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Price-based coordination of interconnected systems with access to external markets

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Abstract

Many industrial processes are coupled via multiple networks of energy and materials to achieve a resource and energy efficient production. In many cases however, setting up an integrated optimization problem for all units or plants that are directly connected to the networks is not possible, especially when not all information can be shared. In such cases, dual decomposition or price-based coordination can be used, where optimal transfer prices are iteratively determined at which the networks are balanced and the resources are allocated optimally between the participants.

In this contribution, price-based coordination is extended to include the situation where limited resources can be bought or sold at predefined prices from external markets (e.g. via pipelines) and the resulting algorithms are demonstrated for a realistic example.

Keywords: Distributed optimization, Price-based coordination, Dual decomposition, External resources, Shared-resource allocation.

1. Introduction

Many industrial production sites consist of individual plants or units, which are tightly coupled by streams of energy and material. Especially in the chemical industry, these streams, e.g., energy in the forms of steam and electricity as well as raw materials and intermediate products, are exchanged via networks that link consuming and producing systems, where the holdups usually are very small. In order to optimize the overall economic performance as well as the resource efficiency of the site, it is not sufficient to optimize each plant or unit individually, but rather a joint optimization of the production plants or site, taking into account the balancing of the resource networks, is necessary.

However, finding the solution to the overall optimization problem in a centralized manner may not be realizable. Due to various reasons, such as robustness and local autonomy or confidentiality, when systems belong to different business units or companies, optimizing each plant independently is preferred. One method to coordinate the individual optimization problems is dual decomposition, which uses transfer prices to balance the interconnecting streams, instead of solving the whole problem at once. An optimal solution is determined by iterating between setting the transfer prices (dual variables) and each system optimizing its response, until equilibrium transfer prices are found (Everett III, 1963). This method has various applications and can be used, for instance, to manage the energy flow in electrical micro grids (Zhang et al., 2013) or to optimally allocate resources in chemical production sites (Wenzel et al., 2016, 2017).

Considering all the networks of such a production process as closed systems is not realistic, since usually at least some of the networks are connected to external sources or sinks, where, on the spot market or according to contracts, amounts can be purchased or sold at certain price levels. The topic of optimal procurement for chemical process networks was already discussed by Calfa and Grossmann (2015) for a centralized optimization. In this contribution, the distributed optimization framework of dual decomposition is extended such that it solves this type of problem in a distributed manner by a modified update of the dual variables. We derive this update mechanism and provide a graphical interpretation for the one-dimensional case. Furthermore, we demonstrate the proposed update scheme using an example.

The following syntax is used: Italic lower case letters represent scalar functions or values. Bold lower case letters represent column vectors or vectored functions and bold upper case letters represent matrices. The transpose of vector $\mathbf{a}$ is denoted by $\mathbf{a}^T$. We use [k] to identify the k-th row of vectors and matrices. Absolute values and norms are denoted by $|a|$ and $||\mathbf{a}||$. To ease the notation, we here assume that all systems have the same number of variables and constraints, which however is not required in general.

2. Problem formulation

The joint optimization of N systems with network constraints can be written as Eqs.(1a) – (1d):

$$\min_{\mathbf{x}_i \forall i} \quad \sum_{i=1}^{N} f_i(\mathbf{x}_i), \tag{1a}$$

$$\text{s.t.:} \quad \mathbf{g}_i(\mathbf{x}_i) \leq \mathbf{0}, \tag{1b}$$

$$\mathbf{x}_{LB,i} \leq \mathbf{x}_i \leq \mathbf{x}_{UB,i}, \tag{1c}$$

$$\sum_{i=1}^{N} \mathbf{A}_i \mathbf{x}_i = \mathbf{0}. \tag{1d}$$

$$\min_{\mathbf{x}_i \forall i, \mathbf{r}_j \forall j} \quad \sum_{i=1}^{N} f_i(\mathbf{x}_i) + \sum_{j=1}^{M} \mathbf{p}_j^T \mathbf{r}_j, \tag{2a}$$

$$\text{s.t.:} \quad \mathbf{g}_i(\mathbf{x}_i) \leq \mathbf{0}, \tag{2b}$$

$$\mathbf{x}_{LB,i} \leq \mathbf{x}_i \leq \mathbf{x}_{UB,i}, \tag{2c}$$

$$\sum_{i=1}^{N} \mathbf{A}_i \mathbf{x}_i - \sum_{j=1}^{M} \mathbf{r}_j = \mathbf{0}, \tag{2d}$$

