returns, a latent variable that captures all movements in the one-period return premium is introduced. By restricting the prices of risk, this variable is constructed to be the only potential source for time variation in the market prices of risk and thus for movements in term premia. The specification of the term structure model follows Dewachter and Iania (2011), Dewachter et al. (2014) and Ireland (2015).

The model section is structured as follows. The first part describes the structural

macroeconomic dynamics and casts the macro model into its state representation. The state variables are then used as pricing factors in the term structure model. Cross-equation restrictions, based on the assumption of no-arbitrage, are employed to tie the movements of yields closely together. Finally, different notion of the term structure premium - the yield and the return premium - are discussed and related to the latent risk variable.

3.3.1 The Macro Part

The macroeconomic dynamics are described by five state variables, three of them are observable - the nominal short-term interest rate r_t, the inflation rate π_t, and the output gap g_t^y - and two variables are unobservable, a risk variable v_t and the central bank's inflation target π_t^*. Following Ireland (2015), the short-term interest rate and the inflation rate do not enter the state equation directly but in form of the interest rate gap and the inflation gap, respectively. Specifically, the interest rate gap g_t^r is defined as the deviation of the interest rate from the inflation target, $g_t^r \equiv r_t - \pi_t^*$, and the inflation gap g_t^π is defined as the deviation of the inflation rate from central bank's inflation target, $g_t^\pi \equiv \pi_t - \pi_t^*$.

Monetary policy consists of choosing an inflation target and setting the short-term nominal interest rate. Precisely, the short-term interest rate is assumed to follow an interest rate rule in the spirit of Taylor (1993),

$$g_t^r - g^r = \rho_r \left(g_{t-1}^r - g^r\right) + (1-\rho_r)\left[\rho_\pi g_t^\pi + \rho_y \left(g_t^y - g^y\right) + \rho_v v_t\right] + \sigma_r \varepsilon_{rt}, \qquad (3.1)$$

where $\rho_r \in [0,1]$ is the interest rate smoothing parameter, $\rho_\pi > 0$ is the central bank's response parameters on inflation, $\rho_y > 0$ is the response parameters on the

output gap, ρ_v is the response parameter on the variation of the risk variable, $\sigma_r > 0$ is a volatility parameter, and g^r and g^y are the steady state values of g_t^r and g_t^y, respectively. The shock ε_{rt} is supposed to be standard normally distributed and represents the interest rate policy shock. The specification of the interest rate rule incorporates the assumption that the steady state value of the inflation gap is zero. Thus, it is assumed that in the steady state the actual inflation rate equals the central bank's target rate. While ρ_π and ρ_y are restricted to be non-negative, the sign of the parameter of the risk variable ρ_v is not constrained. A positive value of ρ_v implies that the central bank tightens monetary policy in response to a rise in term premia. Goodfriend (1993) and McCallum (2005) argue that this should be the case if the central bank regards an increase in premia as an indicator of "inflation scares" or as an indicator of policy laxity.[2] In contrast, the "practitioner" view, as labeled and discussed by Rudebusch et al. (2007), states that monetary policy should response to term premia by adjusting the policy rate in the opposite direction to the change in term premia. Specifically, as noted by Bernanke (2006), to the extent that aggregate demand depends also on long-term interest rates, a rise in the term premium requires the central bank to lower the short-term interest rate in order to offset the effects of the decline in premia and to retain the economic condition, all else being equal. Thus, the coefficient ρ_v should be negative. Apparently, if ρ_v is zero, the central bank does not react at all to changes in the term structure premium.

The incorporation of an unobservable time-varying inflation target is a common approach in the recent macro-finance term structure literature (as in e.g. Dewachter

[2] To be precise, McCallum (2005) suggests that the central bank should tighten monetary policy if the interest rate spread between long-term bond yields and the short-term rate increases, given that the expectation hypothesis holds and that the premium follows an AR(1) process. A rise in the long-short rate spread might be due to two reasons: an increase in future expected short rates or an increase in the term structure premium. In McCallum's (2005) specification of the interest rate rule, the central bank reacts on the long-short spread, and with it, in general, on the fluctuation in the term premium. However, the cause for the rise in the spread is not identified.

and Lyrio, 2006, Hördahl et al., 2006, Rudebusch and Wu, 2008, or Hördahl and Tristani, 2012). It allows, on the one hand, for some variation in the conduction of monetary policy, and it helps, on the other hand, to capture movements in long-term nominal government bond yields which arise due to changes in central bank's inflation target. In fact, Barr and Campbell (1997) for the UK and Gürkaynak et al. (2005) for the US find that movements in long-term interest rates occur mainly due to changes in expected inflation. Also Hördahl et al. (2006), using a macro-finance term structure model with German data, find that changes in the perceived inflation target tend to have a stronger impact on long-term yields than policy rate shocks, inflation shocks, or output shocks. The inflation target π_t^* is supposed to follow a first-order autoregressive process (AR(1)),

$$\pi_t^* = (1 - \rho_{\pi^*})\,\pi^* + \rho_{\pi^*}\pi_{t-1}^* + \sigma_{\pi^*}\varepsilon_{\pi^* t}, \tag{3.2}$$

where π^* is the steady state level of the inflation target, $\rho_{\pi^*} \in [0, 1)$, $\sigma_{\pi^*} > 0$ and the shock $\varepsilon_{\pi^* t}$ is standard normally distributed. As in Hördahl et al. (2006), Rudebusch and Wu (2008), Hördahl and Tristani (2012), or Ireland (2015), this restriction is imposed to ensure stationarity of the inflation target process. A non-stationary inflation target leads to non-stationary inflation and non-stationary nominal short-term interest rate (see e.g. Ireland, 2015). As shown by Campbell et al. (1997, p. 433) or Spencer (2008) for models with homoscedastic shocks, a unit root in the nominal short-term interest rate translates in undefined asymptotic long-term bond yields. Thus, the assumption of the stationarity of the inflation target process ensures that the term structure part of the model is well-behaved.

Similar to Ireland (2015), the dynamics of the remaining three state variables are modeled as in more conventional structural VAR models. The inflation gap, the

output gap, and the risk variable are linear functions of their own lags, the lags of all other state variables, their own innovations, and potentially of the innovations of the other state variables. This specification allows for a fairly high degree of flexibility while restrictions on the contemporaneous relationship of these variables ensure identification of the structural model.

Specifically, the output gap is supposed to depend on own lags, on lags of the interest rate gap, of the inflation gap, and of the risk variable, and on the innovations of inflation $\varepsilon_{\pi t}$, of the inflation target $\varepsilon_{\pi^\star t}$, and on its own innovations ε_{yt},

$$\begin{aligned} g_t^y - g^y &= \sum_{i=1}^{3} \rho_{yr}^i \left(g_{t-i}^r - g^r\right) + \sum_{i=1}^{3} \rho_{y\pi}^i g_{t-i}^\pi + \sum_{i=1}^{3} \rho_{yy}^i \left(g_{t-i}^y - g^y\right) \\ &\quad + \rho_{yv} v_{t-1} + \sigma_{y\pi}\sigma_\pi \varepsilon_{\pi t} + \sigma_{y\pi^\star}\sigma_{\pi^\star}\varepsilon_{\pi^\star t} + \sigma_y \varepsilon_{yt}, \end{aligned} \tag{3.3}$$

where the volatility parameters σ_y and σ_π are assumed to be non-negative, and ε_{yt} and $\varepsilon_{\pi t}$ are both standard normally distributed. The inflation gap is assumed to depend on own lags, on lags of the interest rate gap, of the output gap, and of the risk variable and on innovations of the inflation target $\varepsilon_{\pi^\star t}$ and on its own innovations $\varepsilon_{\pi t}$,

$$\begin{aligned} g_t^\pi &= \sum_{i=1}^{3} \rho_{\pi r}^i \left(g_{t-i}^r - g^r\right) + \sum_{i=1}^{3} \rho_{\pi\pi}^i g_{t-i}^\pi + \sum_{i=1}^{3} \rho_{\pi y}^i \left(g_{t-i}^y - g^y\right) \\ &\quad + \rho_{\pi v} v_{t-1} + \sigma_{\pi\pi^\star}\sigma_{\pi^\star}\varepsilon_{\pi^\star t} + \sigma_\pi \varepsilon_{\pi t}, \end{aligned} \tag{3.4}$$

where the volatility parameter σ_π is non-negative and $\varepsilon_{\pi t}$ is standard normally distributed. Finally, similar to Bekaert et al. (2013) and Ireland (2015), the risk variable is supposed to respond contemporaneously on all distortions of the economy, as bond prices do. Specifically, the risk variable depends on its own lags and lags of all others state variables and on its own innovations ε_{vt} and additionally all innovations in all

other state variables,

$$\begin{aligned} v_t &= \rho_{vr}\left(g^r_{t-1}-g^r\right)+\rho_{v\pi}g^{\pi}_{t-1}+\rho_{vy}\left(g^y_{t-1}-g^y\right)+\rho_{v\pi^\star}\left(\pi^\star_{t-1}-\pi^\star\right) \\ &\quad +\rho_{vv}v_{t-1}+\sigma_{vr}\sigma_r\varepsilon_{rt}+\sigma_{v\pi}\sigma_\pi\varepsilon_{\pi t}+\sigma_{vy}\sigma_y\varepsilon_{yt}+\sigma_{v\pi^\star}\sigma_{\pi^\star}\varepsilon_{\pi^\star t}+\sigma_v\varepsilon_{vt}, \end{aligned} \tag{3.5}$$

where the volatility parameter σ_v is non-negative, and ε_{vt} is standard normally distributed.

The chosen structure imposes restrictions in order to identify structural shocks. Based on Ireland (2015), shocks to the inflation target $\varepsilon_{\pi^* t}$ affect the interest rate gap, the inflation gap, the output gap, and the risk variable only contemporaneously. Thus, Ireland's (2015) specification implies that all further effects of fluctuations in the central bank's inflation target affect the economy only if the change in the inflation gap and interest rate gap are not fully offset by a proportional adjustment of the interest rate and the inflation rate. This specification imposes a form of long-run monetary neutrality. Moreover, as in Ireland (2015), the preceding equations impose exclusion restrictions on the contemporaneous relationship of the model's variables in order to identify structural shocks. In order to separate the effects of the short-term interest rate and term premia movements on output and inflation from the effects of inflation and output on the short-term interest rate and term premia, it is assumed that neither risk variable shocks nor short-term interest rate shocks do affect output and inflation in the same period but only with one period lag. In contrast, the short-term interest rate and the risk variable respond to shocks to the inflation gap and the output gap instantly. Moreover, output gap shocks do not affect the inflation gap in the same period. Finally, the risk variable depends on all structural shocks.

Define the vectors X_t and ε_t containing the state variables and the innovations

by

$$X_t = \left[\begin{array}{cccccccccc} g_t^r & g_{t-1}^r & g_{t-2}^r & g_t^\pi & g_{t-1}^\pi & g_{t-2}^\pi & g_t^y & g_{t-1}^y & g_{t-2}^y & \pi_t^\star & v_t \end{array} \right]',$$

and

$$\varepsilon_t = \left[\begin{array}{cccccccccc} \varepsilon_{rt} & 0 & 0 & \varepsilon_{\pi t} & 0 & 0 & \varepsilon_{yt} & 0 & 0 & \varepsilon_{\pi^\star t} & \varepsilon_{vt} \end{array} \right]',$$

then eq. (3.1) - (3.5) can be expressed by

$$P_0 X_t = \mu_0 + P_1 X_{t-1} + \Sigma_0 \varepsilon_t. \tag{3.6}$$

For the specific form of the matrices P_0, P_1, μ_0, and Σ_0 see Appendix (3.A.1). Eq. (3.6) gives the structural form of the model. Multiplying by P_0^{-1} yields the reduced form representation of the state equation,

$$X_t = \mu + P X_{t-1} + \Sigma \varepsilon_t, \tag{3.7}$$

where

$$\begin{aligned} \mu &= P_0^{-1} \mu_0, \\ P &= P_0^{-1} P_1, \end{aligned}$$

and

$$\Sigma = P_0^{-1} \Sigma_0.$$

3.3.2 The Term Structure Model

Affine term structure models, as developed by Duffie and Kan (1996) and Dai and Singleton (2000), are a particular class of term structure models.[3] In affine term structure models, the time t yield $y_t^{(\tau)}$ of $\tau-$period zero coupon bond is modeled as an affine function of the state vector X_t,

$$y_t^{(\tau)} = A_\tau + B_\tau' X_t,$$

where both coefficients A_τ and B_τ depend on the maturity τ. Though yields are linear affine in the state vector X_t, A_τ and B_τ' are highly non-linear functions of underlying parameters of the state equation and the prices of risk. The particular functional form of these coefficients is derived from cross-equation restrictions, which in turn stem from the assumption of the absence of arbitrage opportunities. These restrictions tie the movements of yields closely together.

The outlined affine term structure model is similar to the one described in Ang and Piazzesi (2003). However, in contrast to Ang and Piazzesi (2003), restrictions are imposed on parameters contained in the matrix of prices of risk which permit the risk variable v_t to be the only source of fluctuations in the prices of risk and with it in the term premium. This subsection is structured as follows: the first part relates the short end of the yield curve to the state vector. The next part discusses the pricing kernel which is used to price bonds. Finally, under the assumption of no-arbitrage, the functional form of the affine yield curve representation is derived and the solution for the coefficients A_τ and B_τ is presented.

[3] More precisely, the discrete-time term structure model presented in this section belongs to the class of essentially affine models of the term structure, as categorized by Duffee (2002), and introduced by Gourieroux et al. (2002) in discrete time.

Short rate equation

The short-term rate, and thus the short end of the yield curve, is from eq. (3.1) under the control of the central bank. The short end of the yield curve can be modeled as an affine function of the state vector X_t,

$$r_t = \delta_0 + \delta_1^{'} X_t, \tag{3.8}$$

where δ_0 is a scalar, and $\delta_1^{'}$ is a 1x11 selection vector indicating the position of g_t^r and τ_t in X_t. The coefficients δ_0 and δ_1 are set to ensure consistency between the macro part and the term structure part of the model. This requires δ_0 to be equal to zero, $\delta_0 = 0$, and

$$\delta_1^{'} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

so that eq. (3.8) corresponds to the definition of the interest rate gap.

Pricing Kernel

The prices of government bonds are supposed to be arbitrage free. As shown in Harrison and Kreps (1979) or in Duffie (2001, pp. 108) the assumption of the absence of arbitrage guarantees for the existence of an "equivalent martingale measure" or "risk-neutral measure" Q.[4] Under the risk-neutral measure Q, the price $P_t^{(\tau)}$ of any zero-coupon asset maturing in τ periods satisfies

$$P_t^{(\tau)} = E_t^Q \left(\exp\left(-r_t\right) P_{t+1}^{(\tau-1)} \right).$$

[4] Moreover, if markets are also complete, then this risk neutral probability measure is also unique (Harrison and Kreps, 1979).

Thus, pricing under the risk-neutral measure implies that the price of an asset is given by the expected discounted future value of the asset, where the discounting takes place with the risk-free short-term interest rate. If market participants are risk-neutral, the risk-neutral probability measure coincides with the data generating measure H. However, in general, the risk-neutral probability measure does not coincide with the data generating process (see Piazzesi, 2010, p. 697). The Radon-Nikodym derivative, which is denoted in the following by ξ_t, $\xi_t \equiv dQ/dH$, provides the link between the risk-neutral measure Q and the data generating measure H (see Duffie, 2001, p. 110). It is used to convert one probability measure into an equivalent measure.[5]

The specification of the pricing kernel is in reduced form. Though it is not explicitly derived from underlying preferences and is in particular not expressed in terms of marginal utility, it is widely used in the finance and macro-finance literature since it does match empirical properties fairly well (see Dai and Singleton, 2002). For discrete time models, following Ang and Piazzesi (2003), the nominal pricing kernel m_{t+1} is defined by

$$m_{t+1} \equiv \exp\left(-r_t\right) \frac{\xi_{t+1}}{\xi_t}, \tag{3.9}$$

and ξ_t is supposed to follow the log-normal process

$$\xi_{t+1} = \xi_t \exp\left(-\frac{1}{2}\lambda_t^{'}\lambda_t - \lambda_t^{'}\varepsilon_{t+1}\right), \tag{3.10}$$

where λ_t is an 11-dimensional vector of time-varying prices of risk. Combining eq.

[5] Given the existence of the risk-neutral measure, for any random variable with finite variance the following holds:

$$E_t^Q\left(Z_{t+1}\right) = \frac{E_t\left(\xi_{t+1} Z_{t+1}\right)}{\xi_t},$$

where $E_t^Q\left(\cdot\right)$ denotes the time $t-$conditional expectations under Q, $E_t\left(\cdot\right)$ the time $t-$conditional expectations under H, and where ξ_t is martingale (see Duffie, 2001, p. 168).

(3.9) and (3.10) yields the pricing kernel,

$$m_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t^{'}\lambda_t - \lambda_t^{'}\varepsilon_{t+1}\right). \tag{3.11}$$

The log-normal pricing kernel depends on the short-term interest rate, the structural shocks and the prices of risk. The prices of risk drive the response of the long-term government bond yields to macro, policy and risk shocks. If all elements in λ_t are equal to zero, pricing takes places under the risk-neutral probability measure.

The prices of risk are supposed to be affine functions of the state variables, taking the functional form

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{3.12}$$

where λ_0 is an 11×1 vector and λ_1 is an 11×11 matrix. For the market prices of risk, I assume that only contemporaneous state variables are priced. The vector of constants λ_0 is given by

$$\lambda_0 = \begin{bmatrix} \lambda_0^r & 0 & 0 & \lambda_0^\pi & 0 & 0 & \lambda_0^y & 0 & 0 & \lambda_0^{\pi^*} & \lambda_0^v \end{bmatrix}'.$$

Note that the coefficients in λ_0 and λ_1 do no vary over time. All fluctuations in the prices of risk λ_t are caused by movements in the state variables in X_t. Evidence by Cochrane and Piazzesi (2005, 2008) indicates that one single factor accounts for a large portion of variation in one-period return premia. In the spirit of this factor, the risk variable v_t is constructed to be the single source for time variation in the prices of risk. Following Dewachter and Iania (2011), Dewachter et al. (2014), and Ireland (2015) the identification of the risk variable is done by setting all elements

in λ_1, except the last column, to be equal to zero,

$$\lambda_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^r \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^\pi \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^y \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^{\pi^*} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^v \end{bmatrix}. \tag{3.13}$$

From eq. (3.12) together with the restrictions in eq. (3.13) all movements in the price of risk arise only from changes in the variable that is ordered as the last element in the vector X_t, that is, the risk variable v_t. As discussed in Section (3.3.3), these restrictions work to attribute movements in term premia to changes in the risk variable v_t.

Bond Prices

Given the pricing kernel, the assumption of the absence of arbitrage opportunities implies that under the data generating probability measure for any gross return R_t of a nominal asset the following equation holds

$$E_t\left(m_{t+1}R_{t+1}\right) = 1. \tag{3.14}$$

Let P_t^τ denote the price of a default-free, zero-coupon bond maturing in τ periods. Then, eq. (3.14) implies that all zero-coupon bond prices can be computed recursively by the no-arbitrage condition

$$P_t^{(\tau)} = E_t\left(m_{t+1}P_{t+1}^{(\tau-1)}\right). \tag{3.15}$$

That is, the time t price of a $\tau+1$-period zero-coupon bond equals the expected discounted price of a τ-period discount bond in period $t+1$, where pricing occurs under the data-generating measure using the stochastic discount factor m_{t+1}.

Given this set-up, Ang and Piazzesi (2003) demonstrate that the price of a zero-coupon bond $P_t^{(\tau)}$ maturing at time $t+\tau$ can be written as an exponentially affine function of the state vector X_t. Thus, the price of a bond maturing in τ-periods is

$$P_t^{(\tau)} = \exp\left(\bar{A}_\tau + \bar{B}_\tau' X_t\right), \tag{3.16}$$

where the coefficients $\bar{A}_t$ and $\bar{B}_\tau$ can be computed recursively by the following ordinary differential equations (see Appendix (3.A.2))

$$\bar{A}_{\tau+1} = \bar{A}_\tau + \bar{B}_\tau'(\mu - \Sigma\lambda_0) + \frac{1}{2}\bar{B}_\tau'\Sigma\Sigma'\bar{B}_\tau - \delta_0, \tag{3.17}$$

$$\bar{B}_{\tau+1}' = \bar{B}_\tau'(P - \Sigma\lambda_1) - \delta_1'. \tag{3.18}$$

Eq. (3.11), (3.15), and $P_{t+1}^0 = 1$ together imply that the log discount bond price of a bond maturing next period is given by $\log(P_t^1) = -r_t$. Consistency of eq. (3.8) and (3.16) for $\tau = 1$, given $\log(P_t^1) = -r_t$, requires then that the initial condition for $\bar{A}_\tau$ and $\bar{B}_\tau$ are given by: $\bar{A}_1 = \delta_0 = 0$, and $\bar{B}_1' = -\delta_1'$. The τ-period zero-coupon

bond yield $y_t^{(\tau)}$ is related to the bond price by

$$y_t^{(\tau)} = -\frac{\log\left(P_t^{(\tau)}\right)}{\tau}. \tag{3.19}$$

Substituting eq. (3.16) into eq. (3.19), yields the affine yield curve representation with functional form

$$y_t^{(\tau)} = A_\tau + B_\tau' X_t. \tag{3.20}$$

where $A_\tau \equiv -\bar{A}_\tau/\tau$ and $B_\tau \equiv -\bar{B}_\tau/\tau$.

3.3.3 Term Structure Premia and the Expectation Hypothesis

Term structure premia can be captured in different forms (see e.g. Cochrane and Piazzesi, 2008, or Joslin et al., 2014). In the following, similar to Dewachter et al. (2014), I will focus on the yield premium and the return premium. The definition of these premia is based on Cochrane and Piazzesi (2008). The yield premium is the most prominent form of the term premium and the one used by Ireland (2015). It can be composed into the average of expected future return premia of declining maturities. The one-period return premium, in turn, is only driven by the risk variable v_t. Before discussing both types of term structure premia, their relationship to each other and their relation to the risk variable, I will review some relevant basic relationships between holding period returns, excess holding returns and bond prices (see e.g. Cochrane, 2005, or Cochrane and Piazzesi, 2008). The holding period return $hpr_{t+1}^{(\tau)}$ is the return from buying a bond at time t that matures in $t+\tau$ periods and

selling this bond the period after. Formally, it is defined by

$$hpr_{t+1}^{(\tau)} \equiv p_{t+1}^{(\tau-1)} - p_t^{(\tau)}, \tag{3.21}$$

where $p_t^{(\tau)}$ is the log price of a zero-coupon bond maturing in $t+\tau$ periods, $p_t^{(\tau)} \equiv \log\left(P_t^{(\tau)}\right)$. The excess holding period return (or short excess return) $hprx_{t+1}^{(\tau)}$ is the return from buying a long term bond in period t and selling it in the subsequent period in excess of the return from buying and holding a short term bond maturing next period,

$$hprx_{t+1}^{(\tau)} \equiv hpr_{t+1}^{(\tau)} - y_t^{(1)}. \tag{3.22}$$

The yield of a τ-period zero-coupon default-free long-term bond $y_t^{(\tau)}$ can be decomposed in an expectation part and a part which is denoted as the *yield* premium $\kappa_t^{(\tau)}$ (see e.g. Cochrane and Piazzesi, 2008):

$$y_t^{(\tau)} = \frac{1}{\tau} E_t \left(\sum_{i=0}^{\tau-1} y_{t+i}^{(1)} \right) + \kappa_t^{(\tau)}. \tag{3.23}$$

The expectation part consists of the average of expected future short rates over the bond's residual maturity. Rearranging eq. (3.23) gives the definition of the yield premium. Thus, the yield premium can be interpreted as the average expected return from buying a τ-period bond and holding this bond until maturity financed by a sequence of short-term debt. It is the compensation that a risk-averse investor demands for holding a long-term bond instead of a sequence of short-term bonds. Under the (pure) expectation hypothesis of the term structure, this premium is (zero) constant.

The yield premium can be written as the average of expected future return premia of declining maturity (as in Cochrane and Piazzesi, 2008, or Ludvigson and Ng, 2009;

for a detailed derivation see Appendix (3.A.3)), where the respective *return* premium is defined as the expected $i+1$-period excess return, $E_t\left(hprx_{t+i+1}^{\tau}\right)$,

$$\kappa_t^{(\tau)} = \frac{1}{\tau}\sum_{i=0}^{\tau-1} E_t\left(hprx_{t+i+1}^{(\tau-i)}\right), \tag{3.24}$$

with

$$E_t\left(hprx_{t+i+1}^{(\tau-i)}\right) = E_t\left(hpr_{t+i+1}^{(\tau-i)} - y_{t+i}^{(1)}\right).$$

Under the expectation hypothesis, these premia are constant but maturity specific. Eq. (3.24) illustrates that the yield- and the return premium (subsumed under the expression "term structure premium") are not the same objects, but both are related and can be derived from the other. While the yield premium reflects the premium in a bond yield over the full lifetime of the bond, the return premium reflects the per-period holding premium. Moreover, if return premia are zero or constant, also the yield premium would be zero or constant.

In order to compute the yield and the return premium, the expectations of future short rates and excess returns have to be calculated. Following Ireland (2015), the expected value of the future short-term rate can be written as

$$E_t\left(y_{t+j}^1\right) = E_t\left(r_{t+j}\right) = \delta_1' E_t\left(X_{t+j}\right),$$

given eq. (3.8). Now define the unconditional expectation of the state vector by $\bar{\mu}$, $\bar{\mu} \equiv E\left(X_t\right)$, then, from eq. (3.7) one can write $\bar{\mu} = \left(I - P\right)^{-1}\mu$. Subtracting $\bar{\mu}$ from both sides of eq. (3.7) yields the (demeaned) state equation:

$$X_{t+1} - \bar{\mu} = P\left(X_t - \bar{\mu}\right) + \Sigma\varepsilon_{t+1}.$$

Then, the time-t conditional expected future short rate for period $t+j$, $\forall j > 0$, can be computed by

$$E_t\left(r_{t+j}\right) = \delta_1'\left(I - \delta' P^j\right)\bar{\mu} + \delta_1' P^j X_t$$

By rearranging eq. (3.23), and using $y_t^{(\tau)} = A_\tau + B_\tau' X_t$ the yield premium is given by

$$\kappa_t^{(\tau)} = A_\tau - \delta_1'\left[I - \frac{1}{\tau}\sum_{j=0}^{\tau-1} P^j\right]\bar{\mu} + \left[B_\tau - \delta_1'\frac{1}{\tau}\sum_{j=0}^{\tau-1} P^j\right] X_t.$$

Using $\Sigma_{j=0}^{\tau-1} P^j = \left(I - P^\tau\right)\left(I - P\right)^{-1}$, the yield premium can be expressed in a computationally more convenient form (as in Ireland, 2015)

$$\begin{aligned} \kappa_t^{(\tau)} &= A_\tau - \delta_1'\left(I - \frac{1}{\tau}\left(I - P^\tau\right)\left(I - P\right)^{-1}\right)\bar{\mu} \\ &\quad + \left(B_\tau' - \delta_1'\frac{1}{\tau}\left(I - P^\tau\right)\left(I - P\right)^{-1}\right) X_t. \end{aligned} \tag{3.25}$$

The return premium can be calculated by plugging the model implied log prices, $p_t^{(\tau)} = \bar{A}_\tau + \bar{B}_\tau' X_t$, into the definition of the $i+1$-period return premium and rearranging terms (see Appendix (3.A.4)),

$$\begin{aligned} E_t\left(hprx_{t+i+1}^{(\tau)}\right) &= \bar{B}_{\tau-1}'\Sigma\left[\lambda_0 + \lambda_1\left(I - P^i\right)\bar{\mu} + \lambda_1 P^i X_t\right] \\ &\quad -\frac{1}{2}\bar{B}_{\tau-1}'\Sigma\Sigma'\bar{B}_{\tau-1} \end{aligned} \tag{3.26}$$

If $i = 0$, then eq. (3.26) is the one-period return premium. Precisely, the one-period return premium of a bond with maturity τ is given by

$$E_t\left(hprx_{t+1}^{(\tau)}\right) = \bar{B}_{\tau-1}'\Sigma\left(\lambda_0 + \lambda_1 X_t\right) - \frac{1}{2}\bar{B}_{\tau-1}'\Sigma\Sigma'\bar{B}_{\tau-1}. \tag{3.27}$$

From the restrictions on the elements in λ_1 in eq. (3.13), eq. (3.27) reveals that

all variation over time in one-period return premia arises solely from fluctuations in v_t for all bond maturities. In contrast, to the extent that the risk variable is not zero over time, the yield premium is affected by all state variables if $\tau > 1$. To see this, recall that the yield premium can be written as the average of expected future return premia of declining maturity. Since the i-period return premium, in general, depends on all state variables, from eq. (3.24) also the yield premium depends on all state variables if $\tau > 1$.

Finally, if all elements in the matrix λ_1 are equal to zero, then the one-period return premium and the yield premium are constant. In this case, in eq. (3.27) the term $\lambda_1 X_t$ disappears, eliminating all time variation in the one-period return premium. Similar, as shown by Ireland (2015), if $\lambda_1 = 0_{11x11}$, eq. (3.18) is given by to

$$B'_\tau = \delta'_1 \frac{1}{\tau} \left(I - P^\tau\right) \left(I - P\right)^{-1}.$$

Plugging B'_τ in eq. (3.25) confirms that $\kappa_t^{(\tau)}$ is constant if all elements in the matrix λ_1 are equal to zero. The discussion of the different types of term premia completes the model section.

3.4 Estimation

The first part of this section discusses the data set that is used for the estimation of the model. The next part presents the state-space system. Then the estimation method is discussed. The last part presents and discusses the choice of the prior distributions for the parameters.

3.4.1 Data

I include Euro area data from September 2004 to April 2014 in my sample. The data set contains macro data and yield data. The data is taken from the Bundesbank and the ECB. The macroeconomic variables are the inflation rate, the output gap, and the nominal short-term interest rate. The financial variables are the yields from an index of risk-free zero-coupon treasury bonds of European countries with maturities of 12, 24, 36, 48, and 60 months. The yield data is only available from the ECB since Fall 2004, restricting effectively the size of the available sample. Due to the short sample size of the dataset - roughly ten years - I use monthly data. This compromises between the high-frequency yield data and the lower frequency macro data. The sample space covers 116 observations per time series. Moreover, data for the risk-free short-term interest rate - the OIS rate for the Eurozone - is only available since mid-2005. During the estimation, the yield data from Fall 2004 until June 2005 are treated as missing observations. The time path of the missing observations is constructed by the Kalman filter.

The output gap variable is defined as the percentage (logarithmic) deviation of actual output from trend output. Since GDP data is only available on a quarterly frequency, I use the seasonally adjusted industrial production index of the Euro area (Euro area 18, fixed composition) as a proxy for output (as e.g. in Ang and Piazzesi, 2003, Clarida et al., 1998, or Favero, 2006). Trend output is constructed by using a linear-quadratic trend (as in Clarida et al., 1998, or Hördahl, 2008). The inflation rate is measured by the annual rate of change of the seasonally adjusted HICP of the Euro area in percentage. For the risk-free zero-coupon yield data, an index of government bonds of countries from the euro area is used. The government bond index consists of all countries of the euro area that are AAA rated. All yields are

continuously compounded. The yield data is taken from the ECB. The yield index of risk-free zero-coupon treasury bonds is not available for bonds with one-month residual maturity. To overcome this shortcoming, the risk-free nominal short-term interest rate is proxied by the Overnight Indexed Swap (OIS) rate. The OIS rate is an interest rate swap with a floating rate indexed to an overnight interbank rate. In the case of the Euro area, this overnight interbank rate is the EONIA. It had become, in particular for the euro area, a lately widely used measure of the risk-free rate (among others by Borgy et al. 2012, Dewachter et al., 2015, Dubeq et al., 2016, Filipović and Trolle, 2013, Finlay and Chambers, 2009, or Joyce et al., 2011), rather than inter-bank rates like the EONIA.[6] The OIS rate data is taken from the Bundesbank.

Table (3.1) provides some summary statistics of the data for the macroeconomic variables and the yield data. The sample average of inflation is around the ECB's announced inflation target of 2 percent. By construction, the mean of the output gap is equal to zero. All macroeconomic variables are persistent, reflected by high first- to third-order autocorrelation. The summary statistics of the yields confirm that the employed yield data are line with stylized facts of yield curves[7] (though the sample space covers the financial crisis): First, the average yield curve is upward sloping. Thus, the longer the residual maturity of a government bond, the higher are yields. Second, the term structure of volatility of yields is downward sloping. The standard deviation of yields declines with maturity. Third, yields are highly autocorrelated. The first- to third-order sample autocorrelations are not below 0.94. Fourth, yields move closely together. The correlation between yields of treasury

[6] Euro area inter-bank rates, which are on unsecured interbank lending, are quite likely to compromise a certain amount of premia for credit risks, in particular, since the onset of the financial crisis in 2007. In contrast, netting and credit enhancement mechanisms of in swap contracts seem to work, also in times of financial turmoil, to mitigate counterparty risk (see Bomfim, 2003).

