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Topology evolution of composite structures based on a phase field model

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The composition of fibers and matrix is of great importance in several fields of engineering, such as steel reinforcement in concrete for civil engineering or lightweight applications in the automotive and aviation industry, as it allows combining the advantages of both materials. If the bond between fibers and matrix is ideally strong enough, the mechanical deformation can be assumed to be equal in both materials. With this assumption we set up a phase field model evolving the topology of reinforcement. The phase field parameter represents regions of reinforcement in the sense of averaged increased stiffness since we do not intend to simulate single fibers. A similar model but for topology optimization based on equivalent stresses was introduced by Muench *et al.* [1]. In many matrix materials, viscoelastic behavior is observed. Therefore, we also consider viscoelasticity in our model for the matrix.

1 Introduction

In our model the topology of reinforcement is described by the phase field parameter φ. It adapts the values $+1$ for reinforced regions and -1 for matrix regions. In the transition zone a sensitivity function controls the growth of the phase. Many topology optimization methods attempt to minimize the compliance of a structure. Here, the intention is to reinforce into the direction of maximum tensile stress. Thus, an algorithm for stress based optimization is used.

2 A phase field model with viscoelastic matrix and fiber reinforcement

The inhomogeneity of maximum principle stress σ_1 is measured by the normalized difference between the tensile strength of the matrix material f_{ct} and σ_1. It yields the objective function

$$F = \int\limits_{\mathfrak{B}} \frac{f_{\mathrm{ct}} - \sigma_1(\boldsymbol{\varepsilon}(\boldsymbol{u}), \varphi)}{f_{\mathrm{ct}}} \varphi \,\mathrm{d}V = \int\limits_{\mathfrak{B}} \gamma(f_{\mathrm{ct}}, \boldsymbol{u}, \varphi)\varphi \,\mathrm{d}V \to \min \text{ w.r.t. } \boldsymbol{u}, \varphi. \quad (1)$$

The purpose of this function is to add reinforcement in regions where σ_1 exceeds f_{ct} of the matrix. The material stiffness of the composite combines the matrix $\mathbf{C}_{\mathrm{M}}$ and fibers $f(\varphi)\mathbf{C}_{\mathrm{F}}$, $f(\varphi) \in [0,1]$. A double-well potential and an inner gradient energy is employed to separate zones of pure matrix and zones with reinforcement. The inner energy density reads

$$\Psi_{\mathrm{int}}(\boldsymbol{\varepsilon}, \varphi, \mathrm{Grad}[\varphi]) = \frac{1}{2}\boldsymbol{\varepsilon}(\boldsymbol{u})[\mathbf{C}_{\mathrm{M}} + f(\varphi)\mathbf{C}_{\mathrm{F}}]\boldsymbol{\varepsilon}(\boldsymbol{u}) + \varphi^6 - \varphi^4 - \varphi^2 + 1 + \frac{1}{2}L_c||\mathrm{Grad}[\varphi]||^2, \quad (2)$$

with $\mathbf{C}_{\mathrm{M}}$ and $\mathbf{C}_{\mathrm{F}}$ from linear elasticity. Incorporating the objective function penalized with c_γ the total energy is given by

$$\Pi = \int\limits_{\mathfrak{B}} \Psi_{\mathrm{int}}(\boldsymbol{\varepsilon}, \varphi, \mathrm{Grad}[\varphi]) \,\mathrm{d}V + \int\limits_{\mathfrak{B}} c_\gamma \gamma \varphi \,\mathrm{d}V - \int\limits_{\partial\mathfrak{B}} (\bar{\boldsymbol{t}} \cdot \boldsymbol{u} + y\,\varphi) \,\mathrm{d}A. \quad (3)$$

