Analyzing the Historical Life Table of Thomas Young[1]

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Abstract: Thomas Young (1773-1829) is one of the greatest thinkers and polymaths. His scientific work includes significant contributions in the fields of medicine, physics, anthropology and ancient history. Less well known, however, is Young's demographic contribution. In 1826, Thomas Young examined graphical curves of mortality of his epoch (decrement tables of the deceased) to see if they matched a formula he had developed. Looking for a law of mortality, he created a high order polynomial for the function of mortality. We use modern demographic methods to analyze and criticize his life table. Young's discrete life table is fitted by a continuous life table function (Lazarus distribution) in order to calculate important parameters. It is shown that Young's formula is an early and successful method of determining a model life table. It corresponds to a particular life table of Coale and Demeny. The article concludes with an exploration of Young's mortality formula of 1816, a concise yet foundational model, showcasing its ability to facilitate calculations of vital functions like life expectancy and the force of mortality, despite its lesser-known status.

1. Introduction

The study of human mortality has been a subject of scientific inquiry for centuries, with scholars seeking to better understand the patterns and trends in the length of human life. In the early 19th century, Thomas Young (1773-1829), a renowned physician, physicist, and linguist, contributed to this field with his work on life expectancy and mortality rates (see, e.g. Peacock, 2013). Young was a prominent figure in his time, making significant contributions to a wide range of fields, including the study of hieroglyphs and the decipherment of the Rosetta Stone[2].

In 1826, Young published a paper in the Philosophical Transactions of the Royal Society titled "A Formula for expressing the Decrement of Human Life," in which he sought to find a law of mortality by creating a high order polynomial for the curve of mortality[3]. His formula consisted of terms having influence in infancy, in youth, in middle age, and in old age. He also constructed a curve to represent the formula, which he believed was more accurate than existing contemporary life tables.

In this paper, we use modern demographic methods to analyze and criticize his life table. Young's discrete life table is fitted by a continuous life table function (Lazarus distribution) in order to calculate important parameters. It is shown that Young's formula is an early and successful method of determining a model life table. We also find that Young's life table corresponds to a particular life table of Coale and Demeny.

2. Young´s Formula

Young created a curve diagram to represent the decrements of life, with age on the x-axis and the corresponding decrements on the y-axis. He used several life tables, including de Moivre,

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[2] In the autumn of 1795, Young travelled to Germany and was awarded a doctorate in medicine from the University of Göttingen in 1796. The choice of the University of Göttingen, apart from its quality, was particularly close for an Englishman because the Kingdom of Hanover was in a personal union with Great Britain (Koelbing, 1974, p. 58).

[3] See also Appendix B.

Finlaison, Carlisle, Northampton, Deparcieux, Morgan, and London Equity, to calculate the mean values of death at each age after eliminating extreme values. He then plotted these mean values on the curve diagram as crosses (see Fig. 1).

In essence, Young writes that the mean obtained from his method could be used as a standard table, but it still displays some minor irregularities that can be seen by examining the line of stars in the graph. To smooth out the variations in the data, it would be most effective to develop a formula that accurately reflects the entire curve. However, Young acknowledges that finding such an expression would be extremely challenging, and that any formula developed would likely be too complex to apply in practice. Despite this difficulty, Young was able to construct a curve that closely follows the line of stars, intersecting it at 10 to 12 different points, using the proposed formula[4].

His formula is

$$dx=368 + 10 \cdot x - 0.11 \cdot \left(156+20 \cdot x-x^2\right)^{1.5} + \frac{10^5}{2.85 + 2.05 \cdot x^2 + 2 \cdot \left(\frac{x}{10}\right)^6} - 5.5 \cdot \left(\frac{x}{50}\right)^{10} + \frac{5 \cdot 5^2}{4000} \cdot \left(\frac{x}{50}\right)^{20} - 5500 \cdot \left(\frac{x}{100}\right)^{40}$$

x=1,2,..96; age = x-1=0,1,2,…95; dx = number of deaths at age x; dx(97)<0;

$0.11 \cdot \left(156+20 \cdot x-x^2\right)^{1.5}$: x=1,2 …26 (The expression is imaginary for x > 26). This term models the low number of deaths during youth.