[7] See for example Ang and Piazzesi, 2003, Campbell, 1995, or Hördahl, Tristani, and Vestin, 2008.

Table 3.1: Descriptive Data Statistics

Obs.:	Moments				Autocorrelation		
116*	Mean	S.D.	Skewness	Kurtosis	1. Lag	2. Lag	3. Lag
π_t	1.9619	0.9031	−0.5073	3.4419	0.9591	0.8970	0.8101
g_t^y	0	4.9900	−0.5449	3.2642	0.9700	0.9281	0.8643
r_t	1.5976	1.5223	0.5976	1.7124	0.9938	0.9791	0.9579
y_t^{12}	1.6825	1.4942	0.4331	1.6909	0.9918	0.9761	0.9547
y_t^{24}	1.8873	1.4262	0.2816	1.7316	0.9892	0.9722	0.9511
y_t^{36}	2.1010	1.3350	0.1095	1.7670	0.9877	0.9705	0.9497
y_t^{48}	2.3142	1.2431	−0.0487	1.8122	0.9862	0.9690	0.9487
y_t^{60}	2.5185	1.1566	−0.1835	1.8681	0.9843	0.9666	0.9468

Source: Yield data, industrial production and inflation: ECB; OIS rate: Bundesbank. The yields are annual zero-coupon bond yields. Inflation is calculated as the percentage year-to-year change of the HICP of the Eurozone. Output is measured by industrial production and the output gap is defined as the deviation of actual output from its trend.
* For the OIS rate, the sample period is 2005:07 to 2014:04, covering 106 observations in total.

bonds with a maturity of 12 and yields of treasuries with a maturity of 36 months is equal to 0.9769 (not displayed in the table) and the correlation between yields of treasury bonds with a maturity of 60 months and yields of treasuries with a maturity of 60 months is equal to 0.9880.

3.4.2 The State-Space System

The macro part and the affine term structure model form a state-space system. The state equation, given by eq. (3.7), describes the dynamic of the state variables, while the observables - output gap, inflation, the short-term interest, and the long-term government bond yields - are linked to the state vector by measurement equations.

For the estimation, a version of the state-space model without constant terms is employed. By dropping the constant terms appearing in eq. (3.7) and (3.20) and using demeaned data the estimation is simplified. Precisely, following Ireland (2015), under the assumption that the central bank is able - on average - to implement

its target inflation rate, so that the average of the actual inflation rate equals the average target inflation rate, the steady state values of g^r, τ and g^y can be calibrated to match the data averages of the short-term interest rate, the output gap and inflation. Moreover, as demonstrated in Ireland (2015), the values of the elements in λ_0 can be calibrated so that the steady state values of yields match the average yields. Thus, the state-space system is given by

$$X_t = PX_{t-1} + \Sigma\varepsilon_t, \tag{3.28}$$

$$Z_t = UX_t + V\eta_t, \tag{3.29}$$

where the vector Z_t containing the eight observables is defined by

$$Z_t \equiv \begin{bmatrix} r_t & \pi_t & g_t^y & y_t^{12} & y_t^{24} & y_t^{36} & y_t^{48} & y_t^{60} \end{bmatrix}',$$

the matrix U is specified by

$$U = \begin{bmatrix} U_r \\ U_\pi \\ U_y \\ B_{12}' \\ B_{24}' \\ B_{36}' \\ B_{48}' \\ B_{60}' \end{bmatrix},$$

with

$$
\begin{aligned}
U_r &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \\
U_\pi &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \\
U_y &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\end{aligned}
$$

the vector B'_n, $n = \{12,\ 24,\ 36,\ 48,\ 60\}$, is determined by eq. (3.18), given the definition $B_\tau \equiv -\bar{B}_\tau/\tau$ and for given starting values $\bar{B}'_1 = -\delta'_1$, and the matrix V contains the volatility parameters of the measurement errors η_t. These errors are attached in order to avoid stochastic singularity. The problem of stochastic singularity arises in this type of models because numerous yield data are observed, but only a few structural shocks of potentially also observable state variables are used, so that the number of observable variables exceeds the number of shocks. Noise or measurement errors are added in order to give the model the ability to fit the high dimensional data vector with a lower dimensional state vector. Two different assumptions on the nature of these measurement errors are commonly drawn: Either only some yields are measured with errors (as e.g. in Ang and Piazzesi, 2003, or Ireland, 2015) or all yields are measured with errors (as e.g. in Ang et al., 2007, or Chib and Ergashev, 2009). Following, Chib and Ergashev (2009), I will treat all yields (except the policy rate) as measured with errors.[8] Specifically, the matrix V

[8] As discussed in Piazzesi (2010, p. 726), supposing that only a certain number of yields - that is, the required number of shocks that needs to be added in order to avoid stochastic singularity - is observed with errors seems to be arbitrary for the particular choice of which yields are observed with error and which not. Data entry mistakes and interpolation methods for construction the zero-coupon yield date might lead to errors that should potentially affect all yields. Thus, if some yields are measured with errors the assumption that possibly all yields are observed with errors seems to be plausible. See Piazzesi (2010, pp. 726) for a more detailed discussion of noise- or measurement errors in the context of affine term structure models.

is given by

$$V = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \sigma_{12} & 0 & 0 & 0 & 0 \\ 0 & \sigma_{24} & 0 & 0 & 0 \\ 0 & 0 & \sigma_{36} & 0 & 0 \\ 0 & 0 & 0 & \sigma_{48} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{60} \end{bmatrix}$$

with σ_{12}, σ_{24}, σ_{36}, σ_{48} and $\sigma_{60} > 0$ and the vector of the corresponding measurement errors η_t is given by

$$\eta_t = \begin{bmatrix} \eta_t^{12} & \eta_t^{24} & \eta_t^{36} & \eta_t^{48} & \eta_t^{60} \end{bmatrix}'.$$

These zero-mean measurement errors are supposed to be standard normally distributed.

3.4.3 Estimation Method

To estimate the state-space model, I apply Bayesian estimation techniques. As often noted in the literature, even the estimation of pure latent affine term structure models is computationally challenging and time-consuming (see e.g. Chib and Ergashev, 2009, or Christensen et al., 2011). Adding the macro-dynamics enhances these difficulties due to the complexity of the macroeconomic interactions with the term structure and vice versa (Rudebusch and Wu, 2008). The parameters in the $B_{(\tau)}$ matrices of the observation equations are highly non-linear functions of the underlying parameters of the state equations and the prices of risk. This non-linearity, as demonstrated by Chib and Ergashev (2009), can produce multimodal likelihood

functions. Applying Bayesian estimation techniques allow employing a priori information which helps to down-weight regions of the parameter space which are not economically reasonable and to rule out economically implausible parameter values. As a result, the posterior distribution can be smoother than the likelihood function (see Chib and Ergashev, 2009). Moreover, the usage of prior information is helpful when dealing with short data sets.

Posterior and Likelihood Function

Formally, let Z denotes the data set, $Z = (Z_1, ..., Z_T)'$, where T is the number of total observations, and let θ denotes the vector of all parameters contained in the matrices P, Σ, Λ, and V, then from Bayes' rule, the joint posterior distribution of θ, $\pi(\theta|X)$, is obtained by combining the likelihood function of the observables, the prior distribution of the parameter vector, and a norming constant. Thus,

$$\pi(\theta|Z) \propto L(Z|\theta)\, p(\theta),$$

where $L(Z|\theta)$ is the likelihood function, and $p(\theta)$ is the prior distribution. Denote by $\mathcal{Z}_{t-1}$ all available information of the observable variables at time $t-1$, $\mathcal{Z}_{t-1} \equiv (Z_1, ..., Z_{t-1})'$. If the initial state X_0 and the innovations $\{\varepsilon_t, \eta_t\}_{t=1}^{T}$ are multivariate Gaussians, then the conditional distribution of the observables Z_t on $\mathcal{Z}_{t-1}$ is also Gaussian (see Hamilton, 1994, p. 385)

$$Z_t|\mathcal{Z}_{t-1} \sim N\left(UX_{t|t-1}, R_{t|t-1}\right),$$

where $X_{t|t-1}$ denotes the one step ahead forecast, $X_{t|t-1} \equiv E\left[X_t|\mathcal{Z}_{t-1}, \theta\right]$, and $R_{t|t-1}$ denotes the conditional variance, $R_{t|t-1} \equiv Var\left(Z_t|\mathcal{Z}_{t-1}, \theta\right)$.[9] Hence, the joint density of the date set Z given θ can be written as

$$\begin{aligned} L\left(Z|\theta\right) &= \prod_{t=1}^{T} \left(2\pi\right)^{-\frac{T}{2}} \left[\det\left(R_{t|t-1}\right)\right]^{-\frac{1}{2}} \\ &\quad \times \exp\left(-\frac{1}{2}\left(Z_t - UX_{t|t-1}\right)' \left(R_{t|r-1}\right)^{-1} \left(Z_t - UX_{t|t-1}\right)\right). \end{aligned}$$

Since two of the state variables are latent, the likelihood $L\left(Z|\theta\right)$ is constructed using the standard Kalman filter recursions (see Harvey, 1991). At the start of the recursions, the initial matrix of the variance of the forecast errors is set equal to the unconditional variance of the state variables.

Since the posterior density is, in general, not known in closed form, I apply Markov Chain Monte Carlo (MCMC) methods (the Adaptive-Metropolis algorithm) in order to simulate draws from the joint posterior distribution.

MCMC Method

The choice of the proposal density of the Metropolis-Hastings algorithm is crucial for the speed of the convergence of the chain. The scaling of the posterior distribution is often done by trial and error. But not only is the scaling of the proposal density "by hand" in general time-consuming, improving the proposal distribution manually also becomes very difficult, if not infeasible, in high-dimensional problems (see Rosenthal, 2011, p. 95). Therefore, I employ the Adaptive Metropolis (AM) algorithm as introduced by Haario et al. (2001) to evaluate the posterior. The main idea of the AM algorithm is to run a chain that alters its own proposal distribution by using all

[9] See Appendix (3.A.5) for the explicit expressions of the prediction and updating equations of the mean and the variance.

information about the posterior cumulated so far. Thus, the algorithm improves on the fly. Precisely, the covariance of the proposal distribution is updated each step using all available information. Apart from the updating scheme, the algorithm is identical to the standard random walk Metropolis-Hastings algorithm. Due to the adaptive nature of the algorithm, it is non-Markovian, but Haario et al. (2001) show that it still has the correct ergodic properties.

Let θ_0, ..., θ_{j-1}, denote the sampled parameters until $j-1$ iterations, where θ_0 is the initial set of parameters. I follow Haario et al. (2001) and let the proposal distribution, denoted by $q(\cdot|\theta_0, ..., \theta_{j-1})$, be a multivariate Gaussian distribution with mean at the current value of the parameter vector θ_{j-1} and a covariance matrix C_t. The algorithm starts with a pre-specified strictly positive proposal distribution covariance C_0. After an initial period n_0 the adaption takes place by updating the covariance of the proposal distribution according to $C_j = s_d Cov(\theta_0, ...\theta_j) + s_d \varepsilon I_d$, where s_d is a parameter that depends only on the dimension d of the parameter vector θ and $\varepsilon > 0$ is a (very small) constant employed to prevent C_j from becoming singular. In practice, the calculation of the covariance C_j is simplified using the following recursion formula (see Haario et al., 2001):

$$C_{j+1} = \frac{j-1}{j} C_j + \frac{s_d}{j} \left(\bar{\theta}_{j-1} \bar{\theta}'_{j-1} - (j+1) \bar{\theta}_j \bar{\theta}'_j + \theta_j \theta'_j + \varepsilon I_d \right).$$

Precisely, the AM algorithm is given by the following steps:

1. Set the number of total iterations n and specify the initial period n_0 $(n_0 < n)$ after which the adaption starts. Chose an (arbitrary) positive definite initial covariance matrix C_0 and specify the initial parameter vector θ_0. Set $C_j = C_0$ and $\theta_{j-1} = \theta_0$.

2. Draw a candidate θ_j^* from $q(\cdot|\theta_{j-1}, C_j)$

3. Compute $\alpha\left(\theta_j^*, \theta_{j-1}\right) = \min\left[1, \frac{\pi\left(\theta_j^*|\cdot\right)}{\pi(\theta_{j-1}|\cdot)}\right]$.

4. Set $\theta_j = \theta_j^*$ with probability $\alpha\left(\theta_j^*, \theta_{j-1}\right)$

   and set $\theta_j = \theta_{j-1}$ with probability $1 - \alpha\left(\theta_j^*, \theta_{j-1}\right)$.

5. Update $C_{j+1} = \begin{cases} C_0, & j \leq n_0 \\ s_d Cov\left(\theta_0, ...\theta_j\right) + s_d \varepsilon I, & j > n_0 \end{cases}$.

6. Repeat step 2-5 until $j = n$.

Haario et al. (2001) note that the choice of an appropriate initial covariance C_0 helps to speed up the algorithm and thus to increase efficiency. Therefore, I use a scaled down version of the inverse of the Hessian matrix computed at the posterior mode for the initial covariance matrix. The initial parameter vector is set to the parameter values at the mode. For the choice of the scaling parameter s_d, I follow Haario et al. (2001), whose choice, in turn, is based on Gelman et al. (1996), and set $s_d = (2.4)^2/d$. The initial period is set to $n_0 = 20,000$ and the number of draws is set to $n = 1,000,000$.

As noted by Chib and Ergashev (2009), the mode of the posterior can in general not be found using Newton-like optimization methods. Therefore, I employ the Covariance Matrix Adaption Evolution Strategy (CMA-ES) algorithm. The CMA-ES is a stochastic method for numerical parameter optimization of non-linear, non-convex functions with many local optima. It belongs to the class of evolutionary optimization algorithms (see Hansen and Ostermeier, 2001). The computation of the mode is conducted by the software package Dynare (Adjemian et al., 2011).

3.4.4 Parameter Restrictions and Prior Distributions

Parameter Restrictions

In addition to restrictions on the interest rate rule parameters and on the parameter of the inflation target process (the non-negativity restrictions of ρ_y and ρ_π, and the restriction that ρ_r and $\rho_\tau \in [0, 1)$, during the estimation the following restrictions are imposed.

To ensure the stationarity of the state equation, the eigenvalues of P are constrained to be less than unity in absolute value, $|eig(P)| < 1$. Likewise, a similar eigenvalue restrictions need to be imposed in order to ensure the stability of the no-arbitrage recursions (see Dai and Singleton, 2000). Specifically, the eigenvalues of $P - \Sigma\lambda_1$ are constrained to be less than unity in absolute value, $|eig(P - \Sigma\lambda_1)| < 1$. For identification, the parameter σ_v of the latent variable needs to be normalized. As well known in the literature of latent factor models (e.g. Dai and Singleton, 2000), multiplicative transformations of the latent factor lead to observationally equivalent systems. In order to fix the scale of the latent variable, $\sigma_v = 0.01$ is imposed. Additionally, the direction in which an increase in the risk variable v_t moves term structure premia needs to be pinned down. Following Ireland (2015), without loss of generality, the constraint $\Lambda^\pi \leq 0$ is imposed during the estimation. Finally, similar to Dewachter et al. (2014) and Ireland (2015), to restrict v_t from being itself a source of priced risk, the constraint $\Lambda^v = 0$ is imposed.

After imposing these restrictions, there are 50 parameters left to estimate in eq. (3.28) - (3.29). The next sub-section presents the prior distributions for these parameters.

Table 3.2: Summary of the Prior Distributions

Taylor Rule							
Parameter	Type	Mean	S. D.	Parameter	Type	Mean	S. D.
ρ_r	$\mathcal{B}$	0.80	0.05	ρ_π	$\mathcal{G}$	1.500	0.250
ρ_v	$\mathcal{N}$	0.00	0.250	ρ_y	$\mathcal{G}$	0.500	0.15
Macro Part							
Parameter	Type	Mean	S. D.	Parameter	Type	Mean	S. D.
$\rho^1_{\pi r}$	$\mathcal{N}$	-0.20	0.150	$\rho^2_{y\pi}$	$\mathcal{N}$	0.00	0.075
$\rho^2_{\pi r}$	$\mathcal{N}$	0.00	0.075	$\rho^3_{y\pi}$	$\mathcal{N}$	0.00	0.050
$\rho^3_{\pi r}$	$\mathcal{N}$	0.00	0.050	ρ^1_{yy}	$\mathcal{N}$	0.90	0.150
$\rho^1_{\pi\pi}$	$\mathcal{N}$	0.90	0.150	ρ^2_{yy}	$\mathcal{N}$	0.00	0.075
$\rho^2_{\pi\pi}$	$\mathcal{N}$	0.00	0.075	ρ^3_{yy}	$\mathcal{N}$	0.00	0.050
$\rho^3_{\pi\pi}$	$\mathcal{N}$	0.00	0.050	$\rho_{v\pi}$	$\mathcal{N}$	0.00	0.150
$\rho^1_{\pi y}$	$\mathcal{N}$	0.00	0.150	ρ_{vv}	$\mathcal{N}$	0.90	0.150
$\rho^2_{\pi y}$	$\mathcal{N}$	0.00	0.075	ρ_{vy}	$\mathcal{N}$	0.00	0.150
$\rho^3_{\pi y}$	$\mathcal{N}$	0.00	0.050	ρ_{vr}	$\mathcal{N}$	0.00	0.150
ρ^1_{yr}	$\mathcal{N}$	-0.20	0.150	$\rho_{v\pi^*}$	$\mathcal{N}$	0.00	0.150
ρ^2_{yr}	$\mathcal{N}$	0.00	0.075	ρ_{π^*}	$\mathcal{B}$	0.90	0.100
ρ^3_{yr}	$\mathcal{N}$	0.00	0.050	ρ_{yv}	$\mathcal{N}$	0.00	0.250
$\rho^1_{y\pi}$	$\mathcal{N}$	0.00	0.150	$\rho_{\pi v}$	$\mathcal{N}$	0.00	0.250
Volatility and co-movement parameters							
Parameter	Type	Mean	S. D.	Parameter	Type	Mean	S. D.
σ_{vr}	$\mathcal{N}$	0.00	2.00	σ_π	$\mathcal{IG}$	0.01	0.200
$\sigma_{v\pi}$	$\mathcal{N}$	0.00	2.00	σ_y	$\mathcal{IG}$	0.01	0.200
σ_{vy}	$\mathcal{N}$	0.00	2.00	σ_{π^*}	$\mathcal{IG}$	0.01	0.200
$\sigma_{v\tau}$	$\mathcal{N}$	0.00	2.00	σ_{12}	$\mathcal{IG}$	0.0001	0.001
$\sigma_{y\pi}$	$\mathcal{N}$	0.00	2.00	σ_{24}	$\mathcal{IG}$	0.0001	0.001
$\sigma_{y\tau}$	$\mathcal{N}$	0.00	2.00	σ_{36}	$\mathcal{IG}$	0.0001	0.001
$\sigma_{\pi\tau}$	$\mathcal{N}$	0.00	2.00	σ_{48}	$\mathcal{IG}$	0.0001	0.001
σ_r	$\mathcal{IG}$	0.01	0.20	σ_{60}	$\mathcal{IG}$	0.0001	0.001
Prices of Risk							
Parameter	Type	Mean	S. D.	Parameter	Type	Mean	S. D.
Λ^r	$\mathcal{N}$	0.00	25.00	Λ^π	$\mathcal{N}$	0.00	25.00
Λ^τ	$\mathcal{N}$	0.00	25.00	Λ^y	$\mathcal{N}$	0.00	25.00

Summary of the prior distributions of the parameters. Type of the distribution is either $\mathcal{N}$, $\mathcal{B}$, $\mathcal{G}$, or $\mathcal{IG}$ where $\mathcal{N}$ denotes the Normal distribution, $\mathcal{B}$ the Beta distribution, $\mathcal{G}$ the Gamma distribution, and $\mathcal{IG}$ the Inverse-Gamma distribution.

Prior Distributions

Using prior information from previous studies and restricting parameters to lie in an economically reasonable region helps to reduce the complexity of the maximization problem by down-weighting economically non-meaningful regions of the parameter space (see Chib and Ergashev, 2009, for a more detailed discussion). The first part of table (3.2) displays the prior distributions of the coefficients of the monetary policy rule. I follow closely Smets and Wouters (2003) for the choice of these priors. The parameter capturing the degree of interest rate smoothing ρ_r is supposed to lay in the interval between 0 and 1. Therefore, for the prior distribution of ρ_r the Beta distribution is employed. I set the prior mean equal to 0.8 and the standard deviation equal to 0.05, assuming a high degree of interest rate inertia. For the prior distribution of the parameter governing central bank's reaction on deviation of the actual inflation rate from its target rate, a Gamma distribution with a mean of 1.5 and a standard deviation of 0.25 is used. I employ the Gamma distribution to ensure that the parameter ρ_π cannot be negative. The prior mean satisfies the Taylor principle. Likewise, I also suppose that the prior for the parameter of central bank's reaction on deviation from the output gap is gamma-distributed. The prior mean is chosen to correspond to the Taylor coefficient of 0.5. Finally, the coefficient of central bank's response to movements in term premia ρ_v is assumed to follow a Gaussian distribution with a mean of 0 and a standard deviation of 0.5, so that the interval $[-1.96; 1.96]$ covers 95% of the probability mass. Given the normalization of σ_v, the choice of the standard deviation implies a relatively uninformative prior. The choice of the prior means implies that monetary policy is, a priori, characterized by a standard Taylor rule.

The choice of the priors of the parameters describing the dynamics of the macro-

economy is displayed in the second part of table (3.2). As described in Section (3.3.1), these dynamics are modeled as in a structural VAR model. The priors for the VAR part (eq. 3.1 - 3.5) are chosen in the spirit of Minnesota (see Litterman, 1986) by assuming that almost all coefficients are Gaussian distributed and by setting the prior means of most of the coefficients equal to zero except for these coefficients corresponding to the first own lags of the dependent variables. These coefficients are set equal to 0.9, as suggested by Koop and Korobilis (2010). The choice of the prior means reflects the assumption that these variables exhibit a high degree of persistence but do not follow a unit root process. The standard deviations of the prior distributions of the parameters are weighted by the lag length, implying that with increasing lag length the coefficients are shrunk towards zero. As in Dewachter et al. (2014), I set the standard deviations for prior distributions of these coefficients on the first lags equal to 0.15. Departing from Minnesota and following Dewachter and Iania (2011) and Dewachter et al. (2014), I choose a negative prior mean for the parameters ρ^1_{yr} and $\rho^1_{\pi r}$. These choices capture the beliefs that an increase in the interest rate dampens economic activity. For the parameters ρ_{yv} and $\rho_{\pi v}$, I choose a relatively uninformative prior. Precisely, I set the prior mean equal to zero and the standard deviation equal to 0.25, assuming that movements in the risk variable do not affect output and inflation a priori. The coefficient of the inflation target process is Beta distributed with a mean of 0.9 and a standard deviation of 0.1. Employing the Beta distribution guarantees that the process of the inflation target is stationary, while it avoids that the central bank's inflation target jumps erratically.

The third part of table (3.2) presents the prior distributions of the volatility parameters of the structural shocks and of the measurement errors and the prior distributions of the co-movement parameters. The prior distributions of the volatility parameters of to the structural shocks and the measurement errors follow, similar to

Dewachter (2008), an Inverse Gamma distribution with a mean of 0.01 and 0.0001, respectively, and a standard deviation of 0.2 and 0.001, respectively. This specification captures the beliefs that measurement errors should be rather small. I employ the Inverse Gamma distribution in order to prevent the volatility parameter from being negative or equal to 0. The prior distributions for the co-movement parameters follow a Gaussian distribution with a mean of 0 and standard deviation of 2.

The last part of table (3.2) presents the priors for the prices of risk. For the choice of the prior distributions of the coefficients Λ^{π}, Λ^{y}, Λ^{r}, and Λ^{τ}, I follow Dewachter and Iania (2011) and Dewachter et al. (2014).

I use relatively uninformative priors, reflected by the choice of large standard deviations. More precisely, each element in the prices of risk is assumed to be Gaussian distributed with a mean of 0 and a standard deviation of 25.

The choice of the priors satisfies the stationarity condition of the state equation and the stability condition of the no-arbitrage recursions. Hence, under the chosen prior specification $|eig\,(P)| < 1$ and $|eig\,(P - \Sigma\lambda_1)| < 1$ hold.

3.5 Results

Table (3.3) and (3.4) list the results of the estimation. They report the posterior modes of the parameters, the posterior means, and the 90% highest posterior density (HPD) interval. While the posterior mode is obtained by maximizing the (log-) posterior distribution, the latter results are obtained by using the Adaptive Metropolis algorithm outlined in Section (3.4.3). First, the estimated values of the interest rate rule parameters are discussed. Then, I will evaluate the estimated mode by plotting impulse response functions (IRF) and decomposing the error forecast variance.

3.5.1 Policy Rule Coefficients

Focusing on the four estimated parameters of the interest rate rule displayed in the first four rows in the table (3.3), the results show that all four parameters are significantly different from zero, including the ECB's response parameter to movements in term structure premia ρ_v. The posterior mean of ρ_v is significantly different from zero and negative, $\rho_v = -0.3693$, implying that the ECB lowered the interest rate in response to a rise in term premia. Thus, in line with the practitioner view, this indicates that the central bank counteracted changes in term premia to retain the overall mix of financial conditions, balancing output and inflation. Carlstrom et al. (2015) demonstrate that in a DSGE model with imperfect financial markets a negative response coefficient on term premia in the monetary policy rule improves welfare. In contrast, Ireland (2015), who estimated the same parameter, but for the Fed with US data from the $1950th$ until 2007 (and for an extended sample until the end of 2014), using a restricted maximum likelihood approach, finds a significant positive coefficient.

The estimated values of the other three parameters of the interest rate rule are similar to those from studies using a more standard interest rate rules specification for the Euro Area (e.g. Andrés et al., 2006, or Smets and Wouters, 2003). The estimate of the interest rate inertia $\rho_r = 0.8730$ reflects a high degree of interest rate smoothing. The estimate of the coefficient measuring central bank's response to changes in the output gap is $\rho_y = 0.1651$. The estimated coefficient of the central bank's response to a change in inflation is larger than one, $\rho_\pi = 1.3681$, satisfying the Taylor principle.

Table 3.3: Results: Posterior Distributions (Part I)

Parameter	Prior Mean	Post. Mode	Post. Mean	90% HPD Interval		Prior
ρ_r	0.8000	0.8435	0.8730	0.8281	0.9161	$\mathcal{B}$
ρ_π	1.5000	1.4399	1.3681	1.0332	1.7358	$\mathcal{G}$
ρ_y	0.5000	0.1372	0.1651	0.0924	0.2434	$\mathcal{G}$
ρ_v	0.0000	-0.2881	-0.3693	-0.4804	-0.2552	$\mathcal{N}$
ρ_{vv}	0.9000	0.9057	0.8765	0.8213	0.9281	$\mathcal{N}$
ρ_{vr}	0.0000	-0.3190	-0.3314	-0.4824	-0.2086	$\mathcal{N}$
$\rho_{v\pi}$	0.0000	-0.0641	-0.0580	-0.2582	0.1349	$\mathcal{N}$
ρ_{vy}	0.0000	0.1533	0.1530	0.0908	0.2104	$\mathcal{N}$
$\rho_{v\pi^*}$	0.0000	-0.2806	-0.3008	-0.3839	-0.2241	$\mathcal{N}$
ρ_{π^*}	0.9000	0.9922	0.9896	0.9822	0.9964	$\mathcal{B}$
$\rho_{\pi v}$	0.0000	-0.0176	-0.0199	-0.0347	-0.0045	$\mathcal{N}$
ρ_{yv}	0.0000	-0.0410	-0.0444	-0.0917	0.0054	$\mathcal{N}$
$\rho^1_{\pi r}$	-0.2000	-0.0247	-0.0187	-0.1394	0.1023	$\mathcal{N}$
$\rho^2_{\pi r}$	0.0000	-0.0036	-0.0118	-0.1077	0.0929	$\mathcal{N}$
$\rho^3_{\pi r}$	0.0000	-0.0393	-0.0352	-0.1047	0.0383	$\mathcal{N}$
$\rho^1_{\pi\pi}$	0.9000	1.0414	1.0212	0.9238	1.1284	$\mathcal{N}$
$\rho^2_{\pi\pi}$	0.0000	0.0099	-0.0025	-0.0984	0.0865	$\mathcal{N}$
$\rho^3_{\pi\pi}$	0.0000	-0.0753	-0.0763	-0.1296	-0.0119	$\mathcal{N}$
$\rho^1_{\pi y}$	0.0000	-0.0306	-0.0238	-0.0597	0.0086	$\mathcal{N}$
$\rho^2_{\pi y}$	0.0000	0.0075	0.0111	-0.0370	0.0577	$\mathcal{N}$
$\rho^3_{\pi y}$	0.0000	0.0466	0.0399	0.0137	0.0693	$\mathcal{N}$
ρ^1_{yr}	-0.2000	-0.0295	0.0111	-0.1664	0.1740	$\mathcal{N}$
ρ^2_{yr}	0.0000	-0.0102	0.0031	-0.0999	0.1165	$\mathcal{N}$
ρ^3_{yr}	0.0000	-0.0389	-0.0252	-0.1124	0.0529	$\mathcal{N}$
$\rho^1_{y\pi}$	0.0000	0.1742	0.1652	-0.0135	0.3340	$\mathcal{N}$
$\rho^2_{y\pi}$	0.0000	-0.0080	0.0006	-0.1177	0.1066	$\mathcal{N}$
$\rho^3_{y\pi}$	0.0000	-0.0117	-0.0088	-0.0902	0.0730	$\mathcal{N}$

Summary of the posterior distributions of the parameters. Type of the distribution is either $\mathcal{N}$, $\mathcal{B}$, $\mathcal{G}$, or $\mathcal{IG}$ where $\mathcal{N}$ denotes the Normal distribution, $\mathcal{B}$ the Beta distribution, $\mathcal{G}$ the Gamma distribution, and $\mathcal{IG}$ the Inverse-Gamma distribution.

Table 3.4: Results: Posterior Distributions (Part II)

Parameter	Prior Mean	Post. Mode	Post. Mean	90% HPD Interval		Prior
ρ_{yy}^1	0.9000	1.1363	1.1275	1.0465	1.2118	$\mathcal{N}$
ρ_{yy}^2	0.0000	-0.0089	-0.0256	-0.1251	0.0627	$\mathcal{N}$
ρ_{yy}^3	0.0000	-0.1765	-0.1732	-0.2309	-0.1117	$\mathcal{N}$
σ_r	0.0100	0.0015	0.0016	0.0014	0.0018	$\mathcal{IG}$
σ_π	0.0100	0.0026	0.0026	0.0023	0.0029	$\mathcal{IG}$
σ_y	0.0100	0.0098	0.0102	0.0092	0.0113	$\mathcal{IG}$
σ_{π^*}	0.0100	0.0013	0.0013	0.0012	0.0015	$\mathcal{IG}$
$\sigma_{\pi\pi^*}$	0.0000	-0.9646	-1.0030	-1.3564	-0.6184	$\mathcal{N}$
$\sigma_{y\pi}$	0.0000	0.7367	0.6767	0.0460	1.3443	$\mathcal{N}$
$\sigma_{y\pi^*}$	0.0000	-0.2204	-0.2148	-1.7942	1.3055	$\mathcal{N}$
σ_{vr}	0.0000	1.0316	1.1640	-0.8603	2.8697	$\mathcal{N}$
$\sigma_{v\pi}$	0.0000	3.9946	3.1414	1.9457	4.4911	$\mathcal{N}$
σ_{vy}	0.0000	-0.3565	-0.2337	-0.5524	0.1260	$\mathcal{N}$
$\sigma_{v\pi^*}$	0.0000	-0.8689	0.0411	-2.3657	2.6599	$\mathcal{N}$
Λ^r	0.0000	1.3260	2.1260	-1.1760	5.4160	$\mathcal{N}$
Λ^π	0.0000	-2.2438	-2.7796	-6.1481	-0.0014	$\mathcal{N}$
Λ^y	0.0000	2.2123	2.1688	-1.2309	6.0037	$\mathcal{N}$
Λ^{π^*}	0.0000	-0.7596	-0.9366	-1.6190	-0.3066	$\mathcal{N}$
σ_{12}	0.0001	0.0005	0.0005	0.0005	0.0006	$\mathcal{IG}$
σ_{24}	0.0001	0.0001	0.0001	0.0001	0.0001	$\mathcal{IG}$
σ_{36}	0.0001	0.0000	0.0000	0.0000	0.0000	$\mathcal{IG}$
σ_{48}	0.0001	0.0000	0.0000	0.0000	0.0000	$\mathcal{IG}$
σ_{60}	0.0001	0.0001	0.0001	0.0001	0.0001	$\mathcal{IG}$

Summary of the posterior distributions of the parameters. Type of the distribution is either $\mathcal{N}$, $\mathcal{B}$, $\mathcal{G}$, or $\mathcal{IG}$ where $\mathcal{N}$ denotes the Normal distribution, $\mathcal{B}$ the Beta distribution, $\mathcal{G}$ the Gamma distribution, and $\mathcal{IG}$ the Inverse-Gamma distribution.