The factor c_γ regulates the sensitivity of the model to its objective function. The first variation with respect to φ yields

$$\delta_\varphi \Pi = \int\limits_{\mathfrak{B}} \underbrace{\left(\frac{1}{2}f'(\varphi)\boldsymbol{\varepsilon}(\boldsymbol{u})\mathbf{C}_{\mathrm{F}}\boldsymbol{\varepsilon}(\boldsymbol{u}) + (6\varphi^5 - 4\varphi^3 - 2\varphi)\right)}_{\chi} \delta\varphi + L_c \mathrm{Grad}[\varphi] \cdot \mathrm{Grad}[\delta\varphi] + c_\gamma \gamma \delta\varphi \,\mathrm{d}V - \int\limits_{\partial\mathfrak{B}} \overline{y}\,\delta\varphi \,\mathrm{d}A, \quad (4)$$

with the Neumann boundary condition $\mathrm{Grad}[\varphi] \cdot \boldsymbol{n} = \overline{y}$ indicating either injection or rejection of fibers via ∂B. The variation of the objective function $\delta\gamma$ is dropped, since γ is given as an integral value. The resulting balance equation of the phase field

$$\chi - L_c \mathrm{Div}[\mathrm{Grad}[\varphi]] + c_\gamma \gamma = \omega \dot{\varphi}, \quad (5)$$

is extended by the rate of the phase field parameter φ and the kinetic coefficient ω, controlling the viscosity of the evolution process. This provides an immediate change of the phase field and ensures the initial equilibrium state. The parameter L_c regulates the size of the "diffuse interface".

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Simo and Hughes [2] describe the standard linear viscoelastic solid as one-dimensional special case of the generalized Maxwell model, which consists of two springs and a dashpot. The inelastic strains $\varepsilon_\mathrm{v}(t)$ must satisfy the evolution equation Eq.(6). The overall stresses are $\sigma(t) = (E_\infty + E_\mathrm{e})\varepsilon - \varepsilon_\mathrm{v}(t)E_\mathrm{e}$ with the initial modulus $E_0 = E_\infty + E_\mathrm{e}$. Applying the implicit Euler time integration scheme to the evolution equation yields the time-discrete version Eq.(7) of Eq.(6)

$$\dot{\varepsilon_\mathrm{v}} + \frac{1}{\tau}\varepsilon_\mathrm{v} = \frac{1}{\tau}\varepsilon, \qquad \lim_{t\to 0} \varepsilon_\mathrm{v}(t) = 0, \qquad (6) \qquad \frac{1}{\Delta t}(\varepsilon^\mathrm{v}_{n+1} - \varepsilon^\mathrm{v}_n) = \frac{1}{\tau}(\varepsilon_{n+1} - \varepsilon^\mathrm{v}_{n+1}), \qquad (7)$$

with the relaxation time $\tau = \frac{\eta}{E^\mathrm{e}}$, the viscosity of the matrix η and the time discretization $t_{n+1} = t_n + \Delta t$.

Transverse isotropic behavior is defined by five independent material parameters: E_1, E_2, G_{12}, G_{23} and ν. In our model it is implemented for the fiber material. Fiber angles are iteratively adjusted to the principle stress direction. This is achieved by a transformation of a reduced material matrix described by Altenbach *et al.* [3] yielding $\boldsymbol{\sigma} = \mathbf{C}_{\mathrm{F,mod}}\boldsymbol{\varepsilon} = (\mathbf{T}^\varepsilon)^T \boldsymbol{C}_{\mathrm{F,red}}\mathbf{T}^\varepsilon\boldsymbol{\varepsilon}$, with the Transformation matrix $\mathbf{T}^\varepsilon$ and the reduced fiber material matrix $\boldsymbol{C}_{\mathrm{F,red}}$.

3 Numerical example

Our simulation considers neutral initial reinforcement with $\varphi \equiv 1$ for the corbel shown in Fig.1a. Three different matrix materials are compared using 10000 finite elements with quadratic Ansatz functions yielding 121203 DOF. The optimized reinforcements in Fig.1b show slight differences in filling levels $K(\sigma_1, f_\mathrm{ct})$, as shown in Fig.1d. However, they differ in distribution of fiber. Observing Fig.1c, the fiber direction θ is increasingly horizontal within the upper reinforcement regions.