The formula in Young's article contains a printing error, as there is a 1 in the numerator of the third term (see also Peacock (1855), footnote p. 372)

The formula consists of 6 terms or components (1-6). Figure 2 shows the curve of the death numbers. The first three terms reflect the trend up to about age 50, while the last three terms model the decline of the number of deceased. (see also Fig. 3). The results for each formula term and the total number of deaths as a function of age x can be found in Table 2 in Appendix A. It should be noted that Young's table contains several printing and calculation errors. The above formula values match many of the values reported by Young (such as the first value). Most deviations are between 1 and 2. Higher deviations are rare, except for the sixth component in the age group above 90, where Young's values are up to 32 deaths lower (apart from age 96).

Using deaths (from age 90 with smoothed values not further explained by him), Young calculated his life table with l(0)=100003 up to a maximum age of 114 (see Young 1826, p. 297). The life table calculated using the correct formula values ends at age 95 (see Table 1 in Appendix A).

As a methodological critique, it should be noted that, besides the large number of parameters to be estimated, creating a life table based on deaths is only possible in a stationary population. Positive population growth would lead to an underestimation of life expectancy or an overestimation of mortality.

[4] Lexis (1877) and Pearson (1897) also applied analytical functions to measure the number of deaths as a function of age. Lexis used the normal distribution for those who died in adulthood.