3.5.2 The Model's Dynamic

The estimation results for the remaining parameters are summarized in table (3.3) and table (3.4). Rather than interpreting each coefficient separately, I will describe the results of the parameter estimates jointly by computing impulse response functions (IRFs) of the model's variables to the fundamental shocks of the economy and by decomposing the forecast error variance. Both methods help to examine the dynamic of the estimated model and to describe the propagation and the relevance of different shocks.

Each of the following figures shows the impulse response of the model's variables to a particular shock. Each shock is of a size of one-standard-deviation. The first column of each figure displays the impulse responses of the macroeconomic variables (the nominal short-term interest rate r_t, the inflation rate π_t, the output gap g_t^y, and central bank's inflation target $\pi_t^\star$). The second column contains the impulse responses of the yield rates (from the 12-month rate to the 60-month rate). The third and fourth column display the IRFs of the one-period return premium $E_t\left(hprx_{t+1}^{(\tau)}\right)$ and the yield premium κ_t, respectively. By construction, the one-period return premium is only driven by the risk variable v_t, while the yield premium, which captures the premium in yields over the full lifetime of the bond, is affected by all state variables. The light gray shaded areas cover the 90 percentage HPD interval while the dark gray shaded areas cover the 68 HPD interval. The IRF (displayed by the blue line) is computed as the mean impulse response. The output gap is depicted in percentage deviations of the steady state, and the inflation- and the yield rate are shown in annualized percentage points. One period corresponds to one month.

Figure (3.1) shows the response to a term premium shock. The increase in the risk variable causes the one-period return premia and the yield premia in yields of

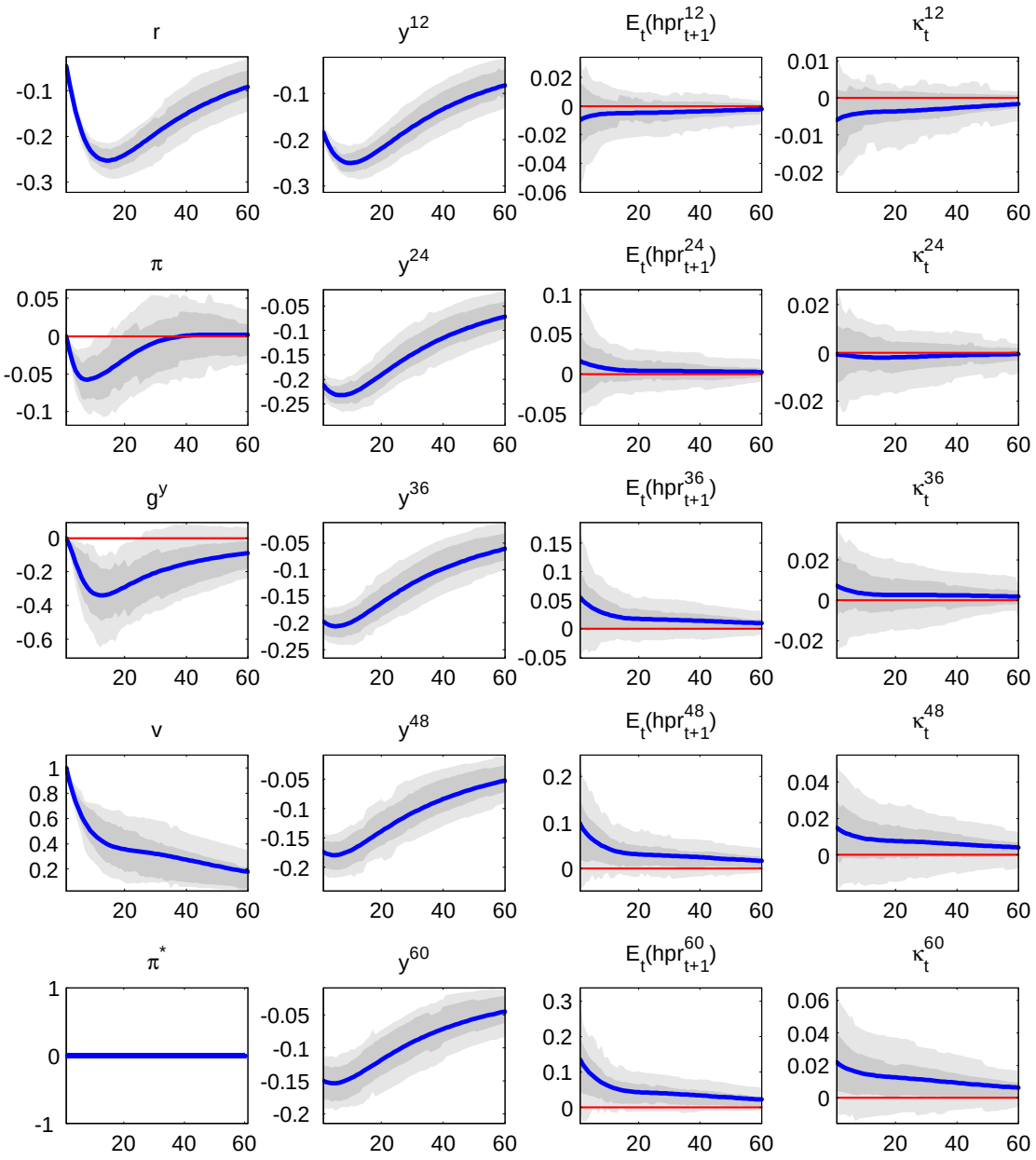


Figure 3.1: Impulse responses of the model's variables to a one-standard-deviation risk variable shock ε_{vt}.

bonds with longer maturities to rise. For term premia incorporated in yields of bonds with shorter, the response is not significantly different from zero. Similar to a negative demand shock, output and inflation drop in response to a term premium shock. In line with the findings of Ireland (2015) for the US, the plots show that an exogenous rise in term premia works to dampen economic activity. According to the interest rate rule, the rise in the risk variable causes the central bank to ease monetary policy. The magnitude of the increase in term premia is larger for premia of bonds with longer maturities. Following the short-term interest rate, long-term yields decline. The decline in long-term yields is mitigated by the increase in term premia. Since the inflation target is given by a univariate autoregressive process, the inflation target is not affected by shocks to the other state variables.

Figure (3.2) displays the response of the economy to a positive interest rate rule shock. The short-term interest rate rises on impact and stays above its steady state level for more than 7 months, converging back to its steady state. The response of the output gap and of the inflation rate to the interest rate shock are in line with previous study and economic theory. The tightening of monetary policy dampens economic activity, leading to a drop in output and inflation though the response of the output gap is not statistically significant from zero on the 90 percent level. The responses of the risk variable and the term premia to the interest rate shock are not significantly different from zero.

The impulse responses to the output shock ε_{yt} are displayed in figure (3.3). The output gap rises sharply on impact and decreases slowly over the next 12 months back to its steady state. The impact response of inflation to the output shock is not significantly different from zero. After roughly 6 months, inflation rises slowly with its peak after 12 months and remains positive for another 12 months. The increase in the output gap and in inflation causes monetary policy to tighten. The rise in

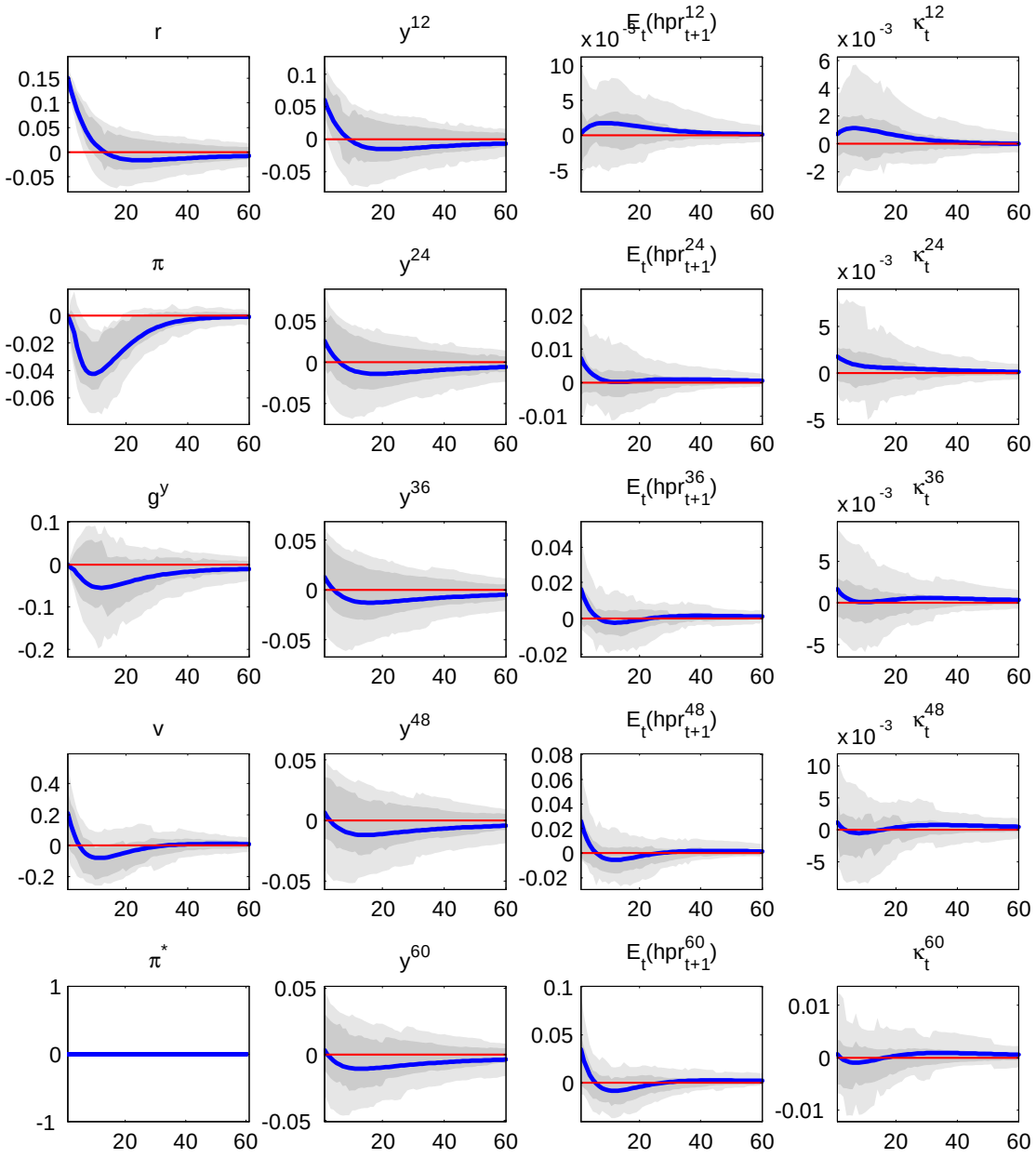


Figure 3.2: Impulse responses of the model's variable to a one-standard-deviation interest rate shock ε_{rt}.

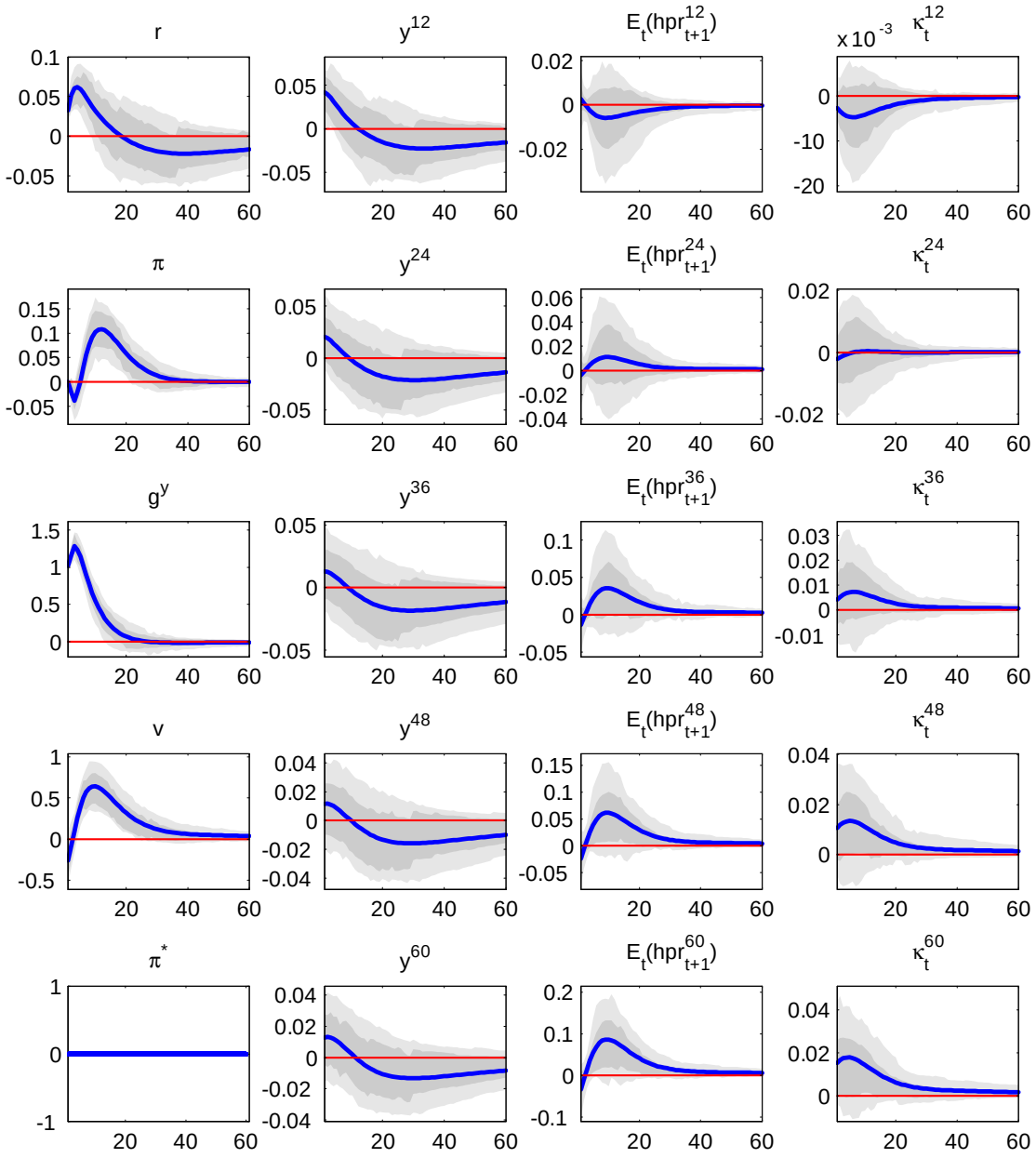


Figure 3.3: Impulse responses of the model's variables to a one-standard-deviation output gap shock ε_{yt}.

short-term interest rate pushes the yield curve upwards. However, the rise in long-term bond yields is only significantly different from zero for the 1-year bond yield. Overall, the effects of this shock work similar to an aggregate demand shock. The response of the risk variable is, on impact, not statistically different from zero. After 4 months the risk variable rises and stays significantly above its steady state value for roughly 16 months.

The impulse responses to an innovation in the inflation rate are shown in figure (3.4). Inflation rises sharply and converges back to its steady state in less than 18 months. According to the interest rate rule, the central bank raises the short-term interest rate in response to the increase in inflation. Yields follow the short-term interest rate. The response of the output gap is not significantly different from zero. In response to the inflation shock, the risk variable rises on impact and stays significantly different from zero for more than 12 months. Again, only the response of the one-period return premium and of the yield premium in yields of bonds with longer residual maturities are significantly different from zero.

Finally, figure (3.5) presents the impulse responses to a shock to the inflation target π_t^*. From the parameter estimates of the inflation target process $\rho_{\pi^*} = 0.9896$, the inflation target process is highly persistent. Actual inflation rises in response to the increase in the inflation target. Also the nominal short-term interest rate and bond yields rise. In line with the findings of Ireland (2015) for the U.S., the inflation target works similar to the level factor observed in latent finance term structure models:[10] It moves bond yields simultaneously and persistently upward, resulting in a higher level of the yield curve. The output gap does not respond on impact but starts to rise slowly after two years. The risk variable drops in response to the to

[10] The first three latent factors commonly studied in affine term structure models in finance, are denoted as level-, slope-, and curvature factor. The factor names refer to the effect that each factor has on the yield curve.

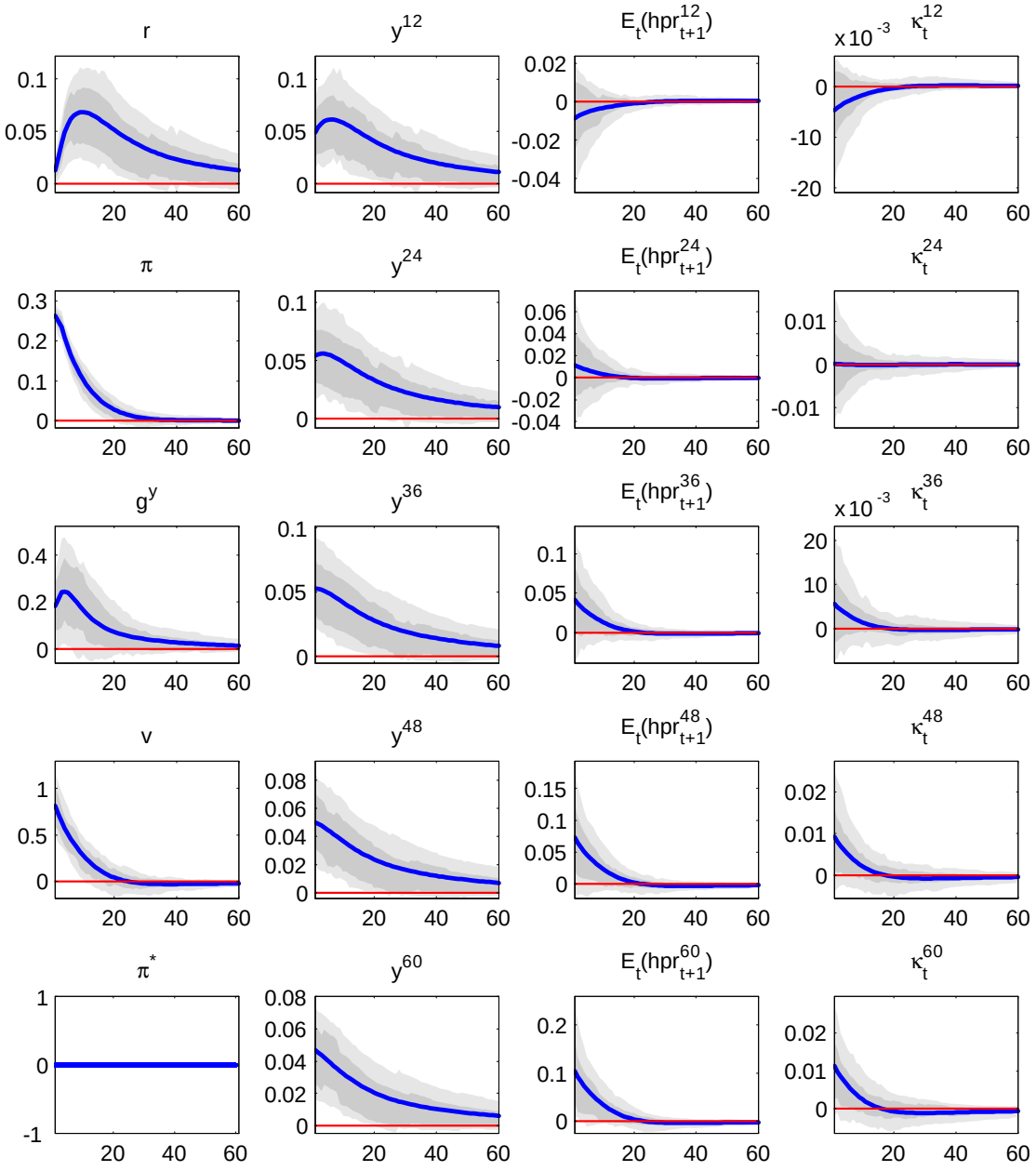


Figure 3.4: Impulse responses of the model's variables to a one-standard-deviation inflation shock $\varepsilon_{\pi t}$.

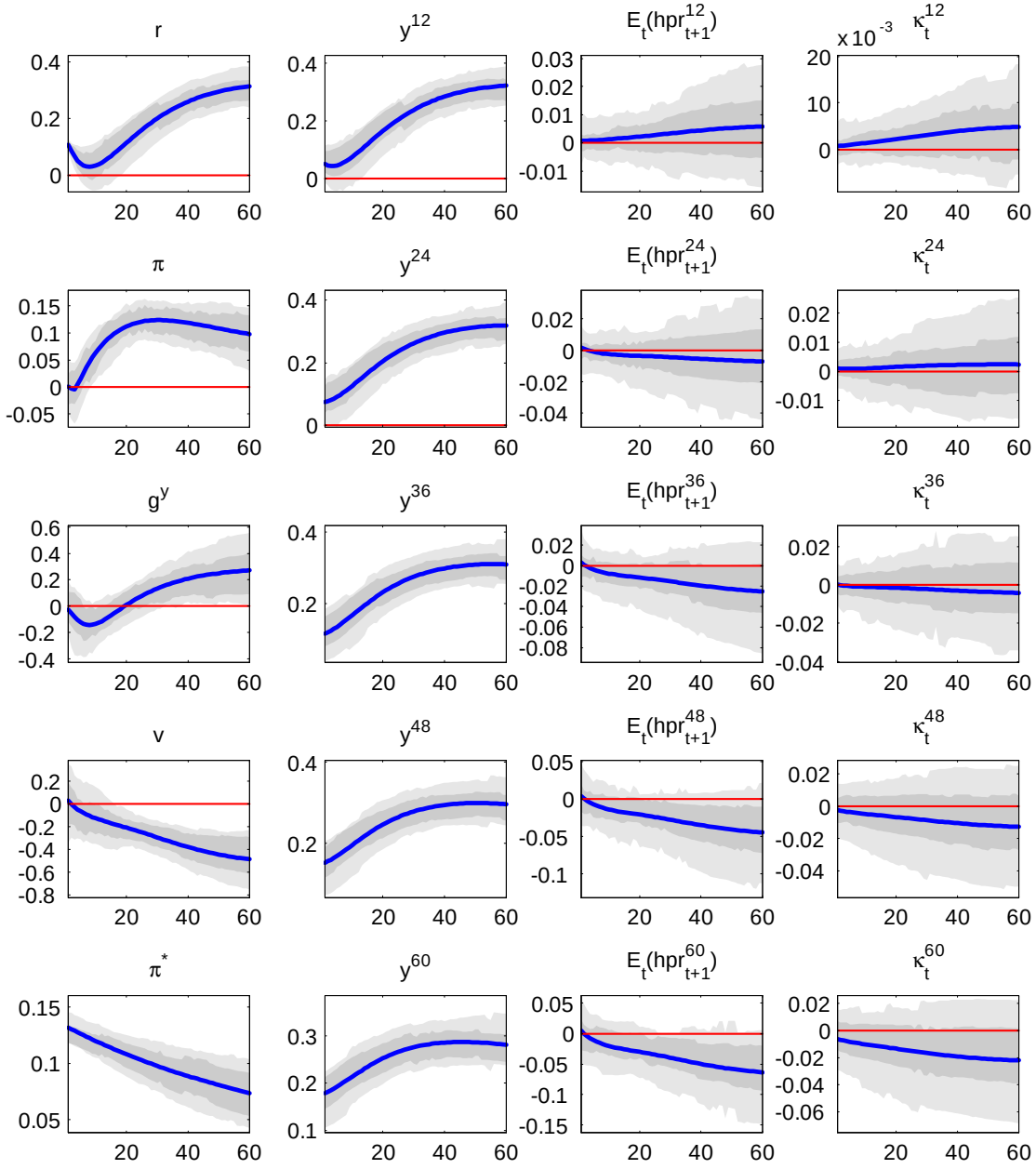


Figure 3.5: Impulse responses of the model's variables to a one-standard-deviation inflation target shock $\varepsilon_{\pi^* t}$.

the inflation target shock.

The displayed results show a rich interaction between the economy and term premia. Previous empirical studies indicate that the bond term premium varies over the business cycle and that this variation is countercyclical (Cochrane and Piazzesi, 2005, Ludvigson and Ng, 2009, or Piazzesi and Swanson, 2008). My results are in line with these findings, but emphasizes that the kind of underlying disturbance is crucial for the sign of the correlation between output gap and term premia, as theory suggests (see e.g. Hördahl et al., 2008, Rudebusch, et al., 2007, or Rudebusch and Swanson, 2012). Shocks to the risk variable move output gap and term premia in opposite directions, leading to a countercyclical relationship. Shocks to the inflation target do not move output gap and term premia on impact, but with a delay. Similar, output gap shocks do not move term premia on impact, but with a delay of 4 months. The results indicate, thus, whether term premia are countercyclical over the business cycle or not depends on the source of the movements.

Next, in order to assess the relative importance of different shocks for the variability of a variable, I compute the forecast error variance decomposition (FEVD). The FEVD helps to quantify the contribution of each of the five structural shocks to the forecast error variance of the model's variables. Formally, the fraction of the forecast error variance of variable i due shock j for horizon h, denoted by $\phi_{i,,j}(h)$, is defined by (see e.g. Lütkepohl, 2005)

$$\phi_{i,,j}(h) = \frac{\omega_{i,,j}(h)}{\Omega_i(h)},$$

where $\omega_{i,,j}(h)$ is the forecast error variance of variable i due to shock j at horizon h and $\Omega_i(h)$ is the total error forecast variance of variable i at horizon h. Table (3.5) and (3.6) present the FEVD of the model's variables for different horizons to the five

Table 3.5: FEVD of Macroeconomic Variables

Short-Term Interest Rate					
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}
1	58.69	0.73	3.92	5.87	30.80
12	8.69	7.69	4.13	73.46	6.04
36	2.84	3.51	1.49	63.83	28.33
60	1.55	1.86	1.21	39.58	55.80
∞	0.45	0.53	0.40	11.95	86.67
Inflation					
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}
1	0.00	99.97	0.00	0.00	0.03
12	2.29	81.74	7.62	4.53	3.81
36	2.36	43.94	14.70	3.91	35.10
60	1.77	33.08	11.07	2.96	51.12
∞	1.20	22.28	7.45	2.00	67.08
Output Gap					
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}
1	0.00	3.67	96.25	0.00	0.08
12	0.41	3.49	91.44	4.08	0.59
36	0.73	3.14	83.37	10.68	2.09
60	0.69	2.91	77.21	11.33	7.86
∞	0.49	2.04	64.02	8.39	25.06
Risk Variable					
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}
1	1.06	48.27	5.47	44.66	0.54
12	0.44	34.95	22.04	41.08	1.500
36	0.38	20.22	30.51	42.03	6.85
60	0.38	15.10	23.01	37.86	23.65
∞	0.16	5.74	8.79	15.48	69.83

structural disturbances. The FEVD of the macroeconomic variables are displayed in table (3.5). Since the inflation target does only react on own innovations, over all horizons 100 percent of the forecast error variance is simply explained by inflation target shocks. Therefore it is omitted from table (3.5).

In the very short run, more than half of the variability of the short-term interest rate is due to interest rate shocks. Term premium shocks ε_{vt} account for between 40 to 75 percent of the error forecast variance of the short-term interest rate at a one- to

five-year horizon. In the long run, inflation target shocks account for more than 86 percent of movements in the interest rate. Term premium shocks do not only move the short term interest rate but do also account for sizeable variations in inflation, output gap and the risk variable itself, revealing a non-negligible influence of term premia shocks on the economy. In line with the "practitioner" view, risk shocks play an important role for economic activity. They account for between 4 and 12 percent of the forecast error variance in the output gap, and also for between 3 and 5 percent in the inflation rate, both at horizons between one and five years. The forecast error variance of the risk variable, in turn, is driven by different disturbances, each differently important at different horizons. At a one- and five-year horizon, term premium shocks account for the bulk of movements in term premia (between 37 and 43 percent). This corresponds to the findings of Dewachter et al. (2014) and Ireland (2015) who find that a large fraction of movements in term premia is not driven by macroeconomic shocks, but by exogenous term premia shocks. In the short run, in addition to term premia shocks, inflation shocks account for a large fraction of the forecast error variance in term premia, while in the long run inflation target shocks account for around 70 percent of the forecast error variance in term premia. At the horizon between one and five years, output gap shocks ε_{yt} account for between 22 and 30 percent of variations in term premia. The results indicate a bidirectional linkage, running from the macroeconomic to term premia and vice versa. According to the estimated model, interest rate shocks did not account for much variance of the other variables over the sample period. Movements in the output gap are mainly driven by own shocks. Variations in the inflation rate are due to inflation shocks and output gap shocks in the short run and inflation target shocks in the long run.

The FEVD of bond yields is presented in table (3.6). In addition to the five fundamental disturbances, also the measurement errors are reported. Term premium

Table 3.6: FEVD of Bond Yields

One Year Yield Rate

Horizon	Structural Shock					Measurement errors				
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}	η_t^{12}	η_t^{24}	η_t^{36}	η_t^{48}	η_t^{60}
1	4.23	7.21	3.65	73.02	5.74	6.14	0	0	0	0
12	0.80	6.01	0.81	85.22	6.78	0.37	0	0	0	0
36	0.83	2.51	0.91	57.40	38.25	0.10	0	0	0	0
60	0.51	1.36	0.83	34.97	62.28	0.05	0	0	0	0
∞	0.16	0.42	0.30	11.30	87.80	0.02	0	0	0	0

Two Year Yield Rate

Horizon	Structural Shock					Measurement errors				
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}	η_t^{12}	η_t^{24}	η_t^{36}	η_t^{48}	η_t^{60}
1	0.11	7.20	0.62	80.32	11.48	0	0.27	0	0	0
12	0.62	4.65	0.17	75.70	18.85	0	0.02	0	0	0
36	0.62	1.80	0.67	45.09	51.82	0	0	0	0	0
60	0.38	0.99	0.61	27.39	70.62	0	0	0	0	0
∞	0.13	0.33	0.24	9.46	89.84	0	0	0	0	0

Three Year Yield Rate

Horizon	Structural Shock					Measurement errors				
h	ε_r	ε_π	ε_y	ε_v	ε_v	η_t^{12}	η_t^{24}	η_t^{36}	η_t^{48}	η_t^{60}
1	0.04	6.49	0.22	66.98	26.27	0	0	0.01	0	0
12	0.67	3.75	0.08	59.32	36.18	0	0	0	0	0
36	0.51	1.36	0.48	33.21	64.44	0	0	0	0	0
60	0.31	0.76	0.44	20.43	78.05	0	0	0	0	0
∞	0.12	0.27	0.18	7.53	91.90	0	0	0	0	0

Four Year Yield Rate

Horizon	Structural Shock					Measurement errors				
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}	η_t^{12}	η_t^{24}	η_t^{36}	η_t^{48}	η_t^{60}
1	0.13	5.65	0.20	50.07	43.94	0	0	0	0.01	0
12	0.62	3.04	0.08	43.30	52.97	0	0	0	0	0
36	0.41	1.07	0.33	23.86	74.32	0	0	0	0	0
60	0.25	0.61	0.32	15.00	83.82	0	0	0	0	0
∞	0.10	0.23	0.14	5.90	93.6	0	0	0	0	0

Five Year Yield Rate

Horizon	Structural Shock					Measurement errors				
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}	η_t^{12}	η_t^{24}	η_t^{36}	η_t^{48}	η_t^{60}
1	0.17	4.80	0.28	35.42	59.24	0	0	0	0	0.09
12	0.52	2.47	0.13	30.71	66.16	0	0	0	0	0.01
36	0.33	0.87	0.24	17.12	81.45	0	0	0	0	0
60	0.20	0.51	0.23	11.03	88.02	0	0	0	0	0
∞	0.08	0.20	0.11	4.60	95.00	0	0	0	0	0

shocks account for sizeable variation in bond yields, in particular, for bonds with shorter terms to maturity and for short forecast horizons. In line with evidence of Barr and Campbell (1997), Gürkaynak et al. (2005) or Ireland et al. (2015), most of the variation in bond yields is caused by inflation target shocks. The contribution of inflation target shocks to the forecast error variance in bond yields is even more pronounced for long-term bonds and increasing in the forecast horizon. Inflation target shocks account for between 64 and 88 percent of movements in yields of bonds of 3- and 5-year residual maturity at forecast horizons between three and five years. But also for bonds with shorter terms to maturity (two years and less), inflation target shocks are important determinants of the forecast error variance. These findings confirm the earlier observation that inflation target shocks work similar to a level shock, moving the entire yield curve upward. Notably, measurement error shocks do not contribute to much movement in bond yields, confirming a good fit of the model. They account for around 5 percent of the one-month ahead forecast error variance in the one-year bond rate, for less than 0.23 percent of the one-month ahead forecast error variance in the two-year bond rate, and even less for rates of bonds with longer terms to maturity. The contribution of measurement errors to the variance of bond yields declines considerably with the forecast horizon.