$$\mathbf{r}_{LB,j} \leq \mathbf{r}_j \leq \mathbf{r}_{UB,j} \tag{2e}$$

The objectives of the sub-problems i are denoted by f_i. The sub-problems are constrained by Eq.(1b) and in Eq.(1c) the states $\mathbf{x}_i$ are bounded by lower and upper bounds. Overall, the network constraint Eq.(1d) has to be satisfied. The interaction of system i with the network is given by $\mathbf{A}_i \mathbf{x}_i$.

In Eqs.(2a) – (2e), the network constraint is extended to include access to M external resources or sinks j, where the amounts $\mathbf{r}_j$, limited by $\mathbf{r}_{LB,j}$ and $\mathbf{r}_{UB,j}$, can be bought or sold at predefined prices $\mathbf{p}_j$.

3. Distributed solution

The problem shown in Eqs.(1a) – (1d) can be solved using dual decomposition, splitting the problem into N independent sub-problems and an overarching coordination problem by relaxation of the network constraints Eq.(1d). The local contributions to the network constraint are multiplied by the corresponding dual variable $\boldsymbol{\lambda}$ and added to the objective of the sub-problem. Locally, Eq.(3) is solved for a given value of $\boldsymbol{\lambda}$:

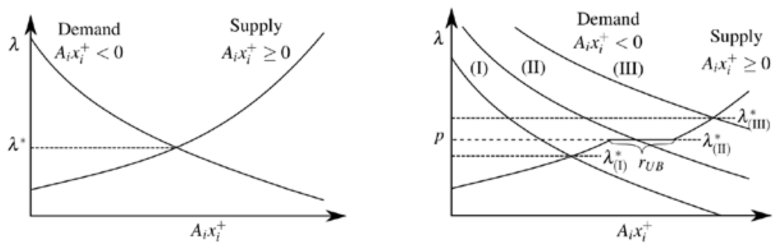
$$\mathbf{A}_i \mathbf{x}_i^+ = \mathbf{A}_i \arg\min_{\mathbf{x}_i} \quad f_i(\mathbf{x}_i) + \boldsymbol{\lambda}^T \mathbf{A}_i \mathbf{x}_i, \tag{3a}$$

$$\text{s.t.:} \quad \mathbf{g}_i(\mathbf{x}_i) = \mathbf{0}, \tag{3b}$$

$$\mathbf{x}_{LB,i} \leq \mathbf{x}_i \leq \mathbf{x}_{UB,i}. \tag{3c}$$

It is assumed that the subsystems do not share their objectives or constraints due to confidentiality reasons, only the responses to transfer prices $\mathbf{A}_i\mathbf{x}_i^+$, are communicated.

In Figure 1a, a one-dimensional example of an aggregated supply, which is defined as the sum of all positive contributions to the network constraint, i.e., $\sum_i \{A_i x_i^+ | A_i x_i^+ \geq 0\}$, and an aggregated demand, defined as $\sum_i \{A_i x_i^+ | A_i x_i^+ < 0\}$, depending on the value of λ are shown.



(a) No external resources and outlets corresponding to Eqs.(1a) – (1d).

(b) Possibility to buy external resources at price p for three different demand scenarios corresponding to Eqs.(2a) – (2e).

Figure 1: Optimal response of the systems to the transfer prices in the one-dimensional case.

Feasibility, i.e., satisfaction of the network constraint Eq.(1d), is achieved by adjusting the dual variables $\boldsymbol{\lambda}$ in an iterative procedure. Economically, this can be interpreted as finding the intersection between supply and demand, cf. Figure 1a. However, in the considered case this cannot be done explicitly, since the individual objectives and constraints are not known in the coordination problem, only the responses to the transfer prices. Hence the sub-gradient method is used for the optimization where transfer prices $\boldsymbol{\lambda}$ are adjusted proportionally to the difference between supply and demand

$$\boldsymbol{\lambda}^+ = \boldsymbol{\lambda} + \alpha \sum_{i=1}^{N} \mathbf{A}_i \mathbf{x}_i^+. \tag{4}$$