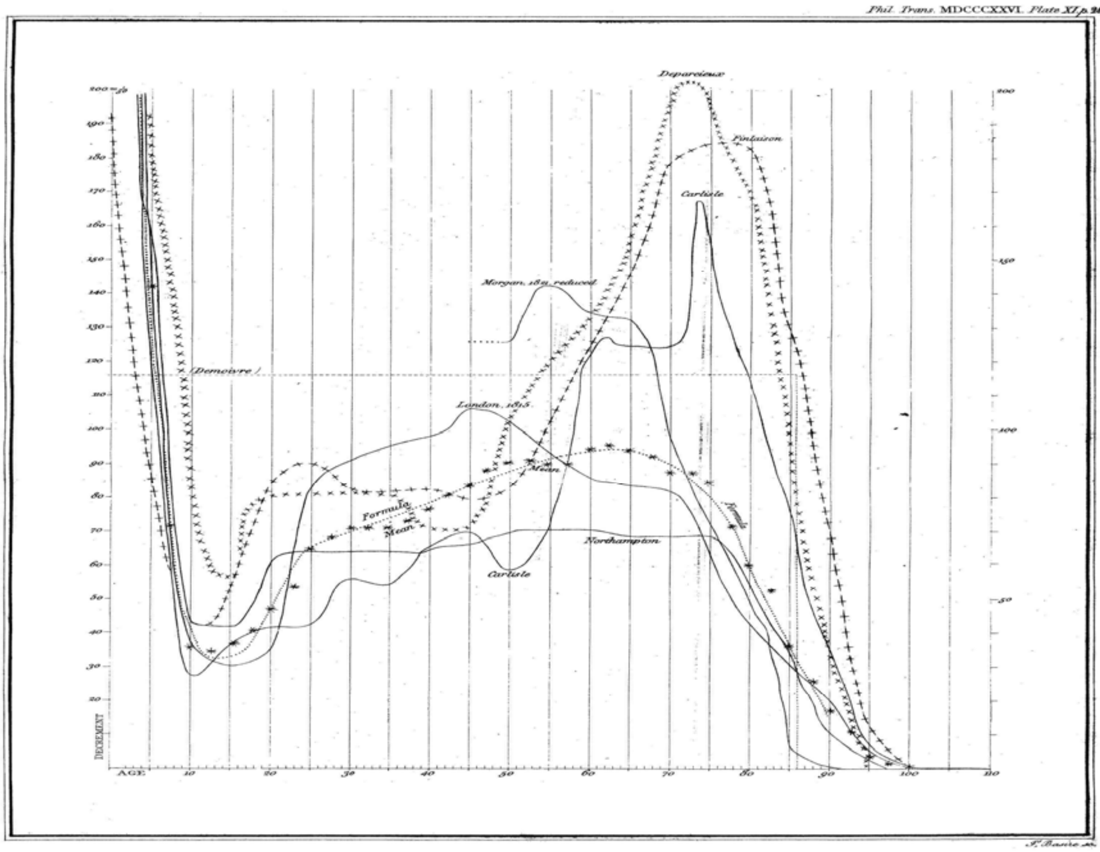

Fig. 1: Decrement Tables and Young´s Graphical Presentation of his Formula
Remarks: with life Tables of: de Moivre, Finlaison, Carlisle, Northampton, Deparcieux, Morgan, and London Equity

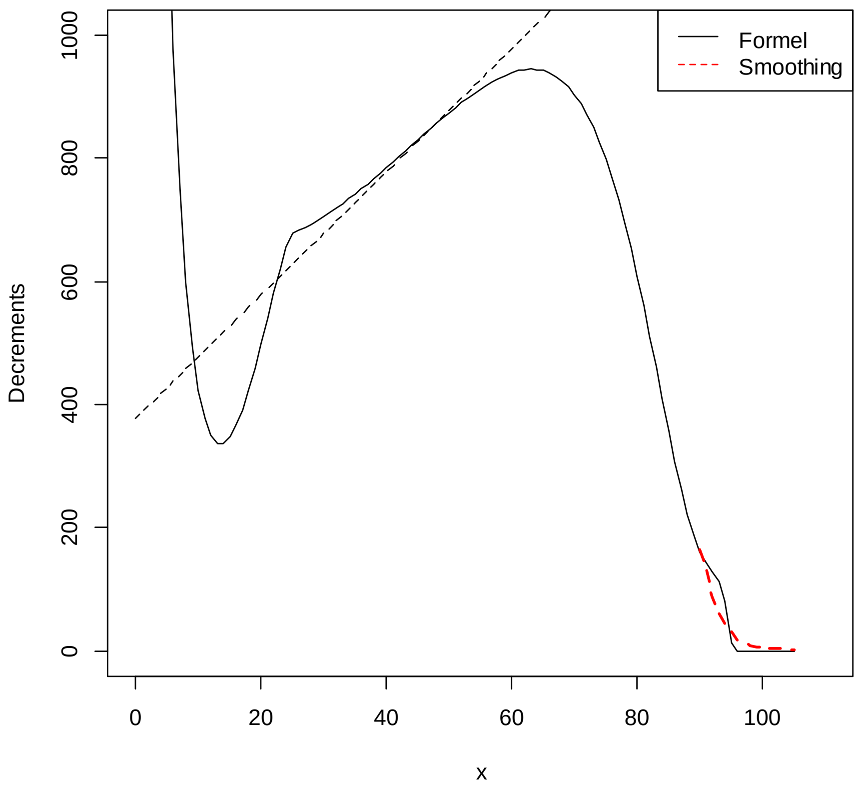

Fig. 2: Mortality Function of Young (1826)
Remarks: Deaths at age x=0: 20532, x=1: 9145, x=2: 4765, x=3: 2853, x=4: 1879; the influence of the first three components of the formula is indicated by the dashed line.