3.6 Conclusion

In this work, I evaluate the interplay of term premia, monetary policy, and the economy in the Euro area. Using a macro-finance model of the term structure, which explicitly allows term premia to affect the economy, my findings reveal a broad interaction among term premia, monetary policy, and the economy. Movements in term premia are captured by an unobservable risk variable which responds to all

other state variables and exhibits an autonomous dynamic. By restricting the prices of risk in the pricing kernel (as in Dewachter and Iania, 2011, Dewachter et al., 2014, and Ireland, 2015) this variable is identified to account for all variations in the one-period return premium. Furthermore, exclusion restrictions on the contemporaneous relationship of the model's variables, similar to those from more conventional VAR models, on the state process of macroeconomic variables are entailed to disentangle the effects of fundamental shocks to the endogenous variables. In line with earlier studies of the term structure and term premia, I find that the term premium is time-varying and that it responds to the state of the economy, contradicting the expectation hypotheses.

I want to emphasize two aspects of my findings. First, a rise in the term premium does affect the economy. Precisely, proving evidence for the practitioner view, a pure exogenous term premium shock dampens output and inflation, similar to an aggregate demand shock. Second, the analysis reveals that the ECB reacts to movements in the term premium. Indeed, in order to counteract the change in the premia, the central bank shifts the policy rate contrary to the change in the premium. Furthermore, this paper does not find evidence for strong effects of conventional monetary policy on term premia.

Examining how term premia movements affect the economy and the effects of conventional monetary policy on term premia is the first step. A natural question arising from these finding is how unconventional monetary policy actions, in particular, "quantitative easing" (QE), affects the term premium. QE intends to stimulate the economy through aggregate demand channels not only by reducing long-term yields, the so-called signaling channel but also by reducing the term premium part in long-term yields, the so-called portfolio-balance channel (International Monetary

Fund Report, 2013). Recent studies[11] find that QE worked to reduce long-term yields though the magnitude of these effects differs greatly and the channel through which large-scale asset purchases affects long-term yields is not clear. If changes in term premia work to affect the economy, what are the qualitative and quantitative effects of QE on term premia? However, the analysis of these effects is beyond the scope of this paper.

[11] Among many others, Bauer and Rudebusch (2015), Carlstrom et al. (2014), Chen et al. (2012), Gagnon et al. (2011), Hamilton and Wu (2012), Krishnamurthy and Vissing-Jorgensen (2011), and Woodford (2012).

3.A Appendix

3.A.1 Parameter Vectors and Matrices

The vectors and matrices P_0, P_1, μ_0, and Σ_0 in eq. (3.7) are defined as

$$P_0 \equiv \begin{bmatrix} 1 & 0 & 0 & -(1-\rho_r)\,\rho_\pi & 0 & 0 & -(1-\rho_r)\,\rho_y & 0 & 0 & 0 & -(1-\rho_r)\,\rho_v \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$P_1 \equiv \begin{bmatrix} \rho_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho^1_{\pi r} & \rho^2_{\pi r} & \rho^3_{\pi r} & \rho^1_{\pi\pi} & \rho^2_{\pi\pi} & \rho^3_{\pi\pi} & \rho^1_{\pi y} & \rho^2_{\pi y} & \rho^3_{\pi y} & 0 & \rho_{\pi v} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho^1_{yr} & \rho^2_{yr} & \rho^3_{yr} & \rho^1_{y\pi} & \rho^2_{y\pi} & \rho^3_{y\pi} & \rho^1_{yy} & \rho^2_{yy} & \rho^3_{yy} & 0 & \rho_{yv} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\pi^\star} & 0 \\ \rho_{vr} & 0 & 0 & \rho_{v\pi} & 0 & 0 & \rho_{vy} & 0 & 0 & \rho_{v\pi^\star} & \rho_{vv} \end{bmatrix},$$

$$\mu_0 \equiv \begin{bmatrix} (1-\rho_r)\left(g^r - \rho_y g^y\right) \\ 0 \\ 0 \\ -\left(\rho^1_{\pi r} + \rho^2_{\pi r} + \rho^3_{\pi r}\right) g^r - \left(\rho^1_{\pi y} + \rho^2_{\pi y} + \rho^3_{\pi y}\right) g^y \\ 0 \\ 0 \\ \left(1 - \left[\rho^1_{yy} + \rho^2_{yy} + \rho^3_{yy}\right]\right) g^y - \left(\rho^1_{yr} + \rho^2_{yr} + \rho^3_{yr}\right) g^r \\ 0 \\ 0 \\ (1-\rho_{\pi^\star})\,\pi^\star \\ -\rho_{vr} g^r - \rho_{vy} g^y - \rho_{v\pi^\star}\pi^\star \end{bmatrix},$$

and

$$\Sigma_0 \equiv \begin{bmatrix} \sigma_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\pi & 0 & 0 & 0 & 0 & 0 & \sigma_{\pi\pi^\star}\sigma_{\pi^\star} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{y\pi}\sigma_\pi & 0 & 0 & \sigma_y & 0 & 0 & \sigma_{y\pi^\star}\sigma_{\pi^\star} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\pi^\star} & 0 \\ \sigma_{vr}\sigma_r & 0 & 0 & \sigma_{v\pi}\sigma_\pi & 0 & 0 & \sigma_{vy}\sigma_y & 0 & 0 & \sigma_{v\pi^\star}\sigma_{\pi^\star} & \sigma_v \end{bmatrix}.$$

3.A.2 Recursive Bond Prices

Following Ang and Piazzesi (2003) or Ireland (2015), the difference equations are derived by induction, using eq. (3.15). Start with $\tau = 0$, then, from $P_{t+1}^0 = 1$, eq. (3.15) implies

$$\begin{aligned} P_t^1 &= E_t\left(m_{t+1}\right) \\ &= E_t\left(\exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\right) \\ &= \exp\left(-r_t\right), \end{aligned}$$

where I used that ε_t is standard normally distributed so that m_{t+1} is log-normal distributed with mean $\mu = -r_t - \frac{1}{2}\lambda_t'\lambda_t$ and variance $\sigma^2 = \lambda_t'\lambda_t$. Now suppose that $P_t^1 = \exp\left(\bar{A}_1 + \bar{B}_1 X_t\right)$ holds, then substituting eq. (3.8) for r_t leads to

$$\exp\left(\bar{A}_\tau + \bar{B}_\tau' X_t\right) = \exp\left(-\delta_1' X\right).$$

Matching coefficients leads to the initial conditions $\bar{A}_1 = 0$ and $\bar{B}_1' = -\delta_1'$. Next, in order to show that the recursions in eq. (3.17) and (3.18) hold for any value of $\tau = 1, 2, ...$, suppose that $P_t^\tau = \exp\left(\bar{A}_\tau + \bar{B}_\tau X_t\right)$. Substitute eq. (3.7), eq. (3.11), (3.12), and (3.16) into eq. (3.15) yields

$$\begin{aligned}
P_t^{\tau+1} &= E_t\left(\exp\left(-\delta_1' X_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\exp\left(\bar{A}_\tau + \bar{B}_\tau' X_{t+1}\right)\right) \\
&= \exp\left(-\frac{1}{2}\lambda_t'\lambda_t + \bar{A}_\tau + \bar{B}_\tau'\mu + \left[\bar{B}_\tau' P - \delta_1'\right] X_t\right) E_t\left(\exp\left(\left[\bar{B}_\tau'\Sigma - \lambda_t'\right]\varepsilon_{t+1}\right)\right) \\
&= \exp\left(\begin{array}{c} -\frac{1}{2}\lambda_t'\lambda_t + \bar{A}_\tau + \bar{B}_\tau'\mu + \left[\bar{B}_\tau' P - \delta_1'\right] X_t \\ +\frac{1}{2}\left[\bar{B}_\tau'\Sigma\Sigma'\bar{B}_\tau - 2\bar{B}_\tau'\Sigma\lambda_t + \lambda_t'\lambda_t\right] \end{array}\right) \\
&= \exp\left(\bar{A}_\tau + \bar{B}_\tau'\mu + \frac{1}{2}\bar{B}_\tau'\Sigma\Sigma'\bar{B}_\tau - \bar{B}_\tau'\Sigma\lambda_0 + \left[\bar{B}_\tau' P - \bar{B}_\tau'\Sigma\lambda_1 - \delta_1'\right] X_t\right),
\end{aligned}$$

where the third equality is obtained by computing the expectation of the exponential function using the normality of ε_{t+1} and

$$E_t\left(\exp\left(\left[\bar{B}_\tau'\Sigma - \lambda_t'\right]\varepsilon_{t+1}\right)\right) = \exp\left(\bar{\mu} + \frac{1}{2}\bar{\sigma}^2\right),$$

with $\bar{\mu} = 0$ and $\bar{\sigma}^2 = \bar{B}_\tau\Sigma\Sigma'\bar{B}_\tau' - 2\bar{B}_\tau\Sigma\lambda_t + \lambda_t'\lambda_t$. Matching coefficients shows that the recursive solution in eq. (3.17) and (3.18) hold.

3.A.3 Yield Premia and Return Premia

This part of the appendix demonstrates that the yield premium can be written as the average of expected future return premia of declining maturity. The yield premium

of τ-period bond $\kappa_t^{(\tau)}$ is given by

$$\begin{aligned}\kappa_t^{(\tau)} &= y_t^{(\tau)} - \frac{1}{\tau}E_t\left[\sum_{i=0}^{\tau-1} y_{t+i}^{(1)}\right] = \frac{1}{\tau}\left[\tau y_t^{(\tau)} - E_t\sum_{i=0}^{\tau-1} y_{t+i}^{(1)}\right] \\ &= \frac{1}{\tau}\left[-p_t^{(\tau)} - E_t\sum_{i=0}^{\tau-1} y_{t+i}^{(1)}\right],\end{aligned}$$

where the last equality uses the relation $y_t^\tau = -p_t^\tau/\tau$. Now add $E_t\sum_{i=0}^{\tau-1} p_{t+i+1}^{\tau-i-1} - E_t\sum_{i=0}^{\tau-1} p_{t+i+1}^{\tau-i-1}$, rearrange terms, and use the definition $E_t\left(hprx_{t+i+1}^{\tau-i}\right) = p_{t+i+1}^{(\tau-i-1)} - p_{t+i}^{(\tau-i)} - y_{t+i}^{(1)}$ to obtain

$$\begin{aligned}\kappa_t^{(\tau)} &= \frac{1}{\tau}\left[-p_t^{(\tau)} - E_t\sum_{i=0}^{\tau-1} y_{t+i}^{(1)}\right] \\ &= \frac{1}{\tau}E_t\left[\sum_{i=0}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=0}^{\tau-1} p_{t+i+1}^{\tau-i-1} - p_t^{(\tau)} - \sum_{i=0}^{\tau-1} y_{t+i}^{(1)}\right] \\ &= \frac{1}{\tau}E_t\left[hprx_{t+1}^{\tau} - p_{t+1}^{\tau-1} + \sum_{i=1}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=1}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=1}^{\tau-1} y_{t+i}^{(1)}\right] \\ &= \frac{1}{\tau}E_t\left[hprx_{t+1}^{\tau} + hprx_{t+2}^{\tau-1} - p_{t+2}^{\tau-2} + \sum_{i=2}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=2}^{\tau-1} p_{t+i+1}^{\tau-i-1} - E_t\sum_{i=2}^{\tau-1} y_{t+i}^{(1)}\right] \\ &= \ldots \\ &= \frac{1}{\tau}E_t\left[\sum_{i=0}^{\tau-2} hprx_{t+i+1}^{\tau-i} + p_{t+\tau}^{(0)} - p_{t+\tau-1}^{(1)} - y_{t+\tau-1}^{(1)} - p_{t+\tau}^{(0)}\right].\end{aligned}$$

Finally, note that $p_{t+\tau}^{(0)} = 0$ (since $P_{t+\tau}^0 = \exp\left(p_{t+\tau}^{(0)}\right) = 1$) and $E_t\left(hprx_{t+\tau}^1\right) = p_{t+\tau}^{(0)} - p_{t+\tau-1}^{(1)} - y_{t+\tau-1}^{(1)}$. Hence,

$$\kappa_t^{(\tau)} = \frac{1}{\tau}\sum_{i=0}^{\tau-1} E_t\left(hprx_{t+i+1}^{\tau-i}\right).$$

3.A.4 Computation of the $i+1$-period Return Premium

The return premium is given by (for $\tau > i$)

$$\begin{aligned} E_t\left(hprx_{t+i+1}^{(\tau)}\right) &= E_t\left(hpr_{t+i+1}^{(\tau)}\right) - E_t\left(y_{t+i}^{(1)}\right) \\ &= E_t\left(p_{t+i+1}^{(\tau-1)} - p_{t+i}^{(\tau)}\right) - E_t\left(y_{t+i}^{(1)}\right). \end{aligned}$$

Plugging the log prices and the expected short rate into the equation above yields

$$E_t\left(hprx_{t+i+1}^{(\tau)}\right) = \bar{A}_{(\tau-1)} + \bar{B}'_{(\tau-1)}E_tX_{t+i+1} - \bar{A}_{(\tau)} - \bar{B}'_{(\tau)}E_tX_{t+i} - \delta_0 - \delta_1'\bar{\mu} - \delta_1'P^i\left(X_t - \bar{\mu}\right).$$

Using $E_tX_{t+j} = \bar{\mu} + P^j\left(X_t - \bar{\mu}\right)$, $\mu = \left(I - P\right)\bar{\mu}$, eq. (3.17), rearranging, and collecting terms yields

$$\begin{aligned} E_t\left(hprx_{t+i+1}^{(\tau)}\right) &= -\bar{B}'_{(\tau-1)}\left(\mu - \Sigma\lambda_0\right) - \frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}_{\tau-1} + \bar{B}'_{(\tau-1)}E_tX_{t+i+1} \\ &\quad -\bar{B}'_{(\tau)}E_tX_{t+i} - \delta_1'\bar{\mu} - \delta_1'P^i\left(X_t - \bar{\mu}\right) \\ &= -\bar{B}'_{(\tau-1)}\left(\mu - \Sigma\lambda_0\right) - \frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}_{\tau-1} + \bar{B}'_{(\tau-1)}\bar{\mu} \\ &\quad -\bar{B}'_{(\tau-1)}P^{i+1}\bar{\mu} - \bar{B}'_{(\tau)}\bar{\mu} + \bar{B}'_{(\tau)}P^i\bar{\mu} - \delta_1'\bar{\mu} + \delta'P^i\bar{\mu} \\ &\quad +\bar{B}'_{(\tau-1)}P^{i+1}X_t - \bar{B}'_{(\tau)}P^iX_t - \delta_1'P^iX_t \\ &= c + \left[\bar{B}'_{(\tau-1)}P^{i+1} - \bar{B}'_{(\tau)}P^i - \delta_1'P^i\right]X_t, \end{aligned}$$

where c is defined by

$$\begin{aligned} c &\equiv -\bar{B}'_{(\tau-1)}\left(\mu-\Sigma\lambda_0\right)-\frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}_{\tau-1}-\bar{B}'_{(\tau-1)}P^{i+1}\bar{\mu} \\ &\quad +\bar{B}'_{(\tau-1)}\bar{\mu}-\bar{B}'_{(\tau)}\bar{\mu}-\delta'\bar{\mu}+\delta'P^i\bar{\mu}+\bar{B}'_{(\tau)}P^i\bar{\mu} \\ &= \bar{B}'_{(\tau-1)}\Sigma\lambda_0-\frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}'_{(\tau-1)} \\ &\quad +\left[\bar{B}'_{(\tau-1)}\left(P-P^{i+1}\right)-\delta'_1+\delta'_1P^i-\bar{B}'_{(\tau)}+\bar{B}'_{(\tau)}P^i\right]\bar{\mu}. \end{aligned}$$

Now use $\bar{B}'_{(\tau)}=\bar{B}'_{(\tau-1)}\left(P-\Sigma\lambda_1\right)-\delta'_1$ to see that

$$c=\bar{B}'_{(\tau-1)}\Sigma\left[\lambda_0+\lambda_1\left(I-P^i\right)\bar{\mu}\right]-\frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}_{(\tau-1)}.$$

and

$$E_t\left(hprx^{(\tau)}_{t+i+1}\right)=c+\bar{B}_{(\tau-1)}\Sigma\lambda_1P^iX_t.$$

Hence,

$$E_t\left(hprx^{(\tau)}_{t+i+1}\right)=\bar{B}_{(\tau-1)}\Sigma\left[\lambda_0+\lambda_1\left[\left(I-P^i\right)\bar{\mu}+P^iX_t\right]\right]-\frac{1}{2}\bar{B}_{(\tau-1)}\Sigma\Sigma'\bar{B}_{(\tau-1)}.$$

Note that the $i+1$-period return premium depends on the state of the economy only due to the term $\lambda_1P^iX_t$. If not only the elements in the last columns of P^i but also other elements in the columns in P^i are different from zero and $P^i\neq I$, all variation in the variables in X_t affect $E_t\left(hprx^{(\tau)}_{t+i+1}\right)$. For $i=0$ follows $P^i=I$ so that the 1-period return premium reads

$$\begin{aligned} E_t\left(hprx^{(\tau)}_{t+1}\right) &= \bar{B}_{(\tau-1)}\Sigma\lambda_0-\frac{1}{2}\bar{B}_{(\tau-1)}\Sigma\Sigma'\bar{B}_{(\tau-1)}+\bar{B}_{(\tau-1)}\Sigma\lambda_1X_t \\ &= \bar{B}'_{\tau-1}\Sigma\left[\lambda_0+\lambda_1X_t\right]-\frac{1}{2}\bar{B}_{(\tau-1)}\Sigma\Sigma'\bar{B}_{(\tau-1)}. \end{aligned}$$

Due to the restricted form of λ_1 the only source of variation in $E_t\left(hprx_{t+1}^{(\tau)}\right)$ is the variable that is ordered at the last position in X_t.

3.A.5 The Likelihood Function

The likelihood function reads

$$\begin{aligned} L\left(Z|\theta\right) &= \prod_{t=1}^{T}\left(2\pi\right)^{-\frac{T}{2}}\left[\det\left(R_{t|t-1}\right)\right]^{-\frac{1}{2}} \\ &\quad \times\exp\left(-\frac{1}{2}\left(Z_t-UX_{t|t-1}\right)'\left(R_{t|r-1}\right)^{-1}\left(Z_t-UX_{t|t-1}\right)\right), \end{aligned}$$

where $R_{t|t-1}$ denotes the conditional variance,

$$R_{t|t-1}\equiv Var\left(Z_t|\mathcal{Z}_{t-1},\theta\right)=U\Xi_{t|t-1}U'+VV',$$

$X_{t|t-1}$ denotes the one step ahead forecast,

$$X_{t|t-1}\equiv E\left[X_t|\mathcal{Z}_{t-1},\theta\right]=PX_{t-1|t-1},$$

with

$$X_{t|t}\equiv X_{t|t-1}+\Xi_{t|t-1}U\left(U'\Xi_{t|t-1}U+VV'\right)^{-1}\left(Z_t-UX_{t|t-1}\right),$$

and $\Xi_{t+1|t}$ denotes the mean squared error of the forecasts,

$$\begin{aligned} \Xi_{t+1|t} &\equiv E\left[\left(X_{t+1}-X_{t|t}\right)\left(X_{t+1}-X_{t+1|t}\right)'\right] \\ &= P\left(\Xi_{t|t-1}-\Xi_{t|t-1}U\left(U'\Xi_{t|t-1}U+VV'\right)^{-1}U'\Xi_{t|t-1}\right)P'+\Sigma\Sigma'. \end{aligned}$$

The Kalman filter is implemented by iterating on $X_{t|t-1}$ and $\Xi_{t|t-1}$ for given initial values $\Xi_{1|0}$ and $X_{1|t}$.

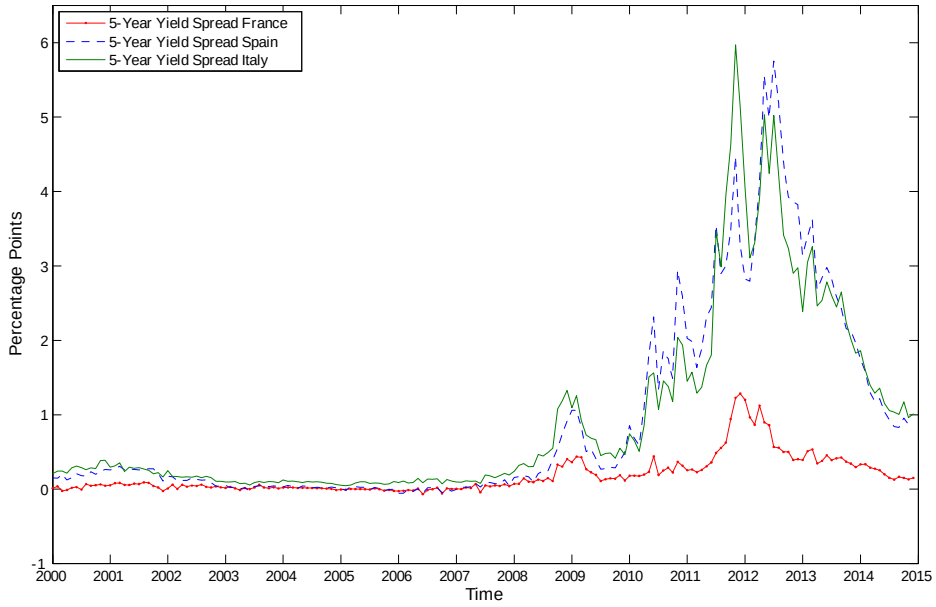
Chapter 4

Risk Aversion, Macro Factors, and non-fundamental Components in Euro Area Yield Spreads: A Macro-Financial Analysis

4.1 Introduction

Since the start of the European Economic and Monetary Union (EMU), a phase of remarkably low spreads between Euro area sovereign bond yields has been observed, despite large differences in fiscal positions among those countries. Though interest rate differentials did not vanish completely, they stabilized around a remarkably low level, indicating that country-specific factors did only play a minor role in this period. However, since the onset of the European debt crisis in late 2009, a dramatic surge between the yield of bonds of Euro area sovereigns vis-à-vis German government bonds did occur. Figure (4.1) illustrates the evolution of the five-year sovereign yield

Figure 4.1: Five-year sovereign bond yield spreads of France, Italy and Spain



Notes: All spreads are calculated with respect to the yield of German government bonds of the same maturity. The yield spreads are shown in annualized percentage points.

spreads of France, Italy, and Spain vis-à-vis Germany. The rise in yield spreads was accompanied by an increase in sovereign debt of several Euro area countries. However, not did only the spreads of sovereign yields of highly indebted countries vis-à-vis Germany rise but also the spreads of countries with solid fiscal fundamentals (see ECB, 2014, p. 75). This suggests that not only credit risk but also other factors account for the rise in yield spreads.

In particular at the beginning of the European debt crisis, in addition to credit risk, the effects of changes in global risk aversion are found to be an important component in yield spreads. However, recent evidence by e.g. De Santis (2015), Dewachter et al. (2015), Di Cesare et al. (2012), or Hördahl and Tristani (2013) suggests that

the surge in sovereign spreads of Euro area countries cannot be fully explained by changes in fundamentals and country-specific fiscal factors. These authors conclude that, in addition to global risk aversion, other common factors, interpreted as contagion or redenomination risk, have played a non-negligible role for the dynamics of yield spreads during the European debt crisis.

This paper investigates the effects of economic fundamentals, among them risk aversion, and a common factor that is unrelated to economic fundamentals on Euro area yield spreads using a macro-finance model of the term structure. In particular, this paper seeks to disentangle the effects of changes in risk aversion and the common non-fundamental risk factor to quantify their respective contribution to yield spreads of Euro area sovereigns vis-à-vis Germany. Since both, risk aversion and redenomination risk, are not directly observable, how these factors are captured is relevant to disentangle their effects on Euro area sovereign yield spreads. In contrast to the existing literature, the risk aversion measure used in this work is directly derived from the Euro area bond market within the macro-finance framework. Specifically, it is identified from changes in the prices of risk of the pricing kernel. Moreover, in order to analyze the drivers of Euro area yield spreads, I account for country-specific fiscal variables, the European business cycle, and monetary policy and their dynamics and interactions.

The results show that the common non-fundamental risk factor played a non-negligible role for yield spreads, accounting for a substantial increase in yield spreads during the financial crisis and the European debt crisis. Notably, the contribution of common non-fundamental risk factor shocks to the yield spreads increased from 2012 onwards. However, the most dominant drivers of yield spreads have been economic shocks. Among the economic shocks, risk aversion shocks were the most important source for variation in sovereign yield spreads, revealing the importance of measuring

risk aversion in Euro area bond markets adequately.

Studying the driver of yields and yield spreads is of interest to practitioners and researchers alike. Not do only play sovereign bonds an important role for asset pricing, sovereign yields are also used as reference rates for key interest rates. Moreover, understanding the determinants of yields is important for understanding the transmission of monetary policy. Likewise, spreads between Euro area sovereign yields may indicate impairments of the transmission process of monetary policy (see ECB, 2014). In addition, higher sovereign yields lead to higher marginal (re-) funding costs of governments and thus have the potential to increase the debt burden of a country.

Since the beginning of the EMU, a large empirical literature analyzes Euro area sovereign yields and yield spreads. Traditionally, the literature focuses on a set of variables describing credit risk, investors' risk aversion, and liquidity risk (see ECB, 2014). A large part of the literature uses regressions of yield spreads of Euro area countries vis-à-vis Germany at a specific maturity on different determinants. In contrast, a small but growing literature relies on affine term structure models to explore the determinants of Euro area sovereign yields (see e.g. Borgy et al., 2011, Dewachter et al., 2015, Geyer et al., 2004, Hördahl and Tristani, 2013, or Monfort and Renne, 2011). By cross-section restrictions derived from no-arbitrage assumptions, these models tie the movements of yields across maturities closely together. They allow to employ information from the cross-section and are suitable to capture the interaction and dynamics of macro variables and the prices of risk. In the empirical literature, investors' risk aversion is usually proxied by U.S. corporate bond spreads or a U.S. stock market volatility index (see e.g. Attinasi et al., 2010, Bernoth et al., 2012, Codogno et al. 2003, Favero et al., 2010, or Favero and Missale, 2012). Although the correlation between risk aversion and these variables is supposed to be high, these measures are unable to infer the underlying determinants that drive risk aver-

sion, as noted by Manganelli and Wolswijk (2009). They suggest using the risk-free short-term interest rate as a proxy for risk aversion. Although evidence (see e.g. by Manganelli and Wolswijk, 2009, or Bekaert et al., 2013) indicates an inverse relationship between the short-term interest rate and risk aversion, risk aversion should potentially also respond to other macroeconomic developments (see e.g. Dewachter et al., 2014) and this response does not necessarily have to coincide with that of the monetary policy authority to macroeconomic developments. Therefore, in this work, the short-term interest rate is, together with other Euro area-wide factors, one potential driver of changes in risk aversion, while the risk aversion measure is derived from European bonds markets with-in the model.

In order to assess the effects of different determinants on the evolution of sovereign yield spreads, I use a multi-country macro-finance model. The model features a unique pricing kernel reflecting the integration of financial markets in a currency union while it still allows for country-specific variables to affect one country's yield curve. Specifically, the yield curve of a country is driven by common variables capturing the European business cycle, a unified monetary policy, the common non-fundamental risk variable, and time-varying risk aversion, and also a country-specific fiscal variable capturing default risk.

The common non-fundamental risk factor captures dynamics in Euro area yield spreads that are unrelated to dynamics in the common economic fundamentals, i.e. a part in Euro area yield spreads that cannot be accounted for by macroeconomic variables (see Dewachter et al., 2015). This factor is identified from information contained in cross-country yield curves. Gathering these information requires estimating the term structure of sovereign yields of different European sovereigns jointly. As in Hördahl and Tristani (2013), the common non-fundamental risk factor is modeled as a latent variable and is, by construction, unrelated to economic fundamentals.

Changes in risk aversion are measured by a risk aversion variable. While the identification of the risk aversion variable follows Dewachter and Iania (2011), Dewachter et al. (2014), and Ireland (2015), the idea behind it is more closely related to Caceres et al. (2010) who use estimates of the prices of risk based on the VIX in order to construct a measure of risk aversion. In contrast, the risk aversion measure in this framework is derived from the pricing kernel of a European investor with-in the macro-finance model. Specifically, following the approach of Dewachter and Iania (2011), Dewachter et al. (2014), and Ireland (2015), by imposing restrictions on the stochastic discount factor, this risk aversion variable is identified from the term structure of default-free government bonds as the only driver of time variation in the prices of risk. Thus, to construct the risk aversion variable, the information contained over the whole yield curve of default-free government bonds are employed. This variable is used to explore the effects of changes in risk aversion on the evolution of yield spreads. The risk aversion variable responds, in turn, to distortions in economic fundamentals, the common non-fundamental risk factor, and exogenous risk aversion shocks.

Monetary policy is described by a policy rate rule in the spirit of Taylor (1993). In addition to output and inflation, monetary policy potentially also responds to movements in the risk aversion variable as in Ireland (2015). To the extent that monetary policy responds to movements in the risk aversion variable, including this channel is important to model expectations of future short-term interest rates and thus for separating changes of risk aversion from changes in expected future short rates. Indeed, as shown by Herrmann (2015) for the Euro area and Ireland (2015) for the U.S., the respective central bank does respond to movements in term premia which are captured by the risk aversion variable. Moreover, the central bank sets the inflation target around which inflation is stabilized. This target is modeled by

a latent variable. The long-run trend component helps to shape the expectation of long-term bond yields. The effects of fundamental shocks on different state variables are identified, similar to structural VAR models, by exclusion restrictions on the contemporaneous relationship of the model's variables.

The model is estimated by Bayesian estimation techniques. The posterior function is evaluated using an Adaptive Metropolis (AM) algorithm in the lines of Haario et al. (2001).

4.2 Literature Review

A broad literature investigates the determinants of Euro area sovereign yield spreads. Traditionally, the literature considers the role of credit risk, liquidity risk, and global risk aversion. More recently, also redenomination risk or systemic risk are considered as drivers of Euro area yield spreads. Credit risk or default risk, measuring a countries creditworthiness, is typically proxied by variables describing the fiscal position of a country (e.g. debt-to-GDP ratio, deficit-to-GDP ratio, the debt maturity, or interest expenditure-to-GDP). The literature finds that the importance of credit risk in sovereign bond yield spreads increased, since the start of the financial crisis and even more since the European sovereign debt crisis (see e.g. Attinasi et al., 2010, or Barrios et al., 2009). Liquidity risk measures the liquidity of sovereign bonds of a specific country. Typically, liquidity risk is proxied by the bid-ask spreads, the amount of outstanding public debt of a country, trading volumes, or turn-over ratios. Global risk aversion is typically proxied by the spread of U.S. Corporate Bonds over U.S. treasury bonds or a volatility index of U.S. stock markets (see e.g. Attinasi et al., 2010, Bernoth et al., 2012, Codogno et al., 2003, Favero et al., 2010, or Favero and Missale, 2012).