Iterating between local optimizations and the update of the transfer prices is done until a stopping criterion is met, e.g., that the maximum residuum $\left\|\boldsymbol{\rho}^+\right\|_\infty = \left\|\sum_i^N \mathbf{A}_i \mathbf{x}_i^+\right\|_\infty$ is less than a predefined tolerance ϵ. The update of the transfer prices is done using the step size parameter $\alpha \in (0, 1)$. Careful selection of the step size parameter is necessary to ensure convergence, especially when local constraints $\mathbf{g}_i(\mathbf{x}_i)$ are active. If all objectives and constraint functions are convex, the solution found upon convergence is the global optimum.

The augmented problem from Eqs.(2a) – (2e) can also be solved using dual decomposition. While the local optimization can still be done using Eq.(3), on the coordination layer, additionally to finding the optimal transfer prices, the cost for interaction with external markets, $\sum_{j=1}^{M} \mathbf{p}_j^T \mathbf{r}_j$, has to be minimized. Finding the optimal values for $\mathbf{r}_j$ can either be done separately from the price adjustment or in a combined step. In Eqs.(5) and (6), the new transfer prices $\boldsymbol{\lambda}$ and the amounts to be bought or sold $\mathbf{r}_j$ are calculated separately using the sub-gradient method:

$$\boldsymbol{\lambda}^{+} = \boldsymbol{\lambda} + \alpha \left[\sum_{i=1}^{N} \mathbf{A}_i \mathbf{x}_i^{+} - \sum_{j=1}^{M} \mathbf{r}_j \right] \tag{5}$$

$$\mathbf{r}_j^{+} = \min\left(\max\left(\mathbf{r}_{LB,j}, \mathbf{r}_j + \mathrm{diag}(\mathbf{p}_j)^{-1}\mathrm{diag}(\mathbf{p}_j - \boldsymbol{\lambda}^{+})\left(\mathbf{r}_{UB,j} - \mathbf{r}_{LB,j}\right)\right) \mathbf{r}_{UB,j}\right). \tag{6}$$

The other approach is to update both variables at the same time. The complete algorithm for balancing multiple networks with several external resources and outlets at different fixed prices is shown in Algorithm 1. In the example in Figure 1a, this corresponds to evaluating different realizations for λ^{+} using Eq.(5), based on the minimum and maximum values for r. If the price range given by $\underline{\Lambda}$ and $\overline{\Lambda}$ is below the price for the external resource p, then r will be at the lower bound, and the maximum value for λ^{+} is used in the next iteration, cf. Case 1 in Algorithm 1. If the price range includes p, Case 2, the difference between supply and demand can be covered by the external resources and therefore $\lambda^{+} = p$. If the price range is above the price for the external resources, Case 1 also applies and the external resource r is at the upper bound. Case 3 is only required, if the step size α is too large. The residuum vector $\boldsymbol{\rho}^{+} = \sum_{i=1}^{N} \mathbf{A}_i \mathbf{x}_i^{+} - \sum_{j=1}^{M} \mathbf{r}_j^{+}$ is used for the evaluation of the stopping criterion in both approaches.

Algorithm 1 Coordinator level update step to determine $\boldsymbol{\lambda}^{+}$ and $\mathbf{r}_j^{+}$