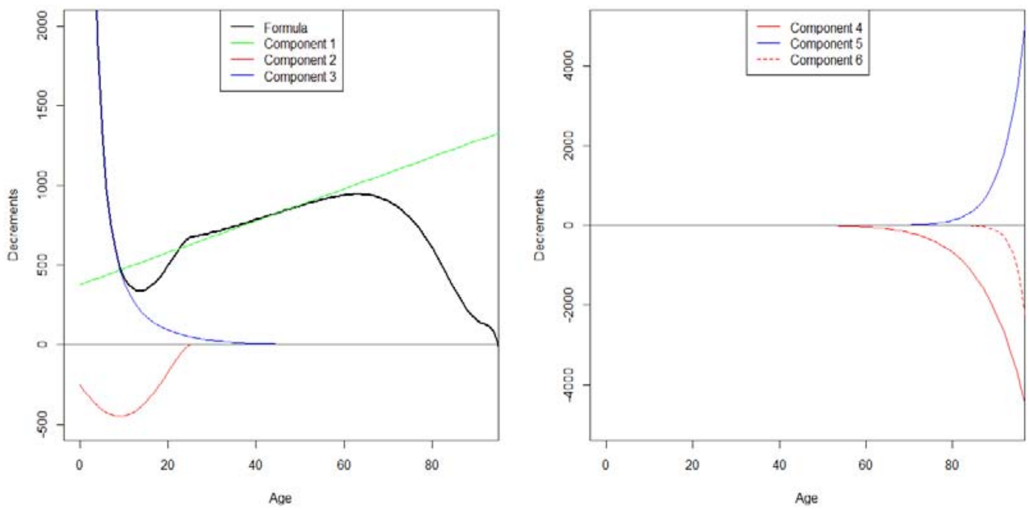

Fig. 3: Terms or Components of Young´s Formula

3. Fitting the Lazarus Distribution

The Gompertz and Makeham laws are partial models, as they do not apply to mortality tables with high mortality in early ages. We now turn to a mortality table proposed by Lazarus (1867). Wilhelm Lazarus (1825-1890) was an actuary in Hamburg and Trieste. His mortality table model is a general mortality law that applies to all age groups (see also Pflaumer, 2015).

Survivor function:

$$l(x) = \exp\left(\frac{A}{k} - \frac{A}{k}\cdot e^{kx} - \frac{B}{g} + \frac{B}{g}\cdot e^{-gx} - C\cdot x\right)$$

Force of mortality function:

$$\mu(x) = B \cdot e^{-g\cdot x} + C + A \cdot e^{k\cdot x}$$

Density of the Lazarus Distribution:

$$-\frac{dl(x)}{dx} = l(x)\cdot\mu(x) = \exp\left(\frac{A}{k} - \frac{A}{k}\cdot e^{kx} - \frac{B}{g} + \frac{B}{g}\cdot e^{-gx} - C\cdot x\right)\cdot\left(B\cdot e^{-g\cdot x} + C + A\cdot e^{k\cdot x}\right)$$

The formula consists of three parts and five parameters, covering the entire age range. The first part represents child mortality, which decreases sharply after birth. The second part describes age-independent mortality, and the third part is the Gompertz law with increasing mortality. This model was also proposed by Siler (1979) and applied to primates. An extension can be found in Thiele (1871), where C is replaced by an age-dependent function C(x). Special cases of the Lazarus model are the Gompertz formula (C = 0, B = 0), the Makeham formula (B = 0), and the Gauss mortality formula[5] (C = 0). This special form of the hazard function is called the bathtub curve in reliability engineering, as it consists of three parts: decreasing, constant, and increasing failure rates. The name comes from the cross-sectional shape of a bathtub.

Fitting the Young life table (1826) with the Lazarus distribution using non-linear least squares with R yields (see also Figs. 4 and 5):

Parameter	Estimate	Std. Error	t-value
A	0.00168	0.0001	12.3
B	0.29385	0.0054	54.6
C	0.00534	0.0004	14.0
g	0.60826	0.0139	43.8
k	0.05285	0.0012	44.3

[5] See, e.g, Pflaumer (2013).