Although a broad spectrum of different modeling approaches is used to assess the determinants of yield spreads, the literature on the determinants of yield spreads can be roughly categorized into two strands. The first strand regresses sovereign bond yields or sovereign bond yield spreads at different maturities on different sets of explanatory variables, representing macro fundamentals, credit risk, liquidity risk, and global risk aversion (see e.g. Attinasi et al., 2010, Barrios et al., 2009, Beber et al., 2009, Bernoth et al., 2012, Di Cesare et al., 2012, Favero et al., 2010, Manganelli and Wolswijk, 2009, or Schuknecht et al., 2011). The second strand of the literature uses no-arbitrage term structure models in order to examine the determinants of euro area sovereign yield spreads. Among these authors are Battestini et al. (2013), Borgy et al. (2012), Dewachter et al. (2015), Geyer et al. (2004), and Hördahl and Tristani (2013). These models tie the yield of bonds of different maturity together by the assumption of the absence of no-arbitrage. This approach helps to employ information contained in the cross-section of yields. While Geyer et al. (2004) employ a purely latent factor model, Borgy et al. (2012) investigate the determinants of yield spreads using only macro variables as factors. Focusing on the effects of fiscal variables on spreads, they find that the importance of these variables for Euro area yield spreads increased since the beginning of the financial crisis. My work shares the closest focus with the work of Dewachter et al. (2015). They use a multi-country affine term structure model with unspanned macro risk factors to analyze the effects of common non-fundamental risk factors, interpreted as redenomination risk, and economic fundamentals on a set of Euro area sovereign yield spreads. Common non-fundamental factors are identified by the dynamic in the first two principal components of all the standardized yield spreads that cannot be accounted for by economic fundamentals, a measure of flight-to-safety motives, and a factor capturing the political uncertainty of the Euro area. In order to measure global tension, they

use the VIX. In contrast to the presented analysis in this paper, they use only observable factors. Moreover, in this work, the risk aversion measure is extracted directly from the Euro area sovereign bond markets. This potentially enhances the model's ability to explain common dynamics in yield spreads.

While the importance of credit risk and global risk aversion, not only during but also before the onset of the European debt crisis, seems to be unambiguous (e.g. Attinasi et al., 2010, Bernoth et al., 2012, Codogno et al., 2003, Favero et al., 2010, Geyer et al., 2004, Laubach, 2011, Manganelli and Wolswijk, 2009, or Schuknecht et al., 2011), the evidence for the relevance of liquidity risks for sovereign bond yields seems to be less striking (as discussed in Borgy et al., 2012, p. 9). Beber et al. (2009) or Haugh (2009) stress the importance of liquidity risk, in particular for smaller European economies, and in particular in times of high market distress. Favero et al. (2010), in contrast, find a negative relationship between their proxy for risk aversion and their proxy for liquidity, suggesting that in times of market stress investors value liquidity less than in "normal" times. Meanwhile, other authors find no or only less explanatory power of liquidity for sovereign yield spreads (e.g. Codogno et al., 2003, Geyer et al., 2004, Pagano and von Thadden, 2004, Favero et al., 2010, or Bernoth et al., 2012). For example, Bernoth et al. (2012) find that liquidity played only a role in European sovereign bond yields before the start of the EMU, but not after the start of the EMU. I follow Borgy et al. (2012) in their conclusion that liquidity risks seem to play, at its best, only a minor role, in particular, for the sovereign bonds of the four largest economies in Europe and during a financial crisis.

Finally, the recent literature finds, in addition to the relevance of global risk aversion, the importance of another common factor, widely interpreted as systemic risk or redenomination risk. Evidence by e.g. Amisano and Tristani (2013), Ang and Longstaff (2013), Caceres et al. (2010), De Santis (2014, 2015), Dewachter et al.

(2015), Di Cesare et al. (2012), Geyer et al. (2004), Giordano et al. (2013), or Hördahl and Tristani (2013) suggests that also systemic risk or redenomination risk and financial contagion effects may be drivers of Euro area sovereign yield spreads. Geyer et al. (2004) find evidence for a common factor in yield spreads which they interpret as "systematic risk". They conclude that systematic risk arises from is a "small but positive probability of a general failure of the EMU" (Geyer et al., 2004, p. 174). Amisano and Tristani (2013) use a Markov switching VAR to examine contagion in euro area sovereign bond spreads. Considering a normal and a crisis regime, they find that the risk of falling into the crisis regime depends on macroeconomic fundamentals, on risk aversion, and on the other countries regime dynamics. Di Cesare et al. (2012) show that the surge in euro area sovereign yield spreads during the debt crisis cannot be fully explained by country-specific fiscal variables and macroeconomic fundamentals but by a common non-fundamental factor. They argue that this common risk factor is the perceived risk of a break-up in the euro area. Giordano et al. (2013) categorize different types of contagion. In contrast to a large part of the literature, using a dynamic panel approach, they do not find evidence for pure contagion, that is, a contagion that is completely unrelated fundamentals in euro area bond markets during the debt crisis. Hördahl and Tristani (2013) construct a quadratic, no-arbitrage term structure model for defaultable sovereign bonds. Using yield spreads of five different Euro area countries vis-à-vis German yields at corresponding maturities, they find that economic fundamentals, but also an unobservable non-fundamental factor contribute significantly to the surge in spreads of most of the considered Euro area countries. De Santis (2015) proposes a measure for redenomination risk in the euro area using CDS spread data. He finds that redenomination risk shocks adversely affect euro area yield spreads. He also provides evidence for spill-over effects of redenomination shocks, concluding that these effects are a source

of systemic risk. Finally, Dewachter et al. (2015) show that economic fundamentals are the dominant drivers of euro area sovereign bond spreads. However, also shocks unrelated to economic fundamentals have played an important role during the European debt crisis.

In the following, I present a multi-country affine term structure model where a common non-fundamental factor which is by construction unrelated to the other fundamental components, risk aversion, and Euro area wide and country-specific economic factors drive Euro area yield spreads. Contributing to the literature, I try to disentangle the effects of this common non-fundamental factor and the risk aversion variable on Euro area yield spreads. Specifically, in contrast to the existing literature, the risk aversion measure used in this work is directly derived from European sovereign bond markets.

4.3 The Model

This section develops a multi-country no-arbitrage affine term structure model for the Eurozone. The model section is structured as follows. The first part describes the structural macroeconomic dynamics and casts the macro part into its state representation. The state variables are then used as pricing factors in the term structure model in the second subsection. Cross-equation restrictions, based on the assumption of no-arbitrage, are employed to tie the movements of yields closely together. The risk aversion variable is identified from restrictions on the prices of risk. Finally, the last subsection discusses the properties of the risk aversion variable. In particular, this subsection demonstrates that the risk aversion variable is the only driver of term premia of the default-free government bonds.

4.3.1 The State Equation

The model state system contains nine variables, six of them are observable, and three are unobservable. The observables are the short-term interest rate r_t, the output gap g_t^y, the inflation rate π_t, and the fiscal variables of the three sovereigns whose sovereign bonds might be subjected to credit risk. The latent variables are a common time-varying central bank's inflation target π_t^*, the risk aversion variable v_t, and a common risk factor C_t. This common risk factor is meant to capture non-fundamental risks, i.e. the part of the spread between yields of a potentially defaultable government bond and of a non-defaultable reference bond of the same maturity that cannot be justified by country-specific economic factors and euro area economic fundamentals. The analysis focuses only on countries belonging to the Euro area. Therefore, monetary policy for all countries is conducted by a single central bank. I follow Ireland (2015) closely in the specification of the dynamics of the Euro area variables while the specification of the country-specific variables is based on Borgy et al. (2012).

The central bank's policy is depicted as choosing an inflation rate target and adjusting the short-term interest rate accordingly. The incorporation of an unobservable time-varying inflation target is a common approach in the recent macro-finance term structure literature (as e.g. in Dewachter and Lyrio, 2006, Hördahl et al., 2006, Ireland, 2015, or Rudebusch and Wu, 2008). It allows for some variation in the conduction of monetary policy, and it helps to capture movements in long-term nominal government bond yields which arise due to changes in central bank's inflation target.[1] The central bank's inflation target is supposed to follow a stationary AR(1) process.

[1] In fact, Barr and Campbell (1997) for the UK and Gürkaynak et al. (2005) for the US find that movements in long-term interest rates occur mainly due to changes in expected inflation.

Specifically,

$$\pi_t^* = (1 - \rho_{\pi^*})\,\pi^* + \rho_{\pi^*}\pi_{t-1}^* + \sigma_{\pi^*}\varepsilon_{\pi^* t}, \tag{4.1}$$

where π^* is the steady state level of the inflation target, $\rho_{\pi^*} \in [0, 1)$, $\sigma_{\pi^*} > 0$, and the shock $\varepsilon_{\pi^* t}$ is standard normally distributed. As e.g. in Hördahl and Tristani (2012), Ireland (2015), or Rudebusch and Wu (2008), the restriction on ρ_{π^*} is imposed to ensure stationarity of the inflation target process.[2]

By defining the inflation gap and the interest rate gap, as in Ireland (2015), the notation is simplified. Specifically, the inflation rate gap is defined as the deviation of the inflation rate from central bank's inflation target, $g_t^\pi \equiv \pi_t - \pi_t^*$, and the interest rate gap is defined as the deviation of the interest rate from the inflation target, $g_t^r \equiv r_t - \pi_t^*$.

The central bank's policy rule for the short-term nominal interest rate is specified in terms of the interest rate gap, the inflation gap, and the output gap. Specifically, the central bank sets the interest rate according to the following interest rate rule in the spirit of Taylor (1993),

$$g_t^r - g^r = \rho_r \left(g_{t-1}^r - g^r\right) + (1 - \rho_r)\left[\rho_\pi g_t^\pi + \rho_y \left(g_t^y - g^y\right) + \rho_v v_t\right] + \sigma_r \varepsilon_{rt}, \tag{4.2}$$

where $\rho_r \in [0, 1]$ is the interest rate smoothing parameter, $\rho_\pi > 0$ is the central bank's response parameter on inflation, $\rho_y > 0$ is the central bank's response parameter on the output gap, ρ_v is the central bank's response parameter on the term premium variable, $\sigma_r > 0$ is a volatility parameter, and g^r and g^y are the steady state values of g_t^r and g_t^y, respectively. The shock ε_{rt} is supposed to be standard normally distributed

[2] A non-stationary inflation target leads to non-stationary inflation and non-stationary nominal short-term interest rate. For models with homoscedastic shocks, a unit root in the nominal short-term interest rate translates in undefined asymptotic long-term bond yields (as discussed in Ireland, 2015, and shown by Campbell, et al., 1997, p. 433). Thus, imposing stationarity of the inflation target process ensures that the term structure part of the model is well-behaved.

and represents the interest rate policy shock. The notation of the policy rule entails the assumption that the central bank is on average able to implement its inflation target. Thus, in the steady state, the actual inflation rate equals the central bank's target rate. While the response parameter ρ_π and ρ_y are restricted to be non-negative, the response parameter ρ_v is unconstrained. By restrictions on the prices of risk, the risk aversion variable is constructed to account for all variations in the prices of risk. As demonstrated in Section (4.3.3), these restrictions imply that the risk aversion variable is also the only source for fluctuations in the one-period return premium. If the coefficient ρ_v is positive, the central bank increases the short-term interest rate in response to a rise in term premia. In contrast, if the coefficient ρ_v is negative the central bank lowers the short-term interest rate in response to a rise in term premia.[3]

To the extent that the central bank does respond on the risk aversion variable, including this response in the monetary policy rule is important for modeling the expectation of future short-term interest rates and thus for separating the movements in long-term bond yields into changes in the expectation of future short-term rates and changes in risk aversion. In fact, Herrmann (2015) for the Euro area and Ireland (2015) for the U.S. find that there is a systematic response of the respective central bank to changes in the risk aversion variable and with-it to term premia movements.

The dynamics of the output gap and the inflation gap are modeled as linear functions of its own lags and lags of the other state variables. Identification is achieved by contemporaneous timing restrictions. Specifically, the output gap depends on its own lags, on lags of the interest rate gap, on lags of the inflation gap, on lags of the risk aversion variable, on the innovations of the inflation target $\varepsilon_{\pi^\star t}$, on the

[3] See Chapter (3.3.1) for a more detailed discussion of the theoretical considerations of the central bank's response to movements in term premia.

innovations of inflation $\varepsilon_{\pi t}$, and on its own innovations ε_{yt},

$$\begin{aligned} g_t^y - g^y &= \rho_{yr}\left(g_{t-1}^r - g^r\right) + \sum_{i=1}^{3} \rho_{y\pi}^i g_{t-i}^{\pi} + \sum_{i=1}^{3} \rho_{yy}^i \left(g_{t-i}^y - g^y\right) \\ &\quad + \rho_{yv} v_{t-1} + \sigma_{y\pi}\sigma_{\pi}\varepsilon_{\pi t} + \sigma_{y\pi^\star}\sigma_{\pi^\star}\varepsilon_{\pi^\star t} + \sigma_y \varepsilon_{yt}, \end{aligned} \tag{4.3}$$

where the volatility parameter σ_y is non-negative, and ε_{yt} is supposed to be standard normally distributed. The evolution of the inflation gap depends on own lags, on lags of the interest rate gap, on lags of the output gap, on lags of the risk aversion variable, on innovations of the inflation target $\varepsilon_{\pi^\star t}$, and on its own innovations $\varepsilon_{\pi t}$,

$$\begin{aligned} g_t^{\pi} &= \rho_{\pi r}\left(g_{t-1}^r - g^r\right) + \sum_{i=1}^{3} \rho_{\pi\pi}^i g_{t-i}^{\pi} + \sum_{i=1}^{3} \rho_{\pi y}^i \left(g_{t-i}^y - g^y\right) \\ &\quad + \rho_{\pi v} v_{t-1} + \sigma_{\pi\pi^\star}\sigma_{\pi^\star}\varepsilon_{\pi^\star t} + \sigma_{\pi}\varepsilon_{\pi t}, \end{aligned} \tag{4.4}$$

where the volatility parameter σ_π is non-negative, and $\varepsilon_{\pi t}$ is standard normally distributed.

The fiscal variable of a country is given by the change in the debt-to-GDP ratio of the respective country. In the choice of the change of the debt-to-GDP ratio as the measure of fiscal sustainability, I follow Borgy et al. (2012) and Dewachter et al. (2015). Similar to Borgy et al. (2012), the dynamics of the fiscal variables are modeled by a stationary AR(1) process,

$$d_t^i = \rho_d^i d_{t-1}^i + \sigma_d^i \varepsilon_{dt}^i \quad \forall i \in \text{fr, it, es} \tag{4.5}$$

where d_t^i denotes the fiscal variable of a country i, $\rho_d^i \in [0, 1)$ is the persistence parameter and $\sigma_d^i > 0$ is the volatility parameter. The shock ε_{dt}^i is standard normally distributed. For parsimonious reasons, the specification supposes that the debt-to-GDP growth rate is exogenous from the other state variables. Omitting feedback

effects from the European business cycle and the common variables to the national fiscal variables help to reduce the number of parameters in an already highly parameterized model.

The model features a latent common non-fundamental risk variable which can affect the yield spreads of the Euro area sovereigns. This factor potentially captures the effects of redenomination risk or contagion on yield spreads. As in Hördahl and Tristani (2013), the common non-fundamental risk variable is supposed to be unrelated to economic fundamentals but is allowed to exhibits an endogenous dynamic through a feedback effect ρ_C and a structural shock ε_{Ct} developments. Specifically, the dynamic of the common non-fundamental risk variable is given by the following AR(1) process

$$C_t = \rho_C C_{t-1} + \sigma_C \varepsilon_{Ct}, \tag{4.6}$$

where $\rho_C \in [0, 1)$, $\sigma_C > 0$, and the shock ε_{Ct} is standard normally distributed.

The risk aversion variable is supposed to be the most endogenous variable in the economy. It does respond to shocks to the common variables (ε_{rt}, $\varepsilon_{\pi t}$, ε_{yt}, $\varepsilon_{\pi^\star t}$, and ε_{Ct}) and the country-specific fiscal variables (ε_{dt}^{fr}, ε_{dt}^{es}, ε_{dt}^{it}) and also to a risk aversion shock (ε_{vt}). This shock is meant to account for not macro related shifts in risk aversion. Moreover, the stochastic process allows for endogenous dynamics, through a feedback effect (ρ_{vv}). The risk aversion variable is identified from the time variation in the prices of risk in the stochastic discount factor. Precisely, by construction, all movements in the prices of risk are attributed to the risk aversion variable (see Section (4.3.3)). Movements in the prices of risk are in turn identified from the default-free reference term structure. Specifically, the evolution of the risk

aversion variable is given by

$$\begin{aligned} v_t &= \rho_{vv} v_{t-1} + \sigma_{vr}\sigma_r \varepsilon_{rt} + \sigma_{v\pi}\sigma_\pi \varepsilon_{\pi t} + \sigma_{vy}\sigma_y \varepsilon_{yt} + \sigma_{v\pi^\star}\sigma_{\pi^\star} \varepsilon_{\pi^\star t} \\ &\quad + \sigma_{vC}\sigma_C \varepsilon_{Ct} + \sigma_{vd}^{fr}\sigma_d^{ft}\varepsilon_{dt}^{fr} + \sigma_{vd}^{es}\sigma_d^{es}\varepsilon_{dt}^{es} + \sigma_{vd}^{it}\sigma_d^{it}\varepsilon_{dt}^{it} + \sigma_v \varepsilon_{vt}, \end{aligned} \tag{4.7}$$

where the volatility parameter σ_v is non-negative, and ε_{vt} is standard normally distributed.

The chosen structure imposes restrictions in order to identify the structural model. As in Ireland (2015), shocks to the inflation target $\varepsilon_{\pi^* t}$ affect the interest rate gap, the inflation gap, the output gap, and the risk aversion variable only contemporaneously. All further effects from the change in the inflation target arise only from changes in the inflation gap and the interest rate gap. Thus, Ireland's (2015) specification imposes a form of long-run monetary neutrality. To disentangle the effects of different fundamental disturbances on the economy's variables, eq. (4.2) - (4.7) contain restrictions on the contemporaneous relationship of some of the model's variables.

The central bank responds immediately to changes in the risk aversion variable while the risk aversion variable only responds to interest rate shocks. While the interest rate responds immediately to fluctuations of the output gap and the inflation gap, changes in the short-term interest rate do not affect the output gap and the inflation gap immediately, but with one period lag. Following Ireland (2015), the output gap shock ε_{yt} does only affect the inflation gap with a lag of one period, while a shock to the inflation gap affects the output gap contemporaneously. Moreover, the fiscal variables are modeled by an autoregressive process, as already discussed above. In addition, as in Borgy et al. (2012) and Hördahl and Tristani (2013), direct feedbacks from the national fiscal variables to the Euro area business cycle are

omitted. However, through their effects on v_t, they can affect the economy.

Define the vectors X_t and ε_t containing the state variables and the structural disturbances, respectively, by

$$X_t = \left[\begin{array}{cccccccccccc} g_t^r & g_t^y & g_{t-1}^y & g_{t-2}^y & g_t^\pi & g_{t-1}^\pi & g_{t-2}^\pi & \pi_t^* & C_t & v_t & d_t^{fr} & d_t^{es} & d_t^{it} \end{array} \right]'$$

and

$$\varepsilon_t = \left[\begin{array}{cccccccccccc} \varepsilon_{rt} & \varepsilon_{yt} & 0 & 0 & \varepsilon_{\pi t} & 0 & 0 & \varepsilon_{\pi^\star t} & \varepsilon_{Ct} & \varepsilon_{vt} & \varepsilon_d^{fr} & \varepsilon_d^{es} & \varepsilon_d^{it} \end{array} \right]',$$

then, eq. (4.1) - (4.6) can be written more compactly,

$$P_0 X_t = \mu_0 + P_1 X_{t-1} + \Sigma_0 \varepsilon_t. \tag{4.8}$$

For the precise definition of the matrices P_0, P_1, μ_0, and Σ_0 see Appendix (4.A.1). Eq. (4.8) gives the structural form of the model. Multiplying by P_0^{-1} yields the reduced form representation of the state equation,

$$X_t = \mu + P X_{t-1} + \Sigma \varepsilon_t, \tag{4.9}$$

where

$$\begin{aligned} \mu &= P_0^{-1} \mu_0, \\ P &= P_0^{-1} P_1, \end{aligned}$$

and

$$\Sigma = P_0^{-1} \Sigma_0.$$

4.3.2 The Term Structure Model

Affine term structure models, as developed by Duffie and Kan (1996) and Dai and Singleton (2000), are a particular class of term structure models where the time t yield $y_t^{(\tau)}$ of $\tau-$period zero coupon bond is modeled as an affine function of the state vector.

The outlined model follows the discrete-time version by Ang and Piazzesi (2003). Restrictions on the prices of risk similar to those in Dewachter and Iania (2011), Dewachter et al. (2014), and Ireland (2015) are imposed to identify the risk aversion variable v_t as the only source of fluctuations in the prices of risk. In order to study the role of default risk in this affine set-up, I employ the extension of affine term structure models to defaultable bond as proposed by Duffie and Singleton (1999). This subsection is structured as follows: the first part derives the default-risk-free bond prices and discusses the restrictions on the prices of risk. The second part derives the prices of defaultable bonds.

Default risk-free Bond Pricing and the Prices of Risk

The short-end of the yield curve, the nominal short-term risk-free interest rate, is modeled as an affine function of the state vector X_t. The short-term interest rate equation is given by

$$r_t = \delta_0 + \delta_1^{'} X_t, \tag{4.10}$$

where δ_0 is a scalar, and $\delta_1^{'}$ is a 1x13 selection vector indicating the position of g_t^r and τ_t in X_t. The short-term rate is from eq. (4.2) under the control of the central bank. The coefficients δ_0 and δ_1 are restricted to ensure consistency between the

macro part and the term structure part of the model. This requires $\delta_0 = 0$ and

$$\delta_1^{'} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

so that eq. (4.10) corresponds to the definition of the interest rate gap.

The prices of government bonds are supposed to be arbitrage free. As shown in Harrison and Kreps (1979) or Duffie (2001, pp. 108) this assumption guarantees for the existence of a positive stochastic discount factor. Following, among many others, Ang and Piazzesi (2003), the stochastic discount factor which is used to price all bonds in the economy is given by the following log-normal process

$$m_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t^{'}\lambda_t - \lambda_t^{'}\varepsilon_{t+1}\right), \tag{4.11}$$

where λ_t are the time-varying prices of risk. If all elements in λ_t are equal to zero, investors are risk neutral. The prices of risk are supposed to be affine functions of the state variables, taking the functional form

$$\lambda_t = \lambda_0 + \Lambda_1 X_t, \tag{4.12}$$

where λ_0 is a 13×1 vector and Λ_1 is a 13×13 matrix.

In the following, restrictions on the matrix λ_1 are imposed. First, similar to Dewachter and Iania (2011), Dewachter et al. (2014), and Ireland (2015), in order to identify the risk aversion variable v_t as the only source for time-variation in the prices of risk, all elements in Λ_1, except the 10th column, are restricted to be equal to zero. Second, I assume that only contemporaneous state variables can be priced.

After applying the restrictions on the matrix Λ_1, Λ_1 reads

$$\begin{aligned}\Lambda_1(*,i) &= \begin{bmatrix} \Lambda^r & \Lambda^y & 0 & 0 & \Lambda^\pi & 0 & 0 & \Lambda^{\pi^*} & \Lambda^C & \Lambda^v & \Lambda^{dfr} & \Lambda^{des} & \Lambda^{dit} \end{bmatrix}' \text{ for } i = 10, \\ \Lambda_1(*,i) &= 0 \text{ for } i \neq 10,\end{aligned}$$

and the corresponding vector λ_0 reads

$$\lambda_0 = \begin{bmatrix} \lambda_0^r & \lambda_0^y & 0 & 0 & \lambda_0^\pi & 0 & 0 & \lambda_0^{\pi^*} & \lambda_0^C & \lambda_0^v & \lambda_0^{dfr} & \lambda_0^{des} & \lambda_0^{dit} \end{bmatrix}'.$$

From eq. (4.12) these restrictions work to attribute all movements in the prices of risk λ_t to the variable that is ordered at the 10th position in X_t, that is, the risk aversion variable v_t. As demonstrated in section (4.3.3), from the restricted form of Λ_1 also all time-variations in the one-period return premium are attributed to the risk aversion variable.

Let $P_t^{\tau+1}$ denote the price of a risk-free zero-coupon bond maturing at time $t+\tau$, then, given the no-arbitrage assumption, the pricing kernel m_{t+1}, and the affine prices of risk λ_t, from the no-arbitrage condition

$$P_t^{\tau+1} = E_t\left(m_{t+1} P_{t+1}^{\tau}\right),$$

the bond price $P_t^{\tau+1}$ can be written as an exponentially affine function of the state vector X_t. Specifically, the price of a $t+\tau$-period risk-free zero-coupon bond $P_t^{\tau+1}$ at period t is given by

$$P_t^{\tau+1} = \exp\left(\bar{A}_{\tau+1} + \bar{B}'_{\tau+1} X_t\right). \tag{4.13}$$

The coefficients $\bar{A}_{\tau+1}$ and $\bar{B}_{\tau+1}$ can be computed by the standard recursive formulas as provided by Ang and Piazzesi (2003).

Pricing of Defaultable Bonds

Following Duffie and Singleton (1999), the no-arbitrage affine term structure model can be extended to price also defaultable bonds. Duffie and Singleton (1999) show that under the assumption that the recovery value of a defaulting bond is a fraction of the bond's value conditional on no default would occur (the so-called "recovery of market value" assumption), there exists some recovery-adjusted default intensity process $s_{j,t}$ (see Appendix (4.A.2)). Defaultable bonds can then be priced using the same formulas, simply by replacing the risk-free short-term interest rate r_t by the default-adjusted short-term interest rate $r^*_{j,t+1} = r_t + s_{j,t+1}$, Then, bond prices can be expressed by

$$\tilde{P}^{\tau+1}_{j,t} = E_t\left(\exp\left(-r^*_{j,t+1} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\tilde{P}^{\tau}_{j,t+1}\right),$$

where $\tilde{P}^{\tau+1}_{j,t}$ denotes the time-t price of a $\tau+1$-period defaultable bond of country j. If the "recovery-adjusted default intensity" (see e.g. Monfort and Renne, 2011) $s_{j,t}$ of a country j is also an affine function of the state vector,

$$s_{j,t} = \psi_{j,0} + \psi_{j,1}X_t,$$

then one can proceed as in standard valuation models for default-risk free bonds. Hence, the price of a zero-coupon defaultable bond can be expressed by

$$\tilde{P}^{\tau+1}_{j,t} = \exp\left(\bar{A}_{j,\tau+1} + \bar{B}'_{j,\tau+1}X_t\right) \tag{4.14}$$

where the specific solution of the pricing matrices $\bar{A}_{j,\tau+1}$ and $\bar{B}'_{j,\tau+1}$ can be computed by the standard recursive formulas. However, these formulas come along with intense computational costs since the pricing matrices have to be calculated for each period

$\tau = 1, ..., 60$, each country j and each evaluation of the log-posterior function. Therefore, I apply an algorithm based by Borgy et al. (2012). Instead of computing the pricing matrices $\bar{A}_{j,\tau+1}$ and $\bar{B}'_{j,\tau+1}$ recursively, this algorithm computes only selected nested bond maturities and concatenates country-specific pricing matrices to compute parts of the pricing matrices for all countries simultaneously. As demonstrated by Borgy et al. (2012), this algorithm reduces computation time considerably. The solution for the pricing matrices $\bar{A}_{j,\tau+1}$ and $\bar{B}'_{j,\tau+1}$and the algorithm are discussed in Appendix (4.A.3).

Finally, the dependence of the adjusted default intensities of country j on the state variables, that is, the elements in the vector $\psi_{j,1}$ need to be specified. Instead of estimating all elements in $\psi_{j,1}$, I follow, among others, Borgy et al. (2012) and impose restrictions on $\psi_{j,1}$. This helps to conserve the number of parameters that need to be estimated. First, the German term structure is supposed to be free of default risk, thus $\psi_{ger,1} = 0_{13x1}$. Noteworthy, in this case, the solution of $\bar{A}_{ger,\tau+1}$ and $\bar{B}'_{ger,\tau+1}$ reduces to the solution for pricing matrices of the risk-free bonds $\bar{A}_{\tau+1}$ and $\bar{B}_{\tau+1}$, respectively. Thus, the German term structure is taken as the default-free reference term structure. It is used to identify the time variation in the prices of risk. Second, as in Borgy et al. (2012) and Dewachter et al. (2015), the spread between risk-free and defaultable bonds depends on common and country-specific factors. In particular, the spread between the yield on a defaultable bond of country j and the yield of a risk-free bond with the same maturity is assumed to depend on the common, euro area economic fundamentals, the common non-fundamental risk factor, and the country-specific fiscal variable of country j. However, it does not depend on the fiscal variables of the other countries. Finally, only contemporaneous variables are allowed to affect spreads.

Bond Yields

The continuously compounded bond yields $y_{j,t}^{(\tau)}$ are defined by

$$y_{ger,t}^{(\tau)} = -\frac{\log\left(P_t^{\tau}\right)}{\tau},$$

and

$$y_{j,t}^{(\tau)} = -\frac{\log\left(\tilde{P}_{j,t}^{\tau}\right)}{\tau} \quad \forall j \neq ger.$$

Given the bond prices $P_t^{\tau+1}$ and $\tilde{P}_{j,t}^{\tau+1}$ from eq. (4.13) and eq. (4.14), respectively, the yields are given by

$$y_{ger,t}^{(\tau)} = A_\tau + B_\tau' X_t, \tag{4.15}$$

and

$$y_{j,t}^{(\tau)} = A_{j,\tau} + B_{j,\tau}' X_t \quad \forall j \neq ger, \tag{4.16}$$

respectively, where $A_\tau = -\bar{A}_\tau/\tau$, $B_\tau' = -\bar{B}_\tau'/\tau$, $A_{j,\tau} = -\bar{A}_{j,\tau}/\tau$ and $B_{j,\tau}' = -\bar{B}_{j,\tau}'/\tau$.

4.3.3 Term Premia

This section demonstrates how the restrictions on the prices of risk affect the one-period return premium. It can be shown that all movements in the one-period return premium can be to attribute to the risk aversion variable v_t. Term structure premia can be captured in different forms (see e.g. Cochrane and Piazzesi, 2008, or Joslin et al., 2014). Similar to Dewachter and Iania (2011), I focus in this analysis on the return premium (as classified by Cochrane and Piazzesi, 2008). For a more detailed discussion of different types of term premia and their relation see Section (3.3.3).

The one-period *return* premium is defined as the expected excess holding period return (or short expected excess return). It is the expected return from buying a

long-term bond in period t and selling it in the subsequent period $t+1$ in excess of the expected return from buying a one-period bond. Formally, the one-period *return* premium is defined as

$$E_t\left(hprx_{t+1}^{(\tau)}\right) = E_t\left(hpr_{t+1}^{(\tau)} - y_t^{(1)}\right),$$

where $hpr_{t+1}^{(\tau)}$ is the holding period return defined by $hpr_{t+1}^{(\tau)} \equiv p_{t+1}^{(\tau-1)} - p_t^{(\tau)}$, where $p_t^{(\tau)}$ is the log price of a zero-coupon bond maturing in $t+\tau$ periods, $p_t^{(\tau)} \equiv \log\left(P_t^{(\tau)}\right)$ and $y_t^{(1)}$ is the yield of a one-period bond. The holding period return $hpr_{t+1}^{(\tau)}$ is the return from buying a bond at time t that matures in $t+\tau$ periods and selling this bond the period after.

To bring the one-period return premium in a computationally more tractable form, the expected holding period return and the expected short rate have to be calculated. The expected future short rate is given from eq. (4.10) by $E_t(r_{t+1}) = \delta_1' E_t(X_{t+1})$. To calculate the expected future short-term interest rate, it proves to be helpful to demean the state equation, eq. (4.9). Let $\bar{\mu}$ be the unconditional mean of the state vector, then from eq. (4.9) $\bar{\mu}$ is given by $\bar{\mu} = (I-P)^{-1}\mu$, and the demeaned state equation reads $X_{t+1} - \bar{\mu} = P(X_t - \bar{\mu}) + \Sigma\varepsilon_{t+1}$. Then, the time-$t$ conditional expected future short rate for period $t+1$, can be computed by

$$E_t(r_{t+1}) = \delta_1'(I - \delta' P)\bar{\mu} + \delta_1' P X_t.$$

The expected holding period return can be calculated by plugging the model implied log prices, $p_t^{(\tau)} = \bar{A}_\tau + \bar{B}_\tau' X_t$, into the definition of the one-period holding period return. Plugging the expected short-term interest rate and the expected holding period return into the definition of the one-period return premium and rearranging

terms (see Appendix (4.A.4)) yields

$$E_t\left(hprx_{t+1}^{(\tau)}\right) = \bar{B}'_{\tau-1}\Sigma\left(\lambda_0 + \Lambda_1 X_t\right) - \frac{1}{2}\bar{B}'_{\tau-1}\Sigma\Sigma'\bar{B}_{\tau-1}. \tag{4.17}$$

Due to the restricted form of Λ_1 the only source of variations in $E_t\left(hprx_{t+1}^{(\tau)}\right)$ is the variable that is ordered at the 10^{th} position in X_t, that is, the risk aversion variable v_t. Thus, eq. (4.17) reveals that all variation over time in one-period return premia arises solely from fluctuations in v_t. If all elements in the matrix Λ_1 are equal to zero, then the one-period return premium is constant. Likewise, if v_t is constant over time, the return premium is constant.