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r_j^+ ← r_{LB,j} ∀j
for all λ[k] in λ do
  π ← [−∞, p_1[k], p_2[k], ..., p_M[k], ∞]^T
  Λ_ ← λ[k] + α ( Σ_{i=1}^{N} A_i[k]x_i − Σ_{j=1}^{M} r_{LB,j}[k] )
  for l = 1 to M+1 do
    Λ‾ ← λ[k] + α ( Σ_{i=1}^{N} A_i[k]x_i − Σ_{j=1}^{l} r_{UB,j}[k] − Σ_{j=l+1}^{M} r_{LB,j}[k] )
    if π[l] ≤ Λ_ < π[l+1] then
      λ^+[k], r_j^+[k] ← Λ_, r_{UB,j}^+[k]                                   Case 1
      Break
    else if Λ‾ ≤ π[l+1] < Λ_ then
      if Σ_{j=1}^{l−1} r_{j,max}[k] ≤ Σ_{i=1}^{N} A_i[k]x_i < Σ_{j=1}^{l} r_{j,max}[k] then
        λ^+[k], r_j^+[k] ← π[l+1], ( Σ_{i=1}^{N} A_i[k]x_i − Σ_{j=1,j≠l}^{M} r_j^+[k] )   Case 2
      else
        λ^+[k], r_j^+[k] ← Λ_, r_{UB,j}^+[k]                                 Case 3
      end if
      Break
    end if
    Λ_ ← Λ‾
  end for
end for
```

4. Example

In the following, an example is used to demonstrate the proposed extension of the dual decomposition approach. Quadratic objective functions with positive diagonal scaling matrices and affine constraints of the following form are assumed:

$$f_i(\mathbf{x}_i) = \left(\mathbf{x}_i - \mathbf{x}_{i,Target}\right)^T \mathbf{B}_i \left(\mathbf{x}_i - \mathbf{x}_{i,Target}\right), \tag{7a}$$

$$\mathbf{g}_i(\mathbf{x}_i) = \mathbf{C}_i\mathbf{x}_i - \mathbf{d}_i = \mathbf{0}, \tag{7b}$$

Five subsystems ($N = 5$) are connected via three shared resources ($n_{Networks} = 3$). For each resource, there are three different sources from which they can be bought ($M = 3$). Each subsystem has four independent variables $\mathbf{x}_i \in \mathbb{R}^4$ and is subject to two local constraints $\mathbf{g}_i : \mathbb{R}^4 \to \mathbb{R}^2$. The transfer prices $\boldsymbol{\lambda} \in \mathbb{R}^3$ are initialized with $\mathbf{0}$, the step size is chosen as $\alpha = 0.03$, and the tolerance is set to $\epsilon = 10^{-6}$. The matrices and vectors $\mathbf{x}_{i,Target}$, $\mathbf{B}_i$, $\mathbf{C}_i$, $\mathbf{d}_i$, $\mathbf{A}_i$, $\mathbf{p}_j$, and $\mathbf{r}_{UB,j}$ are generated from a random seed and the lower bound for flows of external resources is $\mathbf{r}_{LB,j} = \mathbf{0}$.

$$\mathbf{x}_{Target,i} = \begin{bmatrix}-2 & -7 & 4 & 6\end{bmatrix};\begin{bmatrix}-3 & 9 & -10 & 5\end{bmatrix};\begin{bmatrix}-7 & 2 & 8 & -7\end{bmatrix};\\ \begin{bmatrix}-8 & 1 & 4 & 9\end{bmatrix};\begin{bmatrix}1 & 2 & -7 & -6\end{bmatrix},$$

$$\mathbf{B}_i = diag\left(\begin{bmatrix}3 & 5 & 7 & 8\end{bmatrix}\right); diag\left(\begin{bmatrix}6 & 4 & 8 & 9\end{bmatrix}\right); diag\left(\begin{bmatrix}1 & 3 & 4 & 5\end{bmatrix}\right);\\ diag\left(\begin{bmatrix}4 & 7 & 5 & 9\end{bmatrix}\right); diag\left(\begin{bmatrix}3 & 4 & 7 & 7\end{bmatrix}\right),$$

$$\mathbf{C}_i = \begin{bmatrix}9 & 7 & 9 & 7\\ 0 & 5 & 4 & 4\end{bmatrix};\begin{bmatrix}4 & 1 & 8 & 5\\ 0 & 0 & 5 & 4\end{bmatrix};\begin{bmatrix}8 & 7 & 2 & 3\\ 0 & 0 & 8 & 1\end{bmatrix};\\ \begin{bmatrix}0 & 2 & 1 & 8\\ 1 & 6 & 2 & 8\end{bmatrix};\begin{bmatrix}5 & 7 & 6 & 6\\ 9 & 3 & 9 & 9\end{bmatrix},$$

$$\mathbf{d}_i = \begin{bmatrix}9 & 6\end{bmatrix}^T;\begin{bmatrix}1 & 3\end{bmatrix}^T;\begin{bmatrix}6 & 1\end{bmatrix}^T;\begin{bmatrix}3 & 6\end{bmatrix}^T;\begin{bmatrix}5 & 6\end{bmatrix}^T,$$

$$\mathbf{A}_i = \begin{bmatrix}-8 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 6 & 0\end{bmatrix};\begin{bmatrix}2 & 0 & 0 & 0\\ 0 & 3 & 0 & 0\\ 0 & 0 & -9 & 0\end{bmatrix};\begin{bmatrix}-9 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\end{bmatrix};\\ \begin{bmatrix}9 & 0 & 0 & 0\\ 0 & -5 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix};\begin{bmatrix}4 & 0 & 0 & 0\\ 0 & -7 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix},$$