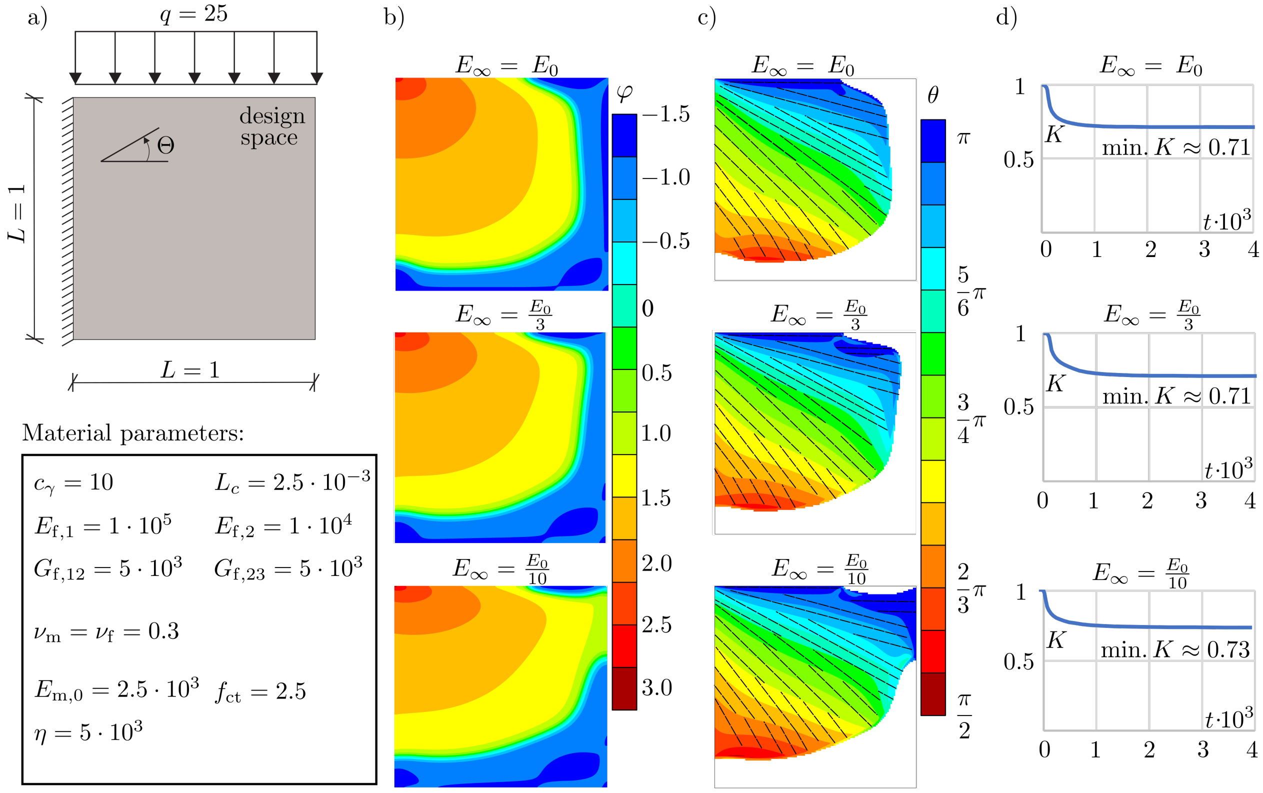

Fig. 1: a) Design space, boundary conditions, loading and element discretization of the tests. b) Optimized reinforcement with different topologies. c) Fiber direction of the optimized reinforcement. d) Filling level development over time.

4 Conclusion

The phase field model allows for course but stable simulation of fiber matrix problems even in case of viscous material effects. Relaxation of viscous matrix materials influences the distribution and direction of fibers in composite structures. The design might benefit from taking these effects into account in order to increase the reliability of the load-bearing element.

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