4.4 Estimation

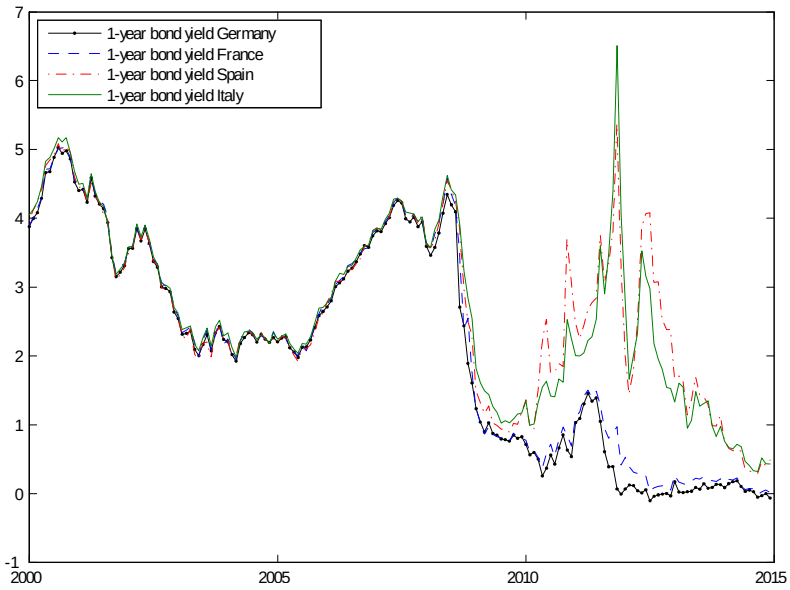
4.4.1 Data

My sample contains monthly data on the Euro area from the beginning of 2000 until the End of 2014. I use government bonds from the four biggest economies in the Euro area: Germany, France, Italy, and Spain. The German term structure is taken as the reference term structure and considered to be free of default risk.[4] Not only is the country relatively large and plays a central role in the Euro area, but also, as shown by De Santis (2014) does the German Bund yield co-move with the OIS rate. The sample contains data for the country-specific fiscal variables, the Euro area business cycle, and the risk-free short-term interest rate.

The model requires zero-coupon yield data. However, government bonds with maturities of more than one year usually do pay coupons. The zero-coupon yields need to be constructed from these data. All zero-coupon yields are constructed using

[4] As noted by De Santis (2015), the expected probability of a credit event in Germany is considered to be negligible.

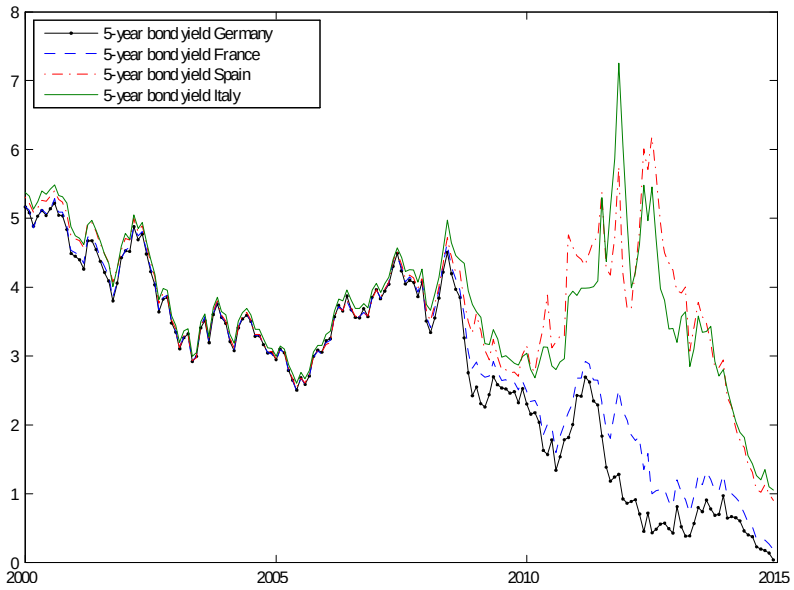
Figure 4.2: One-year bond yields of Euro area sovereigns



the same method to ensure the comparability of yields across countries. Specifically, I estimate the zero-coupon bond yields from the prices of government bonds of each of the four countries using the Nelson-Siegel (1987) model. The data for the end-of-month government bond prices of each country is taken from Datastream. Appendix (4.A.5) describes the data selection and the estimation of the zero-coupon yields in more detail. After constructing the zero-coupon yield data, for the subsequent estimation, yields with maturities of 3 months and 1, 2, 3, 4, and 5 years for the German term structure are selected and yields with maturities of 1 and 5 years for the French, Italian and Spanish term structure are selected. Figure (4.2) and (4.3) depict the estimated one-year and five-year yields of the government bonds of France, Germany, Italy and Spain.

The Euro area variables are the inflation rate, the output gap, and the short-term interest rate. While the first two variables capture the European business cycle, the latter captures monetary policy. The inflation rate is measured by the annual rate of

Figure 4.3: Five-year bond yields of Euro area sovereigns



change of the seasonally adjusted HICP of the euro area. The output gap is defined as the percentage (logarithmic) deviation of actual output from trend output. Since GDP data is only available at a quarterly frequency, I use the seasonally adjusted industrial production index of the Euro area as a proxy for output (as e.g. Clarida et al., 1998, or Favero, 2006). Trend output is constructed using the one-sided HP filter with a smoothing parameter equal to 14,400. The Euro area-wide, risk-free monetary policy rate is proxied by the 3-month rate of zero-coupon German government bonds. In choosing the 3-month rate as the rate with the shortest maturity, I follow the practice of the Bundesbank (see Schich, 1997).[5]

The fiscal variable of a country is measured by the change in the debt-to-GDP ratio of the respective country. The data for the debt-to-GDP ratio is taken from Datastream. Since the debt-to-GDP ratio is only available on a quarterly basis,

[5] The trading volume of government bonds decreases considerably for short residual maturities so that their prices seem to be significantly influenced by low liquidity (see BIS, 2005, p.9). Therefore, prices of bonds with residual maturities shorter than one month are excluded.

the missing observations need to be constructed. Instead of simply interpolating the data, I follow Hördahl and Tristani (2013) and suppose an autoregressive law of motion for the debt-to-GDP ratio. Specifically, in a preceding step, by presuming the autoregressive law of motion the time-path of the missing observations is constructed by the Kalman filter. Finally, I suppose that the long-run mean of the change in the debt-to-GDP ratio is zero (as in Borgy et al., 2012).

4.4.2 The State-Space System

The state equation (4.9) and the measurement equations (4.15) and (4.16) form the state-space system. For the estimation, a version of the state-space model without constant terms is employed. Following Ireland (2015), by using demeaned data and dropping the constant terms appearing in eq. (4.9), (4.15), and (4.16), the estimation is simplified. In particular, under the assumption that the central bank is, on average, able to implement its target inflation rate, the steady state values of g^r, τ, and g^y can be calibrated to match their data averages. More precisely, by setting the steady-state inflation target equal to the steady-state value of the inflation rate, the steady-state value of the interest rate gap g^r can be calculated from the average of the short-term interest rate net the average of the inflation rate. The steady state value of the output gap is set equal to the sample mean of the output gap. As in Borgy et al. (2012), the fiscal factors are assumed to have a mean of zero. This implies that the debt-to-GDP ratio of each country is stationary. Finally, as shown by Ireland (2015), the values of the elements in λ_0 can be set so that the steady state values of yields match the average yields. The state equation then reads

$$X_t = PX_{t-1} + \Sigma\varepsilon_t,$$

and the measurement equation can then be written by

$$Z_t = UX_t + V\eta_t,$$

where Z_t is a vector of observables, U is a matrix connecting the observables to the state vector, η_t is a vector of $i.i.d.$ distributed errors and V is a matrix capturing the volatility parameters of these errors. The vector of observables Z_t consists of the government bond yields of the four countries, the country-specific fiscal variables and the three observables capturing the European business cycle and monetary policy. The vector of observables reads

$$\begin{aligned} Z_t \equiv \Big[& y^{12}_{ger,t} \quad y^{24}_{ger,t} \quad y^{36}_{ger,t} \quad y^{48}_{ger,t} \quad y^{60}_{ger,t} \quad y^{12}_{fr,t} \quad y^{60}_{fr,t} \\ & y^{12}_{es,t} \quad y^{60}_{es,t} \quad y^{12}_{it,t} \quad y^{60}_{it,t} \quad d^{fr}_t \quad d^{es}_t \quad d^{it}_t \quad r_t \quad g^y_t \quad \pi_t \Big]'. \end{aligned}$$

The definition of the matrix U is presented in Appendix (4.A.1).

The matrix V contains the volatility parameters of the yield errors. The errors are attached to avoid stochastic singularity. The problem of stochastic singularity arises in macro-finance term structure models because a high dimensional vector of observables (the yield data and the observable macro variables) is fitted to a lower dimensional state vector. Instead of attaching errors to some selected yields, I assume that all yields are affected by error terms, as in Chib and Ergashev (2009). The last columns of V are equal to zero, reflecting that the short-term interest rate, the output gap, the inflation rate, and the changes in the debt-to-GDP ratios of the three countries are not measured with errors.

4.4.3 Estimation Method

The model captures the effect of national fiscal variables, investors' risk aversion, the European business cycle, a time-varying inflation target, and a common non-fundamental risk factor on sovereign yields. Changes in risk aversion are identified from the default-free term structure. The non-fundamental risk variable is given by the part of sovereign yields that cannot be accounted for by economic fundamentals and country-specific fiscal factors.

In order to account for the interaction of risk aversion and the non-fundamental risk factor, it is not possible to split the estimation into separate steps (e.g. estimating first the risk-free term structure and the macro dynamics together and then the term structure of each of the other counties separately). Instead, the term structures of the four countries under consideration need to be estimated jointly. This complicates the estimation considerably.

Due to the non-linearity of the parameters in the $B_{j,\tau}$-matrices of the measurement equations, even the estimation of pure latent affine term structure model is computationally challenging and time-consuming (see e.g. Chib and Ergashev, 2009, or Christensen et al., 2011). Due to the dynamics and interactions of the macroeconomic variables in the state system, the estimation becomes even more challenging (as discussed e.g. in Rudebusch and Wu, 2008). The non-linearity of the parameters in the $B_{j,\tau}$ matrices can produce multimodal likelihood functions, as demonstrated by Chib and Ergashev (2009). In this case, Bayesian estimation techniques help to down-weight regions of the parameter space which are not economically reasonable and to rule out economically implausible parameter values by employing a priori information. As a result, the joint posterior distribution can be smoother than the joint likelihood function (see Chib and Ergashev, 2009). Moreover, the usage of

prior information is helpful when dealing with short data sets in highly parameterized models like this one.

Posterior and Likelihood function

Formally, let Z denotes the data set, $Z = (Z_1, ..., Z_T)'$, where T is the number of total observations, and let θ denotes the vector of all parameters contained in the matrices P, Σ, Λ, and V, then from Bayes rule, the joint posterior distribution of θ, $\pi(\theta|X)$, is obtained by combining the likelihood function of the observables, the prior distribution of the parameter vector and a norming constant, thus,

$$\pi(\theta|Z) \propto L(Z|\theta)\, p(\theta),$$

where $L(Z|\theta)$ is the likelihood function, and $p(\theta)$ is the prior distribution. If the initial state X_0 and the innovations $\{\varepsilon_t, \eta_t\}_{t=1}^{T}$ are multivariate Gaussians, then the distribution of the observables in Z_t conditional on all information of the observables available at time $t-1$ is also Gaussian (see Hamilton, 1994, p. 385). The joint density of the date set Z given θ can be written as

$$\begin{aligned} L(Z|\theta) &= \prod_{t=1}^{T} (2\pi)^{-\frac{T}{2}} \left[\det\left(R_{t|t-1}\right)\right]^{-\frac{1}{2}} \\ &\quad \times \exp\left(-\frac{1}{2}\left(Z_t - UX_{t|t-1}\right)'\left(R_{t|r-1}\right)^{-1}\left(Z_t - UX_{t|t-1}\right)\right), \end{aligned}$$

where $X_{t|t-1}$ denotes the one step ahead forecast, $X_{t|t-1} \equiv E[X_t|\mathcal{Z}_{t-1}, \theta]$, and $R_{t|t-1}$ denotes the conditional variance, $R_{t|t-1} \equiv Var(Z_t|\mathcal{Z}_{t-1}, \theta)$.[6] Since two of the state variables are latent, the likelihood $L(Z|\theta)$ is constructed using the standard Kalman filter recursions (see Harvey, 1991). Since the posterior density is, in general, not

[6] See Section (3.4.3) and the therein mentioned Appendix for the explicit expressions of the prediction and updating equations of the mean and the variance.

known in closed form, I apply Markov Chain Monte Carlo (MCMC) methods, in particular an Adaptive-Metropolis algorithm, to simulate draws from the joint posterior distribution.

MCMC Method

The choice of the proposal density of the Metropolis-Hastings algorithm is important for the speed of the convergence of the Metropolis-Hastings chain. As already discussed in Section (3.4.3), the scaling of the proposal density "by hand", becomes very hard if not infeasible. Therefore, in order to evaluate the posterior, I apply the Adaptive Metropolis (AM) algorithm as introduced by Haario et al. (2001).

The main idea of the AM algorithm is to run a chain that improves "on the fly" by using all information cumulated so far. More precisely, in each step of the algorithm, the covariance of the proposal distribution is updated using the information from of all of the previous states. Therefore, the AM algorithm adapts continuously to the target distribution. Apart from the updating scheme, the algorithm is based on the standard random walk Metropolis-Hastings algorithm. Haario et al. (2001) show that although the AM algorithm is non-Markovian due to its adaptive nature, it still has the correct ergodic properties. A detailed description of the outlined AM algorithm and its implementation are given in Section (3.4.3).

In order to start the algorithm, the initial covariance matrix of the proposal distribution is set equal to a scaled down version of the inverse of the Hessian matrix computed at the posterior mode. The choice of an appropriate initial covariance C_0 helps to speed up the algorithm and thus to increase efficiency (see Haario et al., 2001). The initial parameter vector is set to the parameter values at the mode. For the choice of the scaling parameter s_d, I follow Haario et al. (2001), whose choice, in turn, is based on Gelman et al. (1996), and set $s_d = (2.4)^2 / d$. The initial period is

set to $n_0 = 20,000$, and the number of total draws is set to $n = 1,000,000$.

Since Newton-like optimization routines tend to get stuck in local optima, there are not suitable to find the mode of the posterior function (as discussed by e.g. Chib and Ergashev, 2009). In order to find the mode of the posterior, I employ an evolutionary optimization algorithm. Precisely, the Covariance Matrix Adaption Evolution Strategy (CMA-ES) algorithm is a stochastic method for numerical parameter optimization of non-linear, non-convex functions with many local optima (see Hansen and Ostermeier, 2001). The computation of the mode is conducted by the software package Dynare (Adjemian et al., 2011).

4.4.4 Parameter Restrictions and Prior Distributions

Parameter Restrictions

For the estimation, restrictions are imposed to ensure the stationarity of the macro dynamics, the stability of the arbitrage recursions, and the identification of the model. Stationarity of the state dynamics requires the eigenvalues of the matrix P to be less than unity in absolute value, $|eig(P)| < 1$. A similar restriction has to be imposed to guarantee the stability of the no-arbitrage recursions (see e.g. Dai and Singleton, 2000). Specifically, the eigenvalues of $P - \Sigma\Lambda$ have to be less than unity in absolute value, $|eig(P - \Sigma\Lambda)| < 1$. For identification purposes, the scaling of the latent variables v_t and C_t have to be pinned down, since a multiplicative transformation of each of the latent factors leads to an observationally equivalent system. To pin down the scale of the latent variables, the scaling parameters of these variables are set equal to $\sigma_v = 0.01$ and $\sigma_C = 0.01$. In the same spirit, the direction in which an increase in the risk aversion variable v_t moves the prices of risk, needs to be specified. Following Ireland (2015), without loss of generality, the constraint $\Lambda^{\pi} \leq 0$ is imposed. Finally,

similar to Dewachter et al. (2014) and Ireland (2015), to guarantee that v_t only moves the prices of risk associated with the other four state variable, the restriction $\Lambda^v = 0$ is imposed. This imposes that the risk aversion variable is not itself a sourced for priced risk. After applying the restrictions, there are 83 parameters left to estimate.

Prior Distributions

This section presents the prior distributions. By applying prior distributions to the parameters, economically non-meaningful regions of the parameter space can be down-weighted. This reduces the complexity of the maximization problem (see Chib and Ergashev, 2009, for a more detailed discussion).

The first part of the table (4.1) displays the prior distributions of the parameters of the monetary policy rule and the parameters associated with the endogenous dynamic of the other state variables. I follow closely Smets and Wouters (2003) for the choice of the priors for the monetary policy rule coefficients. The parameter capturing the degree of interest rate smoothing ρ_r is $\mathcal{B}(0.8, 0.05)$ distributed. The choice of the Beta distribution captures the belief that the parameter lies in the interval between 0 and 1. The prior distributions of the response coefficient on inflation gap ρ_π and on the output gap are ρ_y are given by $\mathcal{G}(1.5, 0.25)$ and $\mathcal{G}(0.5, 0.1)$, respectively. The Gamma distribution is employed to capture the assumption that both parameters are not negative. The coefficient of central bank's response to changes in the risk aversion variable ρ_v is assumed to be $\mathcal{N}(0, 0.25)$ distributed. The choice of the prior means implies that monetary policy is, a priori, characterized by a standard Taylor rule.

The prior distributions of the parameters describing the dynamics of the macroeconomy are also displayed in the first part of table (4.1). The prior distributions for the state equation (eq. 4.2 - 4.7) are chosen in the spirit of Minnesota (see Lit-

Table 4.1: Summary of the Prior Distributions

Taylor Rule and Persistence Parameter							
Param.	type	mean	std. dev.	Param.	type	mean	std. dev.
ρ_r	$\mathcal{B}$	0.80	0.050	$\rho^1_{y\pi}$	$\mathcal{N}$	0.00	0.150
ρ_π	$\mathcal{G}$	1.50	0.250	$\rho^2_{y\pi}$	$\mathcal{N}$	0.00	0.075
ρ_y	$\mathcal{G}$	0.50	0.150	$\rho^3_{y\pi}$	$\mathcal{N}$	0.00	0.050
ρ_v	$\mathcal{B}$	0.00	0.250	ρ^1_{yy}	$\mathcal{N}$	0.90	0.150
$\rho_{\pi r}$	$\mathcal{N}$	-0.20	0.150	ρ^2_{yy}	$\mathcal{N}$	0.00	0.075
$\rho^1_{\pi\pi}$	$\mathcal{N}$	0.90	0.150	ρ^3_{yy}	$\mathcal{N}$	0.00	0.050
$\rho^2_{\pi\pi}$	$\mathcal{N}$	0.00	0.075	ρ^i_d	$\mathcal{B}$	0.90	0.100
$\rho^3_{\pi\pi}$	$\mathcal{N}$	0.00	0.050	ρ_C	$\mathcal{B}$	0.90	0.100
$\rho^1_{\pi y}$	$\mathcal{N}$	0.00	0.150	ρ_{π^*}	$\mathcal{B}$	0.90	0.100
$\rho^2_{\pi y}$	$\mathcal{N}$	0.00	0.075	ρ_{yv}	$\mathcal{N}$	0.00	0.250
$\rho^3_{\pi y}$	$\mathcal{N}$	0.00	0.050	$\rho_{\pi v}$	$\mathcal{N}$	0.00	0.250
ρ_{yr}	$\mathcal{N}$	-0.20	0.150				
Volatility and co-movement Parameters							
Param.	type	mean	std. dev.	Param.	type	mean	std. dev.
σ_{vr}	$\mathcal{N}$	0.00	2.00	σ_y	$\mathcal{IG}$	0.01	0.200
$\sigma_{v\pi}$	$\mathcal{N}$	0.00	2.00	σ_{π^*}	$\mathcal{IG}$	0.01	0.200
σ_{vy}	$\mathcal{N}$	0.00	2.00	σ^{fr}_d	$\mathcal{IG}$	0.01	0.200
$\sigma_{v\pi^*}$	$\mathcal{N}$	0.00	2.00	σ^{es}_d	$\mathcal{IG}$	0.01	0.200
σ_{vC}	$\mathcal{N}$	0.00	2.00	σ^{it}_d	$\mathcal{IG}$	0.01	0.200
σ^{fr}_{vd}	$\mathcal{N}$	0.00	2.00	σ^*_{12}	$\mathcal{IG}$	0.1	1.000
σ^{es}_{vd}	$\mathcal{N}$	0.00	2.00	σ^*_{24}	$\mathcal{IG}$	0.1	1.000
σ^{it}_{vd}	$\mathcal{N}$	0.00	2.00	σ^*_{36}	$\mathcal{IG}$	0.1	1.000
$\sigma_{y\pi}$	$\mathcal{N}$	0.00	2.00	σ^*_{48}	$\mathcal{IG}$	0.1	1.000
$\sigma_{y\tau}$	$\mathcal{N}$	0.00	2.00	σ^*_{60}	$\mathcal{IG}$	0.1	1.000
$\sigma_{\pi\tau}$	$\mathcal{N}$	0.00	2.00	σ^{i*}_{12}	$\mathcal{IG}$	0.1	1.000
σ_r	$\mathcal{IG}$	0.01	0.20	σ^{i*}_{60}	$\mathcal{IG}$	0.1	1.000
σ_π	$\mathcal{IG}$	0.01	0.20				
Prices of Risk and Spread Parameters							
Param.	type	mean	std. dev.	Param.	type	mean	std. dev.
ψ^i_d	$\mathcal{N}$	0.00	2.00	Λ^r	$\mathcal{N}$	0.00	25.00
$\psi^i_{g^y}$	$\mathcal{N}$	0.00	2.00	Λ^{π^*}	$\mathcal{N}$	0.00	25.00
ψ^i_π	$\mathcal{N}$	0.00	2.00	Λ^C	$\mathcal{N}$	0.00	25.00
ψ^i_r	$\mathcal{N}$	0.00	2.00	Λ^π	$\mathcal{N}$	0.00	25.00
ψ^i_v	$\mathcal{N}$	0.00	2.00	Λ^y	$\mathcal{N}$	0.00	25.00
ψ^i_C	$\mathcal{N}$	0.00	2.00	$\Lambda^{d,i}$	$\mathcal{N}$	0.00	25.00
$\psi^i_{\pi^*}$	$\mathcal{N}$	0.00	2.00				

Summary of the prior distributions of the Parameters. Type of the distribution is either $\mathcal{N}$, $\mathcal{B}$, $\mathcal{G}$, or $\mathcal{IG}$ where $\mathcal{N}$ denotes the Normal distribution, $\mathcal{B}$ the Beta distribution, $\mathcal{G}$ the Gamma distribution, and $\mathcal{IG}$ the Inverse-Gamma distribution. The prior distribution holds for all countries i, $\forall i = fr, es, it$.

terman, 1986) by assuming that almost all coefficients are normal distributed and by setting the prior means of most of the coefficients equal to zero, except for those coefficients corresponding to the first own lags of the dependent variables. These coefficients are set equal to 0.9 as suggested by Koop and Korobilis (2010). The choice of the prior means reflects the assumption that these variables exhibit a high degree of persistence, but do not follow a unit root process. The standard deviations of the prior distributions of the parameters are weighted by the lag length, implying that with increasing lag length the coefficients are shrunk towards zero. Departing from Minnesota and following Dewachter and Iania (2011) and Dewachter et al. (2014), I choose a negative prior mean for the parameters ρ_{yr}^1 and $\rho_{\pi r}^1$. These choices capture the beliefs that an increase in the interest rate dampens economic activity. For the parameters ρ_{yv} and $\rho_{\pi v}$, I choose a relatively uninformative prior distribution $\mathcal{N}(0, 0.25)$. This specification assumes that movements in the risk variable do not affect output and inflation a priori. The coefficient of the inflation target process is $\mathcal{B}(0.9, 0.1)$ distributed. Employing the Beta distribution guarantees that the process of the inflation target is stationary while avoiding that the central bank's inflation target jumps erratically. Finally, the persistence parameter of the common non-fundamental risk factor ρ_C and the persistence parameters of the change in the debt-to-GDP ratios ρ_d^i, $\forall i \in \{fr, es, it\}$, are also assumed to be $\mathcal{B}(0.9, 0.1)$ distributed.

The second part of table (4.1) presents the prior distributions of the volatility parameters associated with the structural shocks, the yield errors, and the prior distributions of the co-movement parameters. The prior distributions of the volatility parameters of the structural shocks and the yield errors are given by $\mathcal{IG}(0.01, 0.2)$ and $\mathcal{IG}(0.0001, 0.001)$, respectively, corresponding to a mean of 1 percentage point of the structural shocks and a mean of 0.01 percentage points of the yield errors.

This specification captures the beliefs that yield errors should be rather small. I employ the Inverse Gamma distribution to prevent the volatility parameters from being negative or equal to 0. Note that the table (4.1) displays a reparameterized version of the volatility parameters of the yield errors. The reparameterization is performed since the Inverse-Gamma distribution is not very flexible in dealing with very small numbers, as discussed by Chib and Ergashev (2009). Therefore, the transformations $\sigma_j^* \equiv s * \sigma_j$, $\forall j \in \{12, 24, 36, 48, 60\}$, and $\sigma_k^{i*} \equiv s * \sigma_k^i$, $\forall k \in \{12, 60\}$, $\forall i \in \{fr, es, it\}$, are performed, where s is given by $s = 1000$. The prior distributions for the co-movement parameters are $\mathcal{N}(0, 2)$ distributed. For the elements in the vectors ψ^{es}, ψ^{fr}, and ψ^{it}, I use relatively uninformed priors $\mathcal{N}(0, 2)$. Finally, for the prior distributions of the parameters in the matrix of the prices of risk Λ_1, I follow Dewachter and Iania (2011) and Dewachter et al. (2014). The last part of table (4.1) presents the priors for the prices of risk. Specifically, for the parameters in Λ_1, I use a loose, zero mean prior $\mathcal{N}(0, 25)$.

The overall choice of the priors satisfies the stationarity condition of the state equation, $|eig(P)| < 1$, and the stability condition of the no-arbitrage recursions, $|eig(P - \Sigma\lambda_1)| < 1$.

4.5 Results

This section presents the results of the estimation. Table (4.2) - (4.4) list the estimated parameters. The tables report the posterior modes, the posterior means, and the 90% highest posterior density (HPD) interval of the estimated parameters. While the posterior mode is obtained by maximizing the (log-) posterior distribution, the latter results are obtained by using the Adaptive Metropolis algorithm outlined in Section (4.4.3). In this model, a part of the spreads is explained by a common

Table 4.2: Results: Posterior Distributions (Part I)

Param.	Prior Mean	Post. Mode	Post. Mean	90% HPD Interval		Prior
ρ_r	0.800	0.7910	0.7977	0.7682	0.8296	$\mathcal{B}$
ρ_π	1.500	1.0845	1.1030	0.7999	1.3907	$\mathcal{G}$
ρ_y	0.500	0.1682	0.1791	0.1357	0.2195	$\mathcal{G}$
ρ_v	0.000	-0.1609	-0.1905	-0.2393	-0.1390	$\mathcal{N}$
ρ_{vv}	0.900	0.9885	0.9804	0.9650	0.9988	$\mathcal{B}$
ρ_{π^*}	0.900	0.9800	0.9800	0.9770	0.9829	$\mathcal{B}$
ρ_C	0.900	0.9990	0.9987	0.9977	0.9997	$\mathcal{B}$
ρ_d^{fr}	0.900	0.9937	0.9884	0.9785	0.9998	$\mathcal{B}$
ρ_d^{es}	0.900	0.9934	0.9898	0.9813	0.9999	$\mathcal{B}$
ρ_d^{it}	0.900	0.9962	0.9917	0.9835	0.9999	$\mathcal{B}$
$\rho_{\pi v}$	0.000	0.0092	0.0111	0.0062	0.0157	$\mathcal{N}$
ρ_{yv}	0.000	-0.0183	-0.0177	-0.0339	0.0012	$\mathcal{N}$
$\rho_{\pi r}$	-0.200	-0.0211	-0.0130	-0.0348	0.0076	$\mathcal{N}$
$\rho_{\pi\pi}^1$	0.900	0.8590	0.8439	0.7589	0.9301	$\mathcal{N}$
$\rho_{\pi\pi}^2$	0.000	-0.0358	-0.0339	-0.1127	0.0571	$\mathcal{N}$
$\rho_{\pi\pi}^3$	0.000	0.0163	0.0231	-0.0183	0.0660	$\mathcal{N}$
$\rho_{\pi y}^1$	0.000	0.0088	0.0155	-0.0149	0.0471	$\mathcal{N}$
$\rho_{\pi y}^2$	0.000	-0.0137	-0.0175	-0.0489	0.0139	$\mathcal{N}$
$\rho_{\pi y}^3$	0.000	0.0162	0.0114	-0.0088	0.0289	$\mathcal{N}$
ρ_{yr}	-0.200	-0.4444	-0.4500	-0.5582	-0.3508	$\mathcal{N}$
$\rho_{y\pi}^1$	0.000	-0.1335	-0.1628	-0.3327	-0.0064	$\mathcal{N}$
$\rho_{y\pi}^2$	0.000	-0.0305	-0.0454	-0.1593	0.0603	$\mathcal{N}$
$\rho_{y\pi}^3$	0.000	-0.0000	-0.0059	-0.0861	0.0664	$\mathcal{N}$
ρ_{yy}^1	0.900	1.0161	0.9935	0.9155	1.0759	$\mathcal{N}$
ρ_{yy}^2	0.000	0.1073	0.1087	0.0219	0.1907	$\mathcal{N}$
ρ_{yy}^3	0.000	-0.0486	-0.0262	-0.1050	0.0421	$\mathcal{N}$

Summary of the posterior distributions of the parameters. Type of the distribution is either $\mathcal{N}$, $\mathcal{B}$, $\mathcal{G}$, or $\mathcal{IG}$ where $\mathcal{N}$ denotes the Normal distribution, $\mathcal{B}$ the Beta distribution, $\mathcal{G}$ the Gamma distribution, and $\mathcal{IG}$ the Inverse-Gamma distribution.