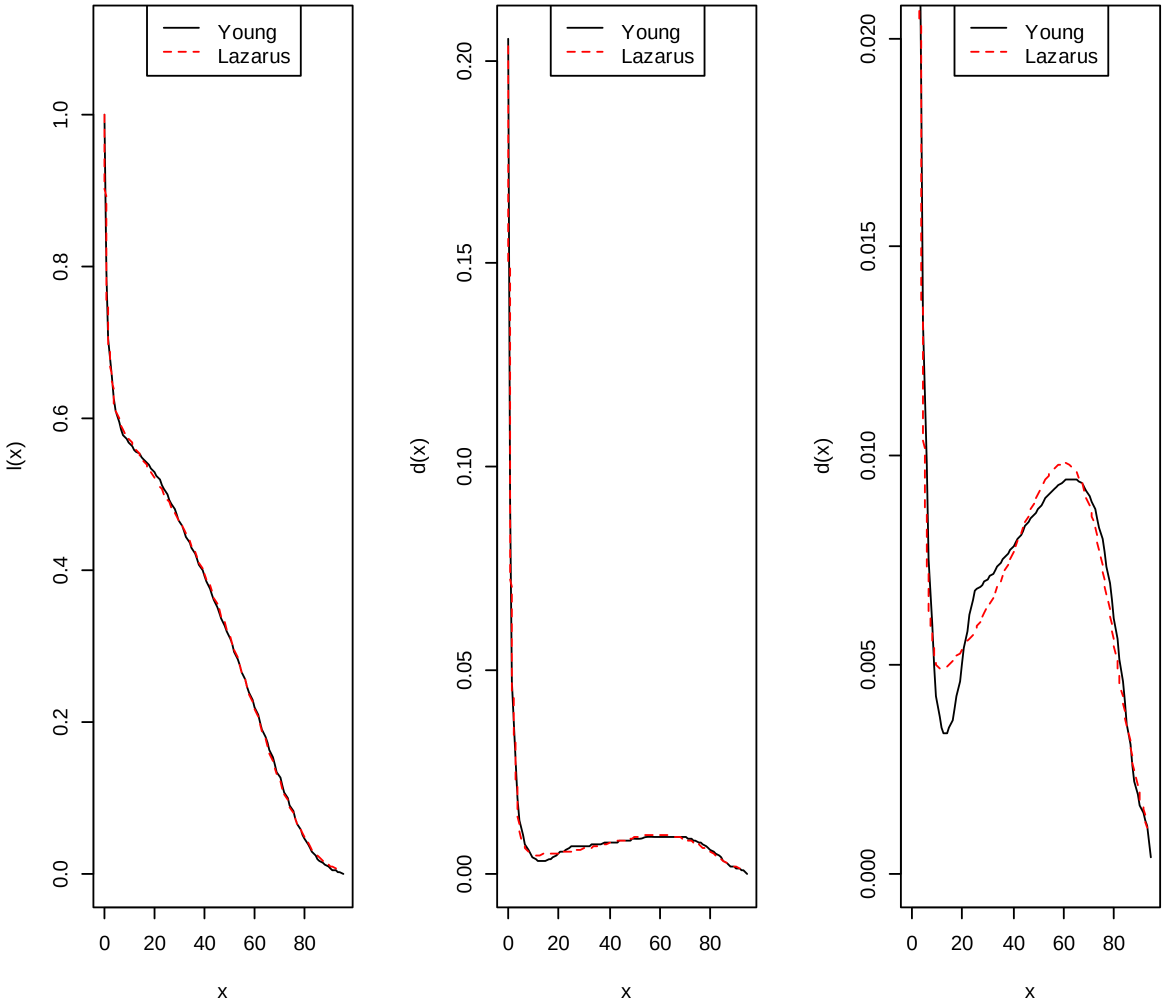

Fig. 4: Fitting Young´s Life Table with the Lazarus Distribution
l(x) und d(x)=l(x)-l(x+1)

Important parameters can only be determined numerically.

The following results were obtained: the normal death age (or modal death age) is 60.6 (compared to 63 in Young's table), life expectancy $\overset{0}{e}(x) = \int_0^\omega l(x)dx$ =30.2 is the same as in Young's table, the average age of the stationary population is $\mu_S = \frac{\int_0^\omega x \cdot l(x)dx}{\int_0^\omega l(x)dx} = 29.2$.