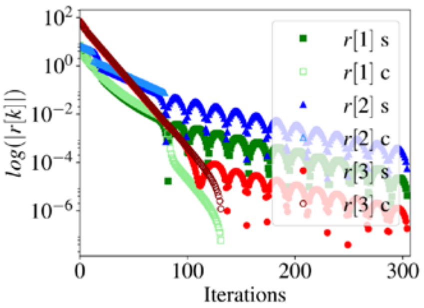
$$\mathbf{p}_j = \begin{bmatrix}5.89 & 2.09 & 1.52\end{bmatrix}^T;\begin{bmatrix}6.28 & 3.1 & 6.95\end{bmatrix}^T;\begin{bmatrix}7.54 & 3.91 & 7.5\end{bmatrix}^T,$$

$$\mathbf{r}_{UB,j} = \begin{bmatrix}3.1 & 4.5 & 3\end{bmatrix}^T;\begin{bmatrix}2.2 & 2.9 & 1.4\end{bmatrix}^T;\begin{bmatrix}4.8 & 1.9 & 4\end{bmatrix}^T,$$

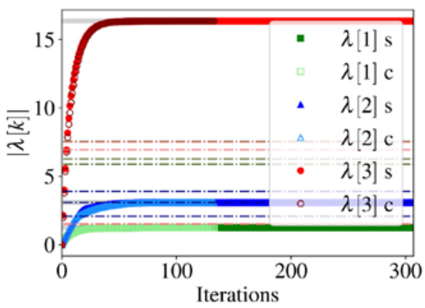
In Figure 2, the change of the residuals $\boldsymbol{\rho}$ and of the transfer prices $\boldsymbol{\lambda}$ over the iterations can be seen for both, the separate (filled) and combined, update rules. The distributed solutions converge towards the optimum of the centralized solution, which can be seen from the vanishing residuals on the left and the matching of the transfer prices at convergence with those from the centralized solution (shown in grey thick lines) on the right. Furthermore, it can be seen that upon convergence $\boldsymbol{\lambda}[1]$ (square) is less than, $\boldsymbol{\lambda}[2]$ (triangle) is equal to, and $\boldsymbol{\lambda}[3]$ (circle) is larger than the prices at which external resources can be bought (dash-dotted lines), i.e., this example covers all three scenarios from Figure 1b. Comparing the two different methods, it becomes evident that the method with separate steps requires more iterations and oscillates towards the optimum.

5. Discussion and Conclusion

The significantly larger number of iterations in the approach with separate steps is due to $\boldsymbol{\lambda}[2]$ being equal to the external price. Since the variables are set consecutively, an



(a) Change of the residuals.



(b) Change of the prices.

Figure 2: Evolution of the resulting trajectories of characterisitc quantities for the separated s and the combined c approach over the iterations.

overshoot is necessary because otherwise the sub-gradient for the usage of the external resource does not change. For the combined approach, after 90 iterations the network constraint of resource 2 is balanced by the external market and hence the residuum is 0.

In general, it can be said that distributed optimization can be used when there is access to external markets. What the authors observed running a suite of test cases with 1000 additional randomly generated examples is that the number of iterations of the combined approach is in a similar range as for a price-based optimization without external markets. Additionally, using this method bounds on the transfer prices of resources can be imposed by setting $\mathbf{r}_{LB,j}$ and $\mathbf{r}_{UB,j}$ to a low/ high values. The authors found this to be necessary to avoid excessive prices in real applications.

6. Outlook

Future work includes investigating the influence of a dynamic adaptation of the step size parameter α, in order to enable an arbitrary starting value for α and to speed up convergence. From a practical perspective, the distribution of the gains from the usage of external markets needs to be investigated, since here these are only realized at the coordination level. Another topic of interest is the implementation of other contracts, such as take or pay, minimum required amounts, discounts for large amounts, etc.

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