Table 4.3: Results: Posterior Distributions (Part II)

Param.	Prior Mean	Post. Mode	Post. Mean	90% HPD Interval		Prior
ψ_d^{fr}	0.000	0.0039	0.0039	0.0023	0.0055	$\mathcal{N}$
ψ_d^{es}	0.000	0.0086	0.0084	0.0055	0.0117	$\mathcal{N}$
ψ_d^{it}	0.000	0.0272	0.0263	0.0137	0.0377	$\mathcal{N}$
ψ_C^{fr}	0.000	0.1203	0.1122	0.0835	0.1400	$\mathcal{N}$
ψ_C^{es}	0.000	0.6130	0.5737	0.4383	0.7154	$\mathcal{N}$
ψ_C^{it}	0.000	0.7608	0.7107	0.5487	0.8828	$\mathcal{N}$
ψ_v^{fr}	0.000	0.0349	0.0339	0.0232	0.0446	$\mathcal{N}$
ψ_v^{es}	0.000	0.2077	0.2071	0.1671	0.2410	$\mathcal{N}$
ψ_v^{it}	0.000	0.2642	0.2655	0.2220	0.3079	$\mathcal{N}$
ψ_r^{fr}	0.000	-0.1376	-0.1341	-0.1874	-0.0780	$\mathcal{N}$
ψ_r^{es}	0.000	-0.6495	-0.6328	-0.8383	-0.4280	$\mathcal{N}$
ψ_r^{it}	0.000	-0.7951	-0.7678	-1.0507	-0.5098	$\mathcal{N}$
$\psi_{g^y}^{fr}$	0.000	0.0759	0.0743	0.0504	0.0971	$\mathcal{N}$
$\psi_{g^y}^{es}$	0.000	0.3909	0.3838	0.2793	0.4802	$\mathcal{N}$
$\psi_{g^y}^{it}$	0.000	0.4968	0.4850	0.3416	0.6126	$\mathcal{N}$
ψ_π^{fr}	0.000	-0.2441	-0.2479	-0.3574	-0.1403	$\mathcal{N}$
ψ_π^{es}	0.000	-1.0391	-1.0664	-1.4487	-0.6555	$\mathcal{N}$
ψ_π^{it}	0.000	-1.7172	-1.7614	-2.2622	-1.2197	$\mathcal{N}$
$\psi_{\pi^*}^{fr}$	0.000	0.1236	0.1184	0.0730	0.1587	$\mathcal{N}$
$\psi_{\pi^*}^{es}$	0.000	0.5553	0.5314	0.3260	0.7453	$\mathcal{N}$
$\psi_{\pi^*}^{it}$	0.000	0.6434	0.6089	0.3384	0.8834	$\mathcal{N}$
Λ^r	0.000	0.3195	0.6509	-0.9197	2.0600	$\mathcal{N}$
Λ^π	0.000	-0.4610	-1.1660	-2.3252	-0.0003	$\mathcal{N}$
Λ^y	0.000	-0.5005	0.2136	-2.4125	2.7543	$\mathcal{N}$
Λ^{π^*}	0.000	0.3421	-0.3222	-2.3236	2.0795	$\mathcal{N}$
Λ^C	0.000	-1.7516	-2.0393	-3.0835	-1.0601	$\mathcal{N}$
$\Lambda^{d,fr}$	0.000	-14.6679	-15.5174	-23.0080	-6.3857	$\mathcal{N}$
$\Lambda^{d,es}$	0.000	-0.6462	-0.8842	-5.3875	3.2281	$\mathcal{N}$
$\Lambda^{d,it}$	0.000	8.8063	10.1227	4.7343	15.0851	$\mathcal{N}$

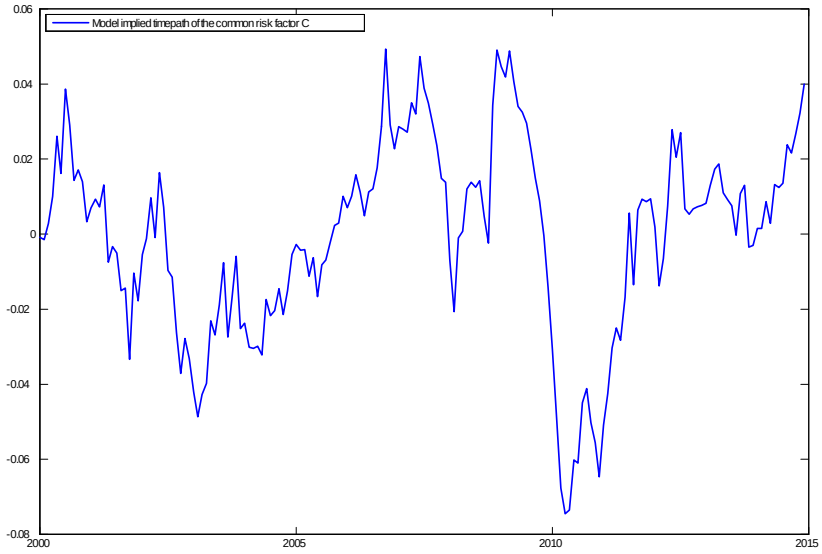
Summary of the posterior distributions of the parameters. Type of the distribution is either $\mathcal{N}$, $\mathcal{B}$, $\mathcal{G}$, or $\mathcal{IG}$ where $\mathcal{N}$ denotes the Normal distribution, $\mathcal{B}$ the Beta distribution, $\mathcal{G}$ the Gamma distribution, and $\mathcal{IG}$ the Inverse-Gamma distribution.

Table 4.4: Results: Posterior Distributions (Part III)

Param.	Prior Mean	Post. Mode	Post. Mean	90% HPD Interval		Prior
σ_r	0.010	0.0026	0.0027	0.0024	0.0029	$\mathcal{IG}$
σ_π	0.010	0.0021	0.0021	0.0019	0.0023	$\mathcal{IG}$
σ_y	0.010	0.0068	0.0071	0.0062	0.0081	$\mathcal{IG}$
σ_{π^*}	0.010	0.0022	0.0022	0.0019	0.0024	$\mathcal{IG}$
σ_d^{fr}	0.010	0.0109	0.0110	0.0101	0.0121	$\mathcal{IG}$
σ_d^{es}	0.010	0.0197	0.0198	0.0180	0.0214	$\mathcal{IG}$
σ_d^{it}	0.010	0.0058	0.0058	0.0053	0.0063	$\mathcal{IG}$
$\sigma_{\pi\pi^*}$	0.100	-0.3524	-0.3424	-0.5105	-0.1750	$\mathcal{N}$
$\sigma_{\pi y}$	0.000	1.7489	1.6673	1.1113	2.1956	$\mathcal{N}$
$\sigma_{y\pi^*}$	0.000	-1.6153	-1.5250	-2.0635	-1.011	$\mathcal{N}$
σ_{vr}	0.000	1.7024	1.7574	0.8551	2.6508	$\mathcal{N}$
$\sigma_{v\pi}$	0.000	4.3751	4.6409	3.1486	6.0133	$\mathcal{N}$
σ_{vy}	0.000	-1.2394	-1.0403	-1.5189	-0.5720	$\mathcal{N}$
$\sigma_{v\pi^*}$	0.000	-2.6718	-2.5953	-3.7861	-1.4208	$\mathcal{N}$
σ_{vC}	0.000	-2.1858	-1.9179	-2.5135	-1.3061	$\mathcal{N}$
σ_{vd}^{fr}	0.000	0.0664	0.1100	-0.3805	0.6736	$\mathcal{N}$
σ_{vd}^{es}	0.000	-0.4222	-0.3974	-0.7357	0.0341	$\mathcal{N}$
σ_{vd}^{it}	0.000	-0.0043	0.1224	-1.3368	1.5694	$\mathcal{N}$
σ_{12}^*	0.100	0.0774	0.0787	0.0711	0.0862	$\mathcal{IG}$
σ_{24}^*	0.100	0.0192	0.0199	0.0156	0.0240	$\mathcal{IG}$
σ_{36}^*	0.100	0.0345	0.0348	0.0313	0.0385	$\mathcal{IG}$
σ_{48}^*	0.100	0.0169	0.0175	0.0142	0.0206	$\mathcal{IG}$
σ_{60}^*	0.100	0.0924	0.0935	0.0846	0.1021	$\mathcal{IG}$
σ_{12}^{fr*}	0.100	1.0827	1.1064	0.9958	1.2065	$\mathcal{IG}$
σ_{60}^{fr*}	0.100	0.8727	0.8975	0.8005	0.9809	$\mathcal{IG}$
σ_{12}^{es*}	0.100	2.6639	2.7297	2.4696	2.9692	$\mathcal{IG}$
σ_{60}^{es*}	0.100	3.3555	3.3956	3.0914	3.6843	$\mathcal{IG}$
σ_{12}^{it*}	0.100	2.8495	2.9152	2.6749	3.1695	$\mathcal{IG}$
σ_{60}^{it*}	0.100	0.0463	0.0809	0.0247	0.1445	$\mathcal{IG}$

Summary of the posterior distributions of the parameters. Type of the distribution is either $\mathcal{N}$, $\mathcal{B}$, $\mathcal{G}$, or $\mathcal{IG}$ where $\mathcal{N}$ denotes the Normal distribution, $\mathcal{B}$ the Beta distribution, $\mathcal{G}$ the Gamma distribution, and $\mathcal{IG}$ the Inverse-Gamma distribution.

Figure 4.4: Estimated time path of the common risk factor



non-fundamental risk factor. Figure (4.4) displays the time path of the common non-fundamental risk factor. The common non-fundamental risk factor has been above its steady state level during the financial crisis, on the onset of the European debt crisis, and from 2012 onwards.

In the literature of macro-finance term structure models, the standard deviations of the yield errors are used to evaluate the fit of the model. The bottom part of table (4.4) presents the standard deviations of these errors. With standard deviations of these errors around 9 and 11 basis points for French bond yields, around 26 and 33 basis points for Spanish bonds yields and around 4 and 29 basis points for Italian bond yields, the fit of the yield curves is reasonably good (see e.g. Borgy et al., 2012, or Hördahl and Tristani, 2013). The model's fit of the German term structure is remarkably good.

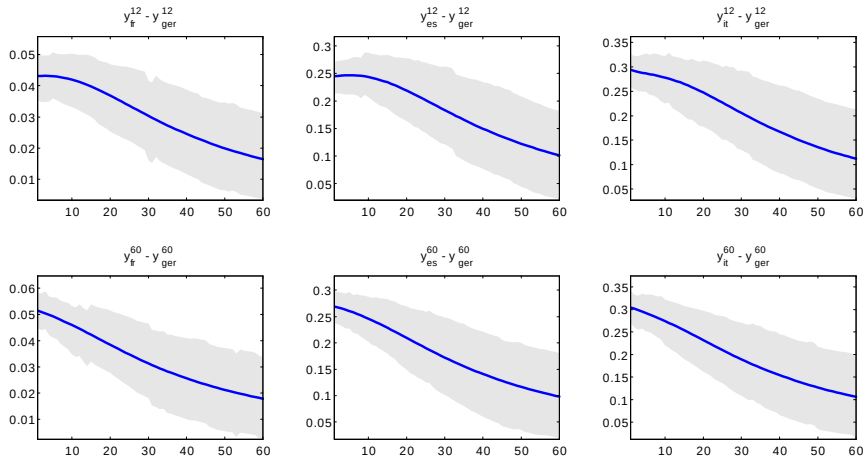
The estimates of the interest rate rule parameters are given in the first four rows in table (4.2). Notably, all four parameter estimates are significantly different from zero, including the ECB's response parameter to movements in the risk aversion

variable ρ_v. The posterior mean of ρ_v is significantly different from zero and negative, $\rho_v = -0.1905$. As demonstrated in Section (4.3.3), all variation in the one-period return premium is attributable to the risk aversion variable. This implies that the ECB lowered the interest rate in response to a rise in term premia. In line with the practitioner view (see Rudebusch et al., 2007), this indicates that the central bank counteracted changes in term premia. Using a macro-finance model and an index of Euro area government bonds, Herrmann (2015) also finds a negative coefficient.

The estimated values of the other three parameters of the interest rate rule are in line with those from studies using a more standard interest rate rules specification for the Euro Area (e.g. Andrés et al., 2006, or Smets and Wouters, 2003). The estimate of the interest rate inertia $\rho_r = 0.7977$ reflects a high degree of interest rate smoothing. The estimate of the coefficient measuring central bank's response to changes in the output gap is $\rho_y = 0.1791$. The estimated coefficient of the central bank's response to a change in inflation is larger than one, $\rho_\pi = 1.1030$, satisfying the Taylor principle.

In the following, rather than interpreting each of the remaining estimates separately, I describe the results of the parameter estimation jointly by computing impulse response functions (IRFs) of the yield spreads to selected shocks, by decomposing the forecast error variance of the yield spreads, and by performing a historical shock decomposition of yield spreads. These methods help to examine the dynamic of yield spreads, to describe the propagation of different shocks, and to reveal the relevance of different shocks for variation in the yield spreads. All yield spreads are calculated with respect to Germany.

Figure 4.5: IRFs to a one-standard-deviation risk aversion shock



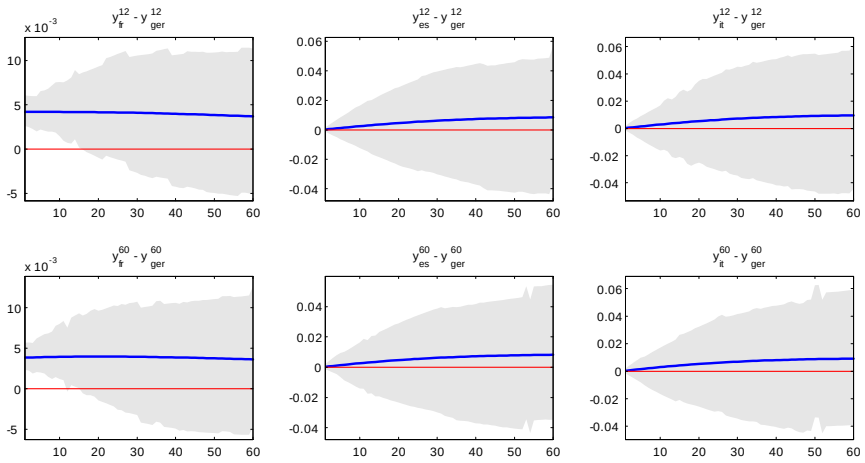
Notes: All spreads are calculated with respect to the yield of German government bonds of the same maturity. The yield spreads are shown in annualized percentage points. The grey shaded areas cover the 90 percent HPD interval.

4.5.1 Impulse Response Functions

Each of the following figures shows the impulse responses of the yield spreads to a particular shock. Each shock is of a size of one-standard-deviation. The first row of each figure gives the graphs of the impulse responses of the one-year spreads of France ($y^{12}_{t,fr} - y^{12}_{t,ger}$), Spain ($y^{12}_{t,es} - y^{12}_{t,ger}$), and Italy ($y^{12}_{t,it} - y^{12}_{t,ger}$). The second row contains the graphs of the impulse responses of the five-year yield spreads of France ($y^{60}_{t,fr} - y^{60}_{t,ger}$), Spain ($y^{60}_{t,es} - y^{60}_{t,ger}$), and Italy ($y^{60}_{t,it} - y^{60}_{t,ger}$). The gray shaded areas cover the 90 percentage HPD interval. The IRF (displayed by the blue line) is computed as the mean impulse response. The yield spreads are shown in annualized percentage points. One period corresponds to one month.

The impulse responses to a risk aversion shock ε_{vt} are presented in figure (4.5). The yield spreads of bonds of both maturities of all countries rise significantly on impact. Over a horizon of five years, the impulse responses of the yield spreads converge slowly back to their steady state. The magnitude of the responses to the

Figure 4.6: IRFs to a one-standard-deviation shock to d_t^{fr}

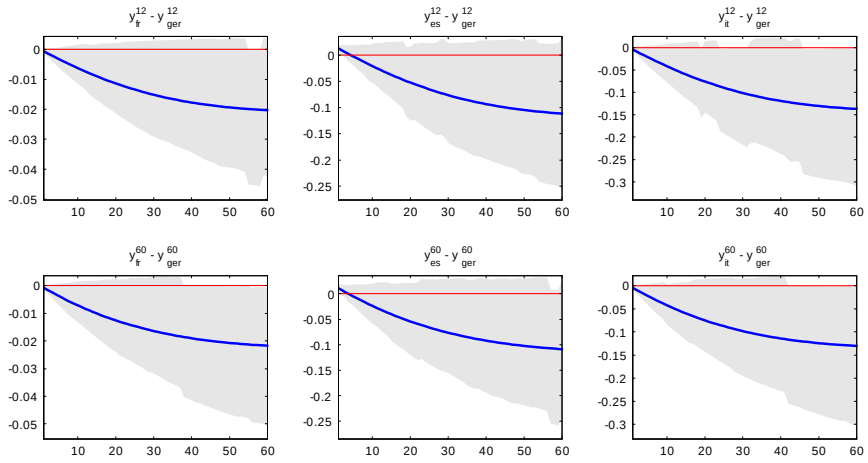


Notes: All spreads are calculated with respect to the yield of German government bonds of the same maturity. The yield spreads are shown in annualized percentage points. The grey shaded areas cover the 90 percent HPD interval.

risk aversion shock is significantly stronger for the spreads of Italy (around 30 basis points for both maturities on impact) and Spain (around 25 basis points for both maturities on impact), than the magnitude of the response of French yield spreads (around four and five basis points, on impact, for the one-year and five-year spread, respectively).

Figure (4.6) displays the impulse responses to a rise in the French debt-to-GDP growth rate. The figure highlights that only the one-year yield spread and the five-year yield spread of France are affected by an increase the debt-to-GDP growth rate of France. The response of all other spreads is not significantly different from zero. The same applies for a shock to the change in the debt-to-GDP ratio of Italy and Spain. Figure (4.7) and (4.8) shows that debt-to-GDP growth rate shocks do only affect the yield spreads of the respective country vis-à-vis Germany. All other spreads do not respond significantly. Thus, the results provide no evidence for effects running from the country-specific fiscal variables to the other countries' yield spreads.

Figure 4.7: IRFs to a one-standard-deviation shock to d_t^{es}



Notes: All spreads are calculated with respect to the yield of German government bonds of the same maturity. The yield spreads are shown in annualized percentage points. The grey shaded areas cover the 90 percent HPD interval.

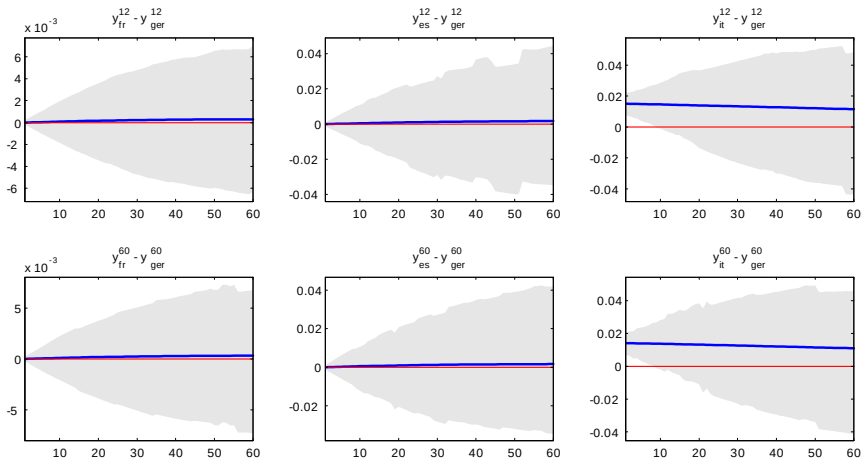
Finally, figure (4.9) shows the impulse responses of the yield spreads to the common non-fundamental risk factor shock. The yield spreads of all countries rise significantly and persistently. The increase in the yield spreads is of stronger magnitude for Spain and for Italy than for France.

4.5.2 Variance Decomposition

To identify the main drivers of movements in bond yield spreads and to assess the relative importance of different shocks for the variability of the yield spreads, I perform a forecast error variance decomposition (FEVD). The FEVD quantifies the contribution of each of the structural shocks to the forecast error variance of the different yield spreads. Formally, the fraction of the forecast error variance of variable i due to shock j for horizon h, denoted by $\phi_{i,,j}(h)$, is defined by (see Lütkepohl, 2005, p. 64)

$$\phi_{i,,j}(h) = \frac{\omega_{i,,j}(h)}{\Omega_i(h)},$$

Figure 4.8: IRFs to a one-standard-deviation shock to d_t^{it}



Notes: All yield spreads are calculated with respect to the yield of German government bonds of the same maturity. The yield spreads are shown in annualized percentage points. The grey shaded areas cover the 90 percent HPD interval.

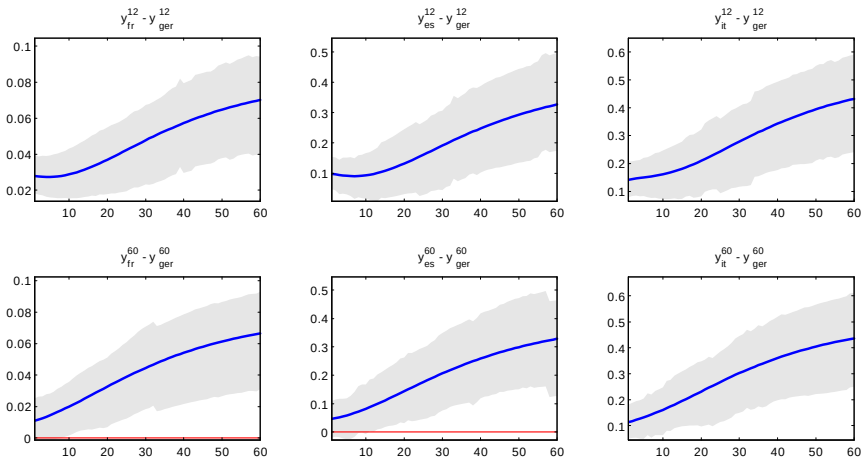
where $\omega_{i,,j}(h)$ is the forecast error variance of variable i due to shock j at horizon h and $\Omega_i(h)$ is the total error forecast variance of variable i at horizon h.

For the sake of clarity, I divide the contribution of the different shocks into two groups: *economic and country-specific factors* and the *common non-fundamental risk factor*. *Economic and country-specific factors* contain the common economic variables, including the risk aversion variable, and country-specific variables.[7] The *common non-fundamental risk factor*, given by C_t, captures common dynamics in yield spreads that are unrelated to the other common economic factors. The FEVD is performed for the one- and five-year yield spreads of France, Italy, and Spain for different horizons. Table (4.5) displays the FEVD of the yield spreads.

Both, economic factors and the common non-fundamental risk factor are important drivers of Euro area sovereign yield spreads. Within the group of economic

[7]For convenience, also the country-specific yield errors are subsumed in this group. They only play a role for short horizons and do not contribute substantially to the forecast error variance of yield spreads for longer forecast horizons.

Figure 4.9: IRFs to a one-standard-deviation common non-fundamental risk factor shock



Notes: All yield spreads are calculated with respect to the yield of German government bonds of the same maturity. The yield spreads are shown in annualized percentage points. The grey shaded areas cover the 90 percent HPD interval.

factors, the risk aversion variable takes a pronounced role. For intermediate forecast horizons (from one year up to three years), it accounts for between 41 and 51 percent of the forecast error variance in the one-year yield spreads of France and for between 44 and 62 percent of the forecast error variance in the five-year yield spreads of France. Risk aversion shocks are also important for the yield spreads of Spain and Italy. They account for between 57 and 72 percent and for between 50 and 68 percent of the variability of the Spanish one-year yield spread and the Spanish five-year yield, respectively, at an intermediate forecast horizon. For the Italian yield spreads, the risk aversion variable accounts for between 50 and 70 percent and for between 45 and 73 percent of the variability in the one-year yield spread and the five-year yield spread, respectively, both at an intermediate forecast horizon. Notably, risk aversion shocks are more pronounced for shorter forecast horizons, while their importance for yield spreads of all maturities decreases with the forecast horizon.

Also common non-fundamental risk factor shocks contribute substantially to the

Table 4.5: Variance Decomposition

	1-year yield spread			**5-year yield spread**		
France						
h	Econ	C	RA	Econ	C	RA
3 months	91.39	08.61	28.19	99.02	00.98	45.16
1 year	84.14	15.86	50.47	96.72	03.28	61.54
3 years	69.30	30.70	41.98	83.55	16.45	44.93
5 years	54.45	45.55	26.85	68.20	31.80	30.45
10 years	33.98	66.02	12.18	43.96	56.04	14.80
Spain						
h	Econ	C	RA	Econ	C	RA
3 months	94.56	5.44	56.12	99.29	00.61	55.63
1 year	93.32	5.68	71.37	98.31	01.69	67.84
3 years	86.21	13.79	57.47	87.97	12.03	50.43
5 years	71.00	29.00	37.70	62.94	27.06	34.51
10 years	45.09	54.91	17.21	37.11	52.89	16.77
Italy						
h	Econ	C	RA	Econ	C	RA
3 months	90.09	09.91	59.46	92.58	07.42	79.58
1 year	86.63	13.27	69.15	89.38	10.62	72.43
3 years	74.70	25.30	50.88	74.50	25.50	45.31
5 years	59.63	40.37	31.70	59.39	40.61	28.82
10 years	37.48	62.52	14.03	37.60	62.40	13.34

Notes: *Econ* denotes the contribution of the economic shocks (including risk aversion shocks) and country-specific shocks to the FEVD. *C* denotes the contribution of common non-fundamental risk factor shocks to the FEVD. *RA* displays the contribution of risk aversion shocks to the FEVD separated from the other economic factors.

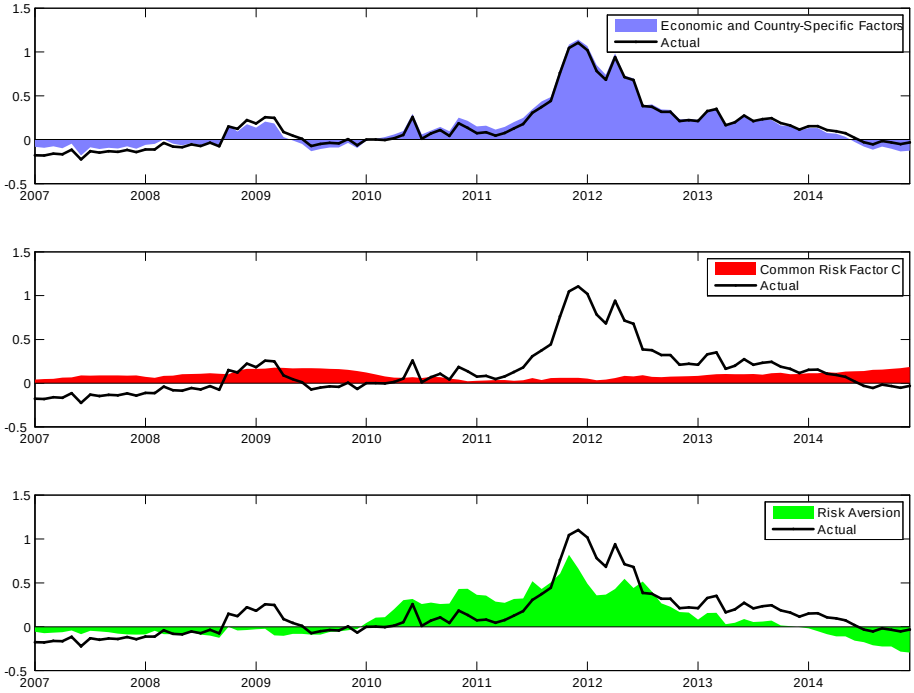
variability of sovereign yield spreads. The effects of variation in the common non-fundamental risk factor are more pronounced for longer forecast horizons. In fact, for longer forecast horizons, common non-fundamental risk factor shocks are the main source of variations in the yield spreads, accounting for between 54 and 66 percent of the variations in the one-year yield spreads and for between 52 and 63 of the variations in the five-year yield spreads at a 10-year forecast horizon. But also for intermediate horizons, shocks to the common non-fundamental risk factor play a non-negligible role. For the one-year yield spread of French government bonds, shocks to the common non-fundamental risk factor account for between 15 and 31

percent of the forecast error variance, and for the five-year yield spread, they account for between 3 and 17 percent of the forecast error variance, both for an intermediate forecast horizon. The same holds true for Spanish and Italian yield spreads. Between 5 and 14 percent of all variation in the one-year yield spread of Spanish government bonds and between 13 and 26 percent of all variations in the one-year yield spread of Italian government bonds are attributable to shocks to the common non-fundamental risk factor. The common non-fundamental risk factor shocks also account for sizeable movements in the five-year yield spreads of both countries. It accounts for between 1 and 12 percent in the Spanish five-year yield spreads and for between 10 and 26 percent in the Italian five-year yield spreads at intermediate horizons.

4.5.3 Historical Shock Decomposition

The historical shock decomposition of the yield spreads is performed to identify the contribution of shocks of each group of factors to the evolution of bond yield spreads. Figure (4.10) - (4.12) presents the historical decomposition of the five-year yield spreads of sovereign bonds of France, Italy, and Spain with respect to the German bond yield of the same maturity. Each figure contains three panels. Each panel shows the historical values of the respective yield spread and the contribution of shocks a factor or a group of factors to the yield spread. The first panel in each figure displays the contribution of shocks to country-specific factors and common economic shocks (including shocks of the risk aversion variable) to the evolution of the respective yield spread. The second panel in each figure depicts the contribution of common non-fundamental risk factor shocks to the evolution of the respective yield spread. The last panel in each figure displays the contribution of risk aversion shocks separated from the contribution of the other economic factors to the respective yield spread. This helps to visualize the importance of risk aversion shocks for the yield

Figure 4.10: Historical Shock Decomposition of the five-year yield spread of France

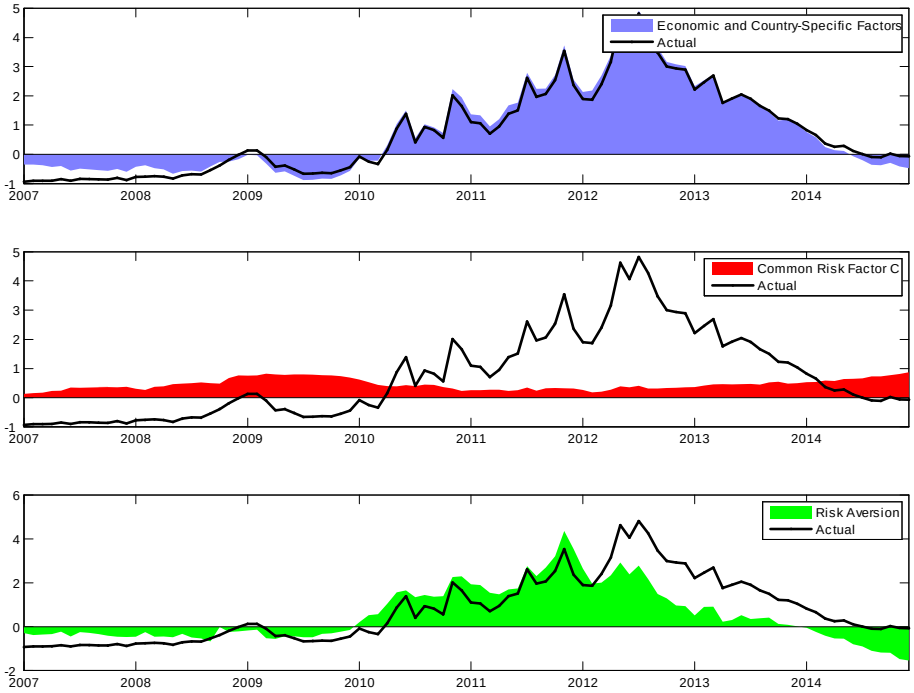


Notes: The spread is shown in annualized percentage points. The figure presents the historical decomposition of the five-year French yield spread with respect to Germany. Economic and Country-Specific Factors contain country-specific factors and Euro area wide economic fundamentals (including risk aversion shocks); Risk Aversion shocks are depicted separately in the last row. The initial values are not displayed.

spreads and to compare the contribution of risk aversion shocks to the contribution of common non-fundamental risk factor shocks. The contribution of the initial values is not plotted. Their contribution to the yield spreads is highly persistent, reflecting the persistence of some of the model's shocks.

In all of the three yield spreads, economic shocks have played the most important role for their evolution. Within the group of economic factors, shocks to the risk aversion variable are the most important drivers. For the Spanish and Italian five-year yield spreads, shocks to the risk aversion variable explain most of the spread between 2010 and the beginning of 2012. From mid-2012 onwards until the end of

Figure 4.11: Historical Shock Decomposition of the five-year yield spread of Spain

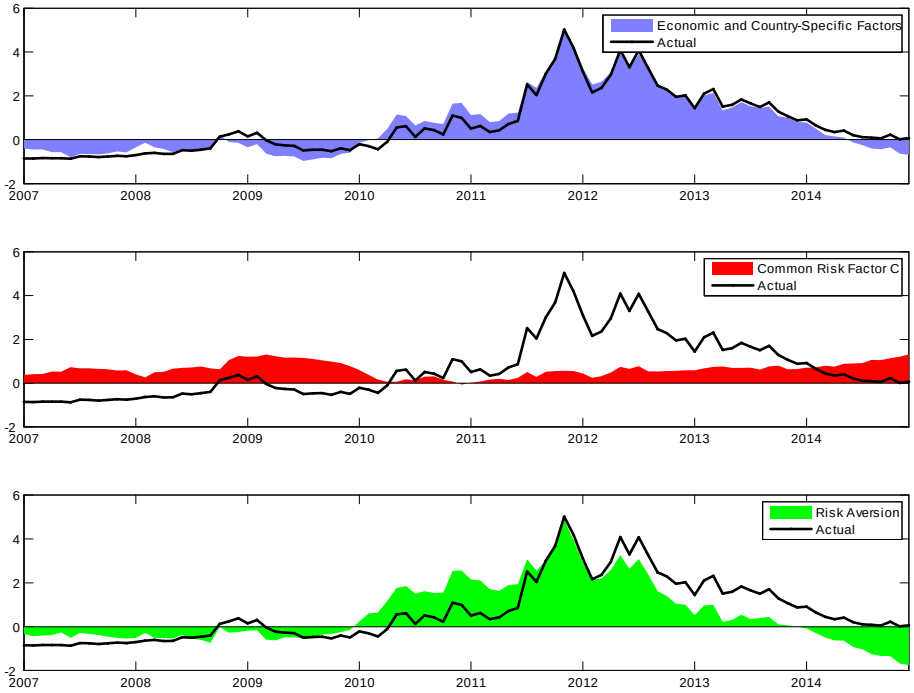


Notes: The spread is shown in annualized percentage points. The figure presents the historical decomposition of the five-year French yield spread with respect to Germany. Economic and Country-Specific Factors contain country-specific factors and Euro area wide economic fundamentals (including risk aversion shocks); Risk Aversion shocks are depicted separately in the last row. The initial values are not displayed.

2014, the importance of shocks to the risk aversion variable for the evolution of yield spreads decreases slowly. Shocks to the risk aversion variable also explain a large part of the 5-year French yield spread, although their contribution to the spread is not as pronounced as to the Spanish and the Italian yield spreads. Within the group of economic factors, shocks to the short-term interest rate had a negative contribution to the yield spreads.

Shocks to the common non-fundamental risk factor also had a substantial impact on yield spreads. In particular, during the financial crisis and the European debt crisis until the end of 2010, common non-fundamental risk factor shocks had a positive

Figure 4.12: Historical Shock Decomposition of the five-year yield spread of Italy



Notes: The spread is shown in annualized percentage points. The figure presents the historical decomposition of the five-year French yield spread with respect to Germany. Economic and Country-Specific Factors contain country-specific factors and Euro area wide economic fundamentals (including risk aversion shocks); Risk Aversion shocks are depicted separately in the last row. The initial values are not displayed.

contribution to the yield spreads of all three countries. From 2012 onwards until the end of the sample at the end of 2014, the contribution of common non-fundamental risk factor shocks to the yield spreads increases slowly. The absolute contribution of common non-fundamental risk factor shocks to the yield spreads is larger for the Spanish and the Italian yield spread than for the French yield spreads. For example, in mid-2013, the common non-fundamental risk factor shock explains 40 basis points in the Spanish yield spread and 70 basis points in the Italian yield spread, highlighting that spreads of Euro area countries cannot be fully justified by economic and country-specific factors only. This result is in line with the findings of previous

studies (see De Santis, 2015, Di Cesare et al., 2012, and Dewachter et al., 2015). However, in contrast to the finding of Dewachter et al. (2015), the contribution of common non-fundamental risk factor shocks to the surge in yield spreads is comparably smaller. Instead, economic shocks are able to explain most of the variation in yield spreads. Moreover, the contribution of the common non-fundamental risk factor to the yield spreads is much smoother over time. Specifically, while Dewachter et al. (2015) find that shocks to common non-fundamental risk factors account for a large fraction in the dramatic surge in yield spreads at the end of 2011, my finding shows that the increase in yield spreads during 2011 can largely be explained by an increase in risk aversion.

Thus, though the common non-fundamental risk factor played a non-negligible role for yield spreads, accounting for a substantial increase in yield spreads during the financial crisis and the European debt crisis, the most important drivers of yield spreads have been economic shocks. In particular, shocks to the risk aversion variable had a huge impact on yield spreads, revealing the importance of measuring risk aversion in Euro area bond markets adequately.

4.6 Conclusion

In this work, I evaluate the effects of economic fundamentals and a common non-fundamental risk factor on Euro area yield spreads. Specifically, using a multi-country macro-finance model of the term structure, where changes in risk-aversion are captured by a single variable, I am interested in disentangling the effects of changes in risk aversion and a common non-fundamental risk factor in Euro area yield spreads. In contrast to the existing literature on Euro area yield spreads, the risk aversion measure used in this work is directly derived from the pricing kernel

of a European investor. Particularly, by restrictions on the prices of risk, one single variable is identified to account for all time-variation in the prices of risk. This risk aversion variable responds contemporaneously to distortions of the economy but also exhibits an autonomous dynamic. The common non-fundamental risk factor is identified as a common factor in Euro area yield spreads that is not related to Euro area economic fundamentals, i.e. the part of the spread that cannot be accounted for by common Euro area economic fundamentals. This common non-fundamental risk factor potentially captures contagion effects or redenomination risk. Furthermore, exclusion restrictions on the contemporaneous relationship of state variables, similar to those from more conventional VARs, are entailed to identify structural shocks.

In line with the results of De Santis (2015), Dewachter et al. (2015), or Di Cesare et al. (2012), a non-negligible part of the Euro area yield spreads cannot be explained by economic fundamentals but is accounted for by the common non-fundamental risk factor. However, although the contribution of the common non-fundamental risk factor has been important for yield spreads, most of the surge in yield spreads during the European debt crisis is explained by economic fundamentals. Within in the group of economic factors, shocks to the risk aversion variable are the most important drivers of yield spreads. This finding underlines the importance of measuring risk aversion in Euro area bond markets adequately.

I like to emphasize two aspects of my findings. First, from the beginning of 2010 onwards until the end of 2011, shocks to the risk aversion variable are able to explain the dramatic surge in yield spreads very well. In fact, for the Spanish and Italian five-year yield spreads shocks to the risk aversion variable explain large parts of the spreads during this time. Shocks to the risk aversion variable also explain a large part of the French yield spreads, although their contribution to the spreads is not as pronounced as for the Spanish and the Italian yield spreads. From 2012 onwards

until 2014 the importance of shocks to the risk aversion variable for the evolution yield spreads decreases. Second, common non-fundamental risk factor shocks had, in particular, during the financial crisis until the beginning of the European debt crisis in 2009, a strong positive contribution to the yield spreads of France, Italy, and Spain. Moreover, from 2012 onwards until the end of the sample in December 2014, the contribution of the common non-fundamental risk factor shocks to the evolution of yield spreads increases.

4.A Appendix

4.A.1 Parameter Matrices

State Equation

The matrix P_0 is given by

$$P_0 = \begin{bmatrix} 1 & \bar{\rho}_y & 0 & 0 & \bar{\rho}_\pi & 0 & 0 & 0 & 0 & \bar{\rho}_v & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

where $\bar{\rho}_y = -(1-\rho_r)\rho_y$, $\bar{\rho}_\pi = -(1-\rho_r)\rho_\pi$, and $\bar{\rho}_v = -(1-\rho_r)\rho_v$. The matrix P_1 is given by

$$P_1 = \left[\begin{array}{cccccccccccc}
\rho_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\rho_{yr} & \rho_{yy}^1 & \rho_{yy}^2 & \rho_{yy}^3 & \rho_{y\pi}^1 & \rho_{y\pi}^2 & \rho_{y\pi}^3 & 0 & 0 & \rho_{yv} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\rho_{\pi r} & \rho_{\pi y}^1 & \rho_{\pi y}^2 & \rho_{\pi y}^3 & \rho_{\pi\pi}^1 & \rho_{\pi\pi}^2 & \rho_{\pi\pi}^3 & 0 & 0 & \rho_{\pi v} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\pi^*} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_C & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{vv} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_d^{fr} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_d^{es} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_d^{it}
\end{array}\right],$$

and the matrix Σ_0 is given by

$$\Sigma_0 = \left[\begin{array}{cc} \Sigma_0^1 & \Sigma_0^2 \end{array}\right],$$

where the sub-matrices Σ_0^1 and Σ_0^2 are given by

$$\Sigma_0^1 = \begin{bmatrix} \sigma_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_y & 0 & 0 & \sigma_{y\pi}\sigma_\pi & 0 & 0 & \sigma_{y\pi^*}\sigma_{\pi^*} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\pi & 0 & 0 & \sigma_{\pi\pi^*}\sigma_{\pi^*} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\pi^*} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{vr}\sigma_r & \sigma_{vy}\sigma_y & 0 & 0 & \sigma_{v\pi}\sigma_\pi & 0 & 0 & \sigma_{v\pi^\star}\sigma_{\pi^\star} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\Sigma_0^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \sigma_C & 0 & 0 & 0 & 0 \\ \sigma_{vC}\sigma_C & \sigma_v & \sigma_{vd}^{fr}\sigma_d^{fr} & \sigma_{vd}^{es}\sigma_d^{es} & \sigma_{vd}^{it}\sigma_d^{it} \\ 0 & 0 & \sigma_d^{fr} & 0 & 0 \\ 0 & 0 & 0 & \sigma_d^{es} & 0 \\ 0 & 0 & 0 & 0 & \sigma_d^{it} \end{bmatrix}.$$

Finally, the vector μ_0 is given by

$$\mu_0 = \begin{bmatrix} (1-\rho_r)\left(g^r - \rho_y g^y\right) \\ -\rho^{\pi r} g^r - \left(\rho^1_{\pi y} + \rho^2_{\pi y} + \rho^3_{\pi y}\right) g^y \\ 0 \\ 0 \\ \left(1 - \left(\rho^1_{yy} + \rho^2_{yy} + \rho^3_{yy}\right)\right) g^y - \rho_{yr} g^r \\ 0 \\ 0 \\ (1-\rho_{\pi^*})\pi^\star \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Measurement Equation

The matrix U is given by

$$U \equiv \begin{bmatrix} B'_{12} \\ B'_{24} \\ B'_{36} \\ B'_{48} \\ B'_{60} \\ B'_{fr,12} \\ B'_{fr,60} \\ B'_{es,12} \\ B'_{es,60} \\ B'_{it,12} \\ B'_{it,60} \\ U^f_{fr} \\ U^f_{es} \\ U^f_{it} \\ U^r \\ U^y \\ U^\pi \end{bmatrix}$$

The elements in the $B-$vectors are given by eq. (4.19) and (4.20). The remaining elements in the $U-$matrix are given by

$$
\begin{aligned}
U^f_{fr} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \\
U^f_{es} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \\
U^f_{it} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \\
U^r &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
U^y &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
U^\pi &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.
\end{aligned}
$$

4.A.2 Pricing of Defaultable Bonds

The derivation of the market value of a defaultable bond follows Borgy et al. (2012) and Monfort and Renne (2011). Consider the time t price of a defaultable zero-coupon bond $\tilde{P}^\tau_{j,t}$ issued by the sovereign of country j maturing in $\tau-$periods that promises to pay a certain amount at maturity. If no default has occurred until time t, the value of this bond is given by the present value of the recovery payment in the case of default between period t and $t+1$ plus the present value of the bond if no default occurred,

$$
\tilde{P}^\tau_{j,t} = E_t\left(m_{t+1}\tilde{P}^{\tau-1}_{j,t+1} \mid D_{j,t+1} = 0\right) + E_t\left(m_{t+1}\tilde{P}^{\tau-1}_{j,t+1} \mid D_{j,t+1} = 1\right) \tag{4.18}
$$

where $D_{j,t}$ is a default indicator variable taking the values 0 in the event of no-default prior to time t and 1 in the event of default at/or prior to time t. Duffie and Singleton (1999) assume that the recovery value of the bond is equal to a fraction ω of what

the bond would have been worth in the event of no-default (the so-called "recovery to market value assumption").

Denote the time t default probability of issuer j that it survives until $t+1$ by $\tilde{s}_{j,t}$[8], then

$$E_t\left(m_{t+1}\tilde{P}^{\tau-1}_{j,t+1} \mid D_{j,t+1}=0\right) = E_t\left(\exp\left(-\tilde{s}_{j,t+1}\right) m_{t+1}\tilde{P}^{\tau-1}_{j,t+1}\right)$$

and

$$E_t\left(m_{t+1}\tilde{P}^{\tau-1}_{j,t+1} \mid D_{j,t+1}=1\right) = E_t\left(\left(1-\exp\left(-\tilde{s}_{j,t+1}\right)\right) m_{t+1}\omega\tilde{P}^{\tau-1}_{j,t+1}\right),$$

and the present value of the bond is given by

$$\begin{aligned}\tilde{P}^{\tau}_{j,t} &= E_t\left(\left(1-\exp\left(-\tilde{s}_{j,t+1}\right)\right) m_{t+1}\omega\tilde{P}^{\tau-1}_{j,t+1} + \exp\left(-\tilde{s}_{j,t+1}\right) m_{t+1}\tilde{P}^{\tau-1}_{j,t+1}\right)\\ &= E_t\left(\left[\left(1-\exp\left(-\tilde{s}_{j,t+1}\right)\right)\omega + \exp\left(-\tilde{s}_{j,t+1}\right)\right] m_{t+1}\tilde{P}^{\tau-1}_{j,t+1}\right).\end{aligned}$$

Finally, define the "recovery-adjusted default intensities" $s_{j,t}$ (see e.g. Monfort and Renne, 2011) by

$$\exp\left(-s_{j,t+1}\right) \equiv \left(1-\exp\left(-\tilde{s}_{j,t+1}\right)\right)\omega + \exp\left(-\tilde{s}_{j,t+1}\right),$$

then the market value of the bond is given by

$$\tilde{P}^{\tau}_{j,t} = E_t\left(\exp\left(-s_{j,t+1}\right) m_{t+1}\tilde{P}^{\tau-1}_{j,t+1}\right).$$

Note, if the recovery rate is equal to zero ($\omega = 0$), then the recovery-adjusted default intensity $s_{j,t}$ would be equal to the default probability $\tilde{s}_{j,t+1}$. However, since the

[8] Thus, the time t survival probability of an issuer j until time $t+1$ is given by $E_t\left(\exp\left(-\tilde{s}_{j,t+1}\right)\right)$.

recovery rate is, in general, larger than zero, $s_{j,t}$ reflects the adjusted default intensity of country j, rather than actual default intensities.

4.A.3 Pricing Matrices

Borgy et al. (2012) depart from the standard formulas for the computation of the matrices $\bar{A}_{i,\tau}$ and $\bar{B}_{i,\tau}$ in eq. (4.14), as provided by Ang and Piazzesi (2003) and suggest an improved algorithm to compute the pricing matrices of the different countries. Instead of computing each of the pricing matrices $\bar{A}_{i,\tau}$ and $\bar{B}_{i,\tau}$ $\forall \tau = 1, ..., 60$, the idea behind their algorithm is to compute only selected nested bond maturities and to concatenate country-specific pricing matrices. As demonstrated by Borgy et al. (2012), this algorithm reduces computation time significantly, in particular for increasing numbers of yield curves and for high frequency data.

Starting from the no-arbitrager condition, pricing of defaultable bonds of a country i under the risk-neutral measure is given by

$$\bar{P}_{i,t}^{\tau+1} = E_t^Q \left(\exp\left(-r_t - s_{t+1}^i\right) \bar{P}_{i,t+1}^{\tau} \right).$$

By iterating, we get

$$\bar{P}_{i,t}^{\tau+1} = E_t^Q \left(\exp\left(-r_t - s_{t+1}^i ... - r_{t+\tau} - s_{t+\tau+1}^i\right) \right).$$

The short-term interest rate r_t and the default intensities s_{t+1}^i are both affine in X_t,

$$r_t = \delta_1 X_t,$$

and

$$s_{t+1}^i = \psi_0 + \psi_1^i X_{t+1}.$$

Moreover, it can be shown (see e.g. Gourieroux et al., 2003) that the pricing factors X_t (under the risk-neutral measure) follow the autoregressive process

$$X_t = \mu^* + P^* X_{t-1} + \Sigma \varepsilon_t^*,$$

where $\varepsilon_t^* \sim N(0, I)$ and

$$\begin{aligned} \mu^* &= \mu - \Sigma\lambda_0, \\ P^* &= (P - \Sigma\Lambda_1). \end{aligned}$$

Thus,

$$\begin{aligned} \bar{P}_{i,t}^{\tau+1} &= E_t^Q \left(\exp \left(\begin{array}{c} -\delta_1 X_t - \left(\psi_0^i + \psi_1^i X_{t+1}\right) - \delta_1 X_{t+1} \\ -\left(\psi_0^i + \psi_1^i X_{t+2}\right) ... - \delta_1 X_{t+\tau} - \left(\psi_0^i + \psi_1^i X_{t+\tau+1}\right) \end{array} \right) \right) \\ &= \exp\left(-\tau\psi_0^i\right) E_t^Q \left(\exp\left(-\delta_1 X_t - \tilde{\psi}_1^i \left(X_{t+1} + ... + X_{t+\tau}\right) - \psi_1^i X_{t+\tau+1} \right) \right), \end{aligned}$$

where $\tilde{\psi}_1^i$ is defined by

$$\tilde{\psi}_1^i = \psi_1^i + \delta_1.$$

Now, define

$$F(i)_{t,t+\tau} \equiv -\delta_1 X_t - \tilde{\psi}_1^i \left(X_{t+1} + ... + X_{t+\tau}\right) - \psi_1^i X_{t+\tau+1}$$

and note that if $X_{t+1}, ..., X_{t+\tau}$ are Gaussian under the risk-neutral measure, then also $F_{t,t+\tau}$ is Gaussian under the risk neutral measure. More precisely, let $F(i)_{t,t+\tau}$

be Gaussian distributed

$$F(i)_{t,t+\tau} \sim N^{Q}\left(\chi^{i}_{0,\tau+1} + \chi^{i}_{1,\tau+1} X_t, \Omega_{i,t}\right),$$

then, one can express the price of an defaultable government bond of country i with maturity τ by $\bar{P}^{\tau+1}_{i,t} = \exp\left(\bar{A}_{i,\tau+1} + \bar{B}_{i,\tau+1} X_t\right)$, where from

$$\begin{aligned}
\bar{P}^{\tau+1}_{i,t} &= \exp\left(-\tau\psi^{i}_{0}\right) E^{Q}_{t}\left(\exp\left(F(i)_{t,t+\tau+1}\right)\right) \\
&= \exp\left(-\tau\psi^{i}_{0} + \chi^{i}_{0,\tau+1} + \frac{1}{2}\Omega_{i,t} + \chi^{i}_{1,\tau+1} X_t\right)
\end{aligned}$$

the coefficients $\bar{A}_{i,\tau+1}$ and $\bar{B}_{i,\tau+1}$ are given by

$$\bar{A}_{i,\tau+1} = \chi^{i}_{0,\tau+1} + \frac{1}{2}\Omega_{i,t}, \tag{4.19}$$

$$\bar{B}_{i,\tau+1} = \chi^{i}_{1,\tau+1}. \tag{4.20}$$

Finally, in order to calculate the coefficients $\bar{A}_{i,\tau+1}$ and $\bar{B}_{i,\tau+1}$, it remains to compute $\chi^{i}_{0,\tau+1}$, $\chi_{1,\tau+1}$ and $\Omega_{i,t}$. However, since I employ a version of the model without constant terms, it is only necessary to calculate $\chi^{i}_{1,\tau+1}$. Computation of the conditional

expectation of $F(i)_{t,t+\tau}$ is done by

$$\begin{aligned}
E_t^Q\left(F(i)_{t,t+\tau}\right) &= E_t^Q\left(-\delta_1 X_t - \tilde{\psi}_1^i\left[X_{t+1}+...+X_{t+\tau}\right] - \psi_1^i X_{t+\tau+1}\right) \\
&= E_t^Q\begin{pmatrix} -\tilde{\psi}_1^i\left[\mu^* + P^* X_t + ... + \sum_{k=0}^{j-1}\mu^*\left(P^*\right)^k + \left(P^*\right)^\tau X_t\right] \\ -\delta_1 X_t - \psi_1^i\left(P^*\right)^{\tau+1} X_t \end{pmatrix} \\
&\quad + E_t^Q\begin{pmatrix} -\tilde{\psi}_1^i\left[\Sigma\varepsilon_t + ... + \sum_{j=0}^{\tau-1}\left(P_1^*\right)^j \Sigma\varepsilon_t\right] \\ -\psi_1^i\sum_{j=0}^{\tau}\left(P_1^*\right)^j\Sigma\varepsilon_t \end{pmatrix} \\
&= -\delta_1 X_t - \tilde{\psi}_1^i\left[\tau I + (\tau-1)P + (\tau-2)P^2 + ... + P^{\tau-1}\right]\mu^* \\
&\quad -\tilde{\psi}_1^i\left[P^* + ... + \left(P^*\right)^\tau\right]X_t - \psi_1^i\left(P^*\right)^{\tau+1}X_t \\
&= -\tilde{\psi}_1^i\left[P^*\left[\left(P^*\right)^\tau - I\right]\left[I-\left(P^*\right)\right]^{-1} - \tau I\right]\left[I-\left(P^*\right)\right]^{-1}\mu^* \\
&\quad -\left[\delta_1 + \tilde{\psi}_1^i P^*\left[\left(P^*\right)^\tau - I\right]\left[I-\left(P^*\right)\right]^{-1} + \psi_1^i\left(P^*\right)^{\tau+1}\right]X_t,
\end{aligned}$$

where I used in the second equality that $E_t^Q X_{t+j} = E_t^Q\left[\sum_{k=0}^{j-1}\mu^*\left(P^*\right)^k\right] + E_t^Q\left[\left(P_1^*\right)^j X_t + \sum_{k=0}^{j-1}\left(P_1^*\right)^k\Sigma\varepsilon_t\right]$ and in the fourth equality that

$$\begin{aligned}
&\left[\tau I + (\tau-1)P + (\tau-2)P^2 + ... + P^{\tau-1}\right]\mu^* \\
= &\left[P^*\left[\left(P^*\right)^\tau - I\right]\left[I-\left(P^*\right)\right]^{-1} - \tau I\right]\left[I-\left(P^*\right)\right]^{-1}\mu^*
\end{aligned}$$

and

$$P^* + ... + \left(P^*\right)^\tau = P^*\left[\left(P^*\right)^\tau - I\right]\left[I-\left(P^*\right)\right]^{-1}.$$

Thus,

$$E_t^Q\left(F(i)_{t,t+\tau}\right) = \chi_{0,\tau+1}^i + \chi_{1,\tau+1}^i X_t,$$

where

$$\chi^i_{1,\tau+1} = -\left[\delta_1 + \tilde{\psi}^i_1 P^* \left[(P^*)^\tau - I\right] \left[(P^*) - I\right]^{-1} + \psi^i_1 (P^*)^{\tau+1}\right], \quad (4.21)$$

$$\chi^i_{0,\tau+1} = -\tilde{\psi}^i_1 \left[P^* \left[(P^*)^\tau - I\right] \left[I - (P^*)\right]^{-1} - \tau I\right] \left[I - (P^*)\right]^{-1} \mu^*. \quad (4.22)$$

Note that the terms $P^* \left[(P^*)^\tau - I\right] \left[(P^*) - I\right]^{-1}$ and $(P^*)^{\tau+1}$ in eq. (4.21) do not depend on the debtor, thus, these terms do not need to be calculated for each debtor separately.

4.A.4 Computation of the one-period Return Premium

The one-period return premium is given by (for $\tau > 1$)

$$\begin{aligned} E_t\left(hprx^{(\tau)}_{t+1}\right) &= E_t\left(hpr^{(\tau)}_{t+1}\right) - E_t\left(y^{(1)}_t\right) \\ &= E_t\left(p^{(\tau-1)}_{t+1} - p^{(\tau)}_t\right) - E_t\left(y^{(1)}_t\right). \end{aligned}$$

Plugging the log prices and the expected short rate into the equation above yields

$$E_t\left(hprx^{(\tau)}_{t+1}\right) = E_t\left[\bar{A}_{(\tau-1)} + \bar{B}'_{(\tau-1)} E_t X_{t+1} - \bar{A}_{(\tau)} - \bar{B}'_{(\tau)} X_t - \delta'_1 \bar{\mu} - \delta_0 - \delta'_1 \left(X_t - \bar{\mu}\right)\right].$$

Next, the pricing matrices $\bar{A}_{(\tau)}$ and $\bar{B}_{(\tau)}$ can be expressed recursively by

$$\bar{A}_{(\tau+1)} = \bar{A}_{(\tau)} + \bar{B}'_{(\tau)} \left(\mu - \Sigma\lambda_0\right) + \frac{1}{2} \bar{B}'_{(\tau)} \Sigma\Sigma' \bar{B}'_{(\tau)} - \delta_0, \quad (4.23)$$

$$\bar{B}'_{(\tau+1)} = \bar{B}'_{(\tau)} \left(P - \Sigma\lambda_1\right) - \delta'_1, \quad (4.24)$$

with initial conditions for $\bar{A}_{(\tau)}$ and $\bar{B}_{(\tau)}$ are given by $\bar{A}_1 = \delta_0 = 0$, and $\bar{B}'_1 = -\delta'_1$ (see e.g. Ang and Piazzesi, 2003). Using $E_t X_{t+1} = \bar{\mu} + P\left(X_t - \bar{\mu}\right)$, $\mu = \left(I - P\right)\bar{\mu}$

and eq. (4.23), rearranging and collecting terms yields

$$\begin{aligned}
E_t\left(hprx_{t+1}^{(\tau)}\right) &= \bar{A}_{(\tau-1)} - \bar{A}_{(\tau)} + \bar{B}'_{(\tau-1)}\left[\bar{\mu} + P\left(X_t - \bar{\mu}\right)\right] \\
&\quad - \left[\bar{B}'_{(\tau-1)}\left(P - \Sigma\lambda_1\right) - \delta'_1\right] X_t - \delta'_1\bar{\mu} - \delta'_1\left(X_t - \bar{\mu}\right) - \delta_0 \\
&= \bar{A}_{(\tau-1)} - \bar{A}_{(\tau)} + \bar{B}'_{(\tau-1)}\left(I + P\right)\bar{\mu} + \bar{B}'_{(\tau-1)}\Sigma\lambda_1 X_t - \delta_0 \\
&= \bar{A}_{(\tau-1)} - \left[\bar{A}_{(\tau-1)} + \bar{B}'_{(\tau-1)}\left(\mu - \Sigma\lambda_0\right) + \frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}_{(\tau-1)} - \delta_0\right] \\
&\quad + \bar{B}'_{(\tau-1)}\left(I + P\right)\bar{\mu} + \bar{B}'_{(\tau-1)}\Sigma\lambda_1 X_t - \delta_0 \\
&= \bar{B}'_{(\tau-1)}\Sigma\left[\lambda_0 + \lambda_1 X_t\right] - \frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}_{(\tau-1)}.
\end{aligned}$$

Due to the restricted form of λ_1 the only source of variation in $E_t\left(hprx_{t+1}^{(\tau)}\right)$ is the variable that is ordered at the last position in X_t.

4.A.5 Zero-Coupon Yield Data

The model uses yield data of zero-coupon government bonds from four European countries (France, Germany, Italy, and Spain). However, usually, most bonds bear coupon payments, in particular, those issued with a maturity of more than one year. Thus, a method to extract zero coupon rates from the prices of coupon-bearing bonds is needed. In order to construct zero-coupon bond data, different methods are in used in practice (see BIS, 2005), which can be broadly categorized into parametric and spline-based approaches.

As in Gürkaynak, Sack, and Wright (2005), I use a parametric model. The basic idea of parametric models is to specify a single function defined over the entire maturity domain. In particular, following Borgy et al. (2012), I choose the Nelson-Siegel (1987) model. In the following, I will briefly discuss the Nelson-Siegel model and the estimation approach. The discussion of the Nelson-Siegel model is based on

BIS (2005).

The Nelson-Siegel function for the instantaneous forward rates f at a given point in time t is defined by

$$f_t^{\tau}(\theta) = \beta_0 + \beta_1 \exp\left(-\frac{\tau}{\tau_1}\right) + \beta_2 \frac{\tau}{\tau_1} \exp\left(-\frac{\tau}{\tau_1}\right),$$

where τ denotes the time to maturity, and $\theta = \left(\beta_0, \beta_1, \beta_{2,}\tau_1\right)'$ denotes the parameters of the Nelson-Siegel Function. It can be shown that to the corresponding spot rate function for a given point in time t is given by

$$y_t^{\tau}(\theta) = \beta_0 + (\beta_1 + \beta_2)\left(1 - \exp\left(-\frac{\tau}{\tau_1}\right)\right) - \beta_2\left(-\frac{\tau}{\tau_1}\right) n,$$

where β_0 can be interpreted as the instantaneous asymptotic rate and the term $(\beta_0 + \beta_1)$ as the asymptotic spot rate.

Consider one particular coupon bearing bond at time t that matures in τ periods. The present value of the coupon-bearing bond is calculated as the discounted sum of coupon payments and the bond's repayment on maturity. Thus, the price of a coupon-bearing bond will be equal to

$$\hat{P}_{t,\tau} = \sum_{i=1}^{\tau} d_{t,i} C + d_{t,\tau} V, \tag{4.25}$$

where C denotes the coupon payment, V is the bond's repayment on maturity, and the discount function which gives the price of a zero-coupon bond paying one Euro at maturity is defined by

$$d_{t,i} = \exp\left(-y_t^i(\theta)\, i\right).$$

For given parameters from the discount function together with eq. (4.25), the model

based bond prices can be computed.

In the estimation process, the parameters of the Nelson-Siegel spot rate function are chosen to minimize the distance between the observed bond prices at time t and the calculated bond prices. Specifically, the minimization problem is given by

$$\hat{\theta}_t = \arg\min_{\theta} \sum_{j=1}^{N} w_j \left(P_t^j - \hat{P}_t^j \right)^2,$$

where N is the total number of observed dirty bond prices at date t, P_t^j denotes the observed dirty prices of coupon bond j at time t, $\hat{P}_t^j$ denotes the model-implied price of the coupon bond j, and w_j is a weighting factor.[9]

A different approach is to minimize the sum of squared yield errors (as opposed to minimizing the sum of squared pricing errors). However, minimizing the sum of yield errors is computationally more time consuming since it requires to solve additionally for the yields after calculating bond prices. However, as noted by BIS (2005), minimizing the squared sum of pricing errors (instead of minimizing the sum of squared yield errors) leads to an unsatisfactory fit of yields of bonds relatively short residual maturity.[10] In order to correct for this shortcoming, different weights are chosen for different residual maturities. In particular, I set the optimization weight, following the practice of e.g. the Belgian central bank or the Spanish central bank (see BIS, 2005) equal to the inverse of the modified duration times the observed dirty price.

The data for the prices of coupon bonds is taken from Datastream. In order to calculate the bonds' cash flows accrued interest and the respective day-count

[9] The dirty price of a bond is defined as the price of a bond including any interest accruing on the next coupon payment.

[10] Intuitively, the smaller (modified) duration (which is the elasticity of bond prices to changes in yield to maturity changes) of bonds with shorter/longer residual maturities makes their prices more/less sensitive to yield changes. Choosing equal weights would lead to an overfitting of the long-end of the yield curve at the expense of the fit of the short-end of the yield curve.

conventions are taken into account. In the spirit of Gürkaynak et al. (2005) and following the practice of the ECB (ECB, 2008) different filters on the bond data are applied in order to detect and remove outliers that would bias the estimation results. In particular, I exclude all bonds from the estimation that are issued before 1990, and prices of bonds with a residual maturity less than 1 month. In order to prevent noise from the yield estimation, outliers are traced separately for a number of residual maturity brackets. Specifically, bond yields that deviate more than two standard deviations from the average yield in this bracket are considered as outliers and excluded. The procedure is iterated in order to account for potentially large outliers in the first round that would distort the average yield and the standard deviation. For the size of each maturity bracket, I follow the specification of the ECB.

Finally, due to the lack of information on the trading volume of bonds, for each point in time at which the estimation has been conducted, the yields are checked manually. Since the trading volume of bonds usually decreases considerably for shorter maturities, this may lead to large outliers at the short end of the yield curve. Moreover, some maturity brackets may not include enough bond yield data to apply the outliers removal algorithm. Checking yields manually helps to eliminate outliers that would otherwise result in unrealistic high or low short-term rates (e.g. short-term rates above 50 percentage points).

Chapter 5

Conclusion

The presented thesis contributes to different important topics of the recent literature of financial and monetary economics.

Chapter 2 analyzed the effectiveness of different unconventional monetary policies in a macro model where private financial intermediation is limited by an agency problem. It shows that, in general, the effects of collateralized lending and direct lending differ and that the effectiveness of both policies depends on how strong the considered financial friction affects the dynamic of the model. In particular, in a setting where the agency problem affects the dynamic of the model substantially, collateralized lending does only work, at its best, modestly to reduce credit spreads.

Chapter 3 conducted an empirical analysis of interplay of monetary policy, term premia movements, and economic activity in the Euro area. Chapter 3 demonstrated that movements in term premia incorporated in long-term bond yields do affect economic activity. Thus, if the central bank wants to influence long-term bond rates by forward guidance of the path of future short-term rates, they have to take changes in term premia into account. Moreover, Chapter 3 provided evidence that the ECB indeed responds to movements in term premia by adjusting the short-term interest

rate.

While Chapter 3 focused on the effects of movements in term premia on the Economy in the Euro area and the response of the ECB on these term premia movements, Chapter 4 analyzed the determinants of Euro area yield spreads and their interplay. In particular, this chapter disentangled the effects of changes in risk aversion and a common non-fundamental risk factor, interpreted as redenomination risk or systemic risk, on Euro area yield spreads. The analysis performed in Chapter 4 did show that although the common non-fundamental factor played a non-negligible role in the dynamics of Euro area yield spreads, economic shocks, in particular, risk aversion shocks, have been the most dominant drivers of Euro area yield spreads. Contributing to the literature on the determinants of yields spreads, Chapter 4 showed that risk aversion shocks are able to explain a substantial fraction of Euro area yield spreads, highlighting the importance of measuring risk aversion adequately.

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