Rectangularization indices are $g = \frac{\overset{0}{e}(x)}{2 \cdot \mu_S} = \frac{30.2}{2 \cdot 29.2} = 0.517$ (Gumbel coefficient),

$$H = -\frac{\int_0^{\omega} l(x) \cdot \ln\left(l(x)\right) dx}{\int_0^{\omega} l(x) dx} = -\frac{\text{-26.4}}{30.2} = 0.874 \text{ (Keyfitz Entropy)},$$

and Gini coefficient $R = \frac{1}{e(0)} \int_0^{\omega} \left(l(x) - l(x)^2 \right) dx = 0.54$.

The minimum of the death density function is at the age of 12.4 (compared to 13.5 in Young's table). The minimum age of death probability function is 11.5 years.

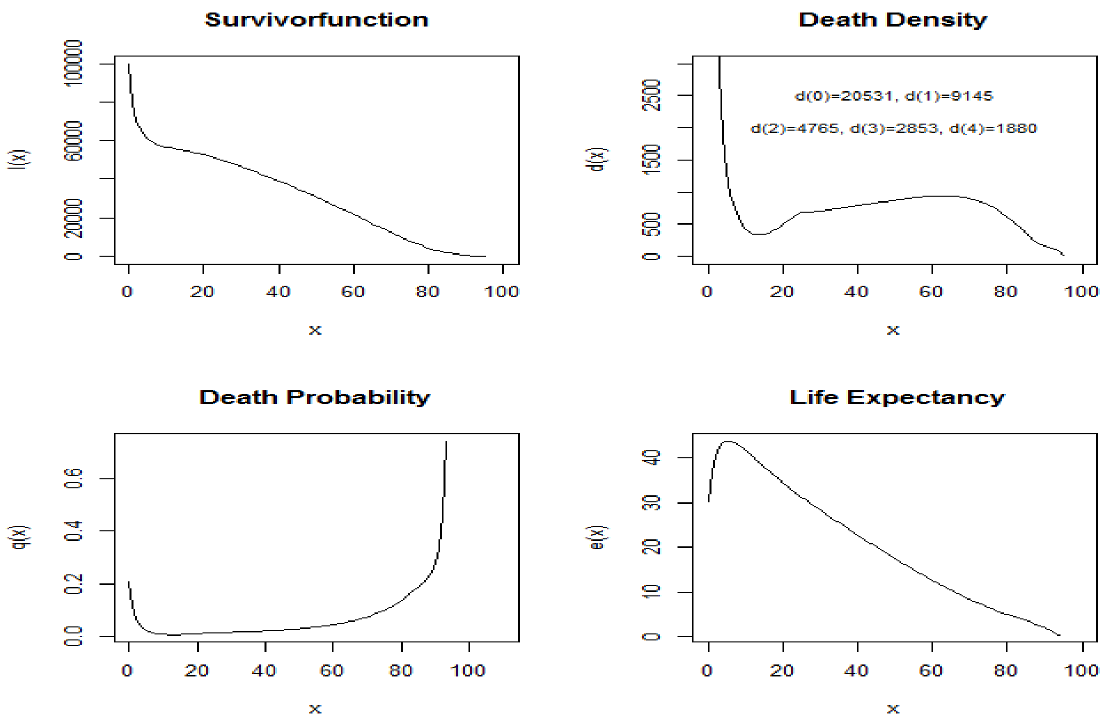

Fig. 5: Important Life Table Function of Young´s Formula

In addition to life expectancies at age x, he also calculated present values for annuities using this formula (Young, 1828)

$$a(x) = \frac{2 \cdot \omega^2}{3 \cdot (\omega + x)} - \frac{x}{3} \quad \text{with } \omega\text{: maximum age.}$$

Young (1828) modified the de Moivre formula $l(x) = 1 - \frac{x}{\omega}$ in 1829 (see also Peacock, 1855, pp. 392 ff.) by

$$l(x) = 1 - \frac{x^2}{\omega^2}.$$

Compared to other historical life tables, Young´s table is characterized by relatively high mortality (see Fig. 6). The life expectancies are: Zillmer: 41.1; Germany (males): 35.6; Halley: 33.4; Young: 30.2; Süßmilch: 28.5; Russia (males): 26.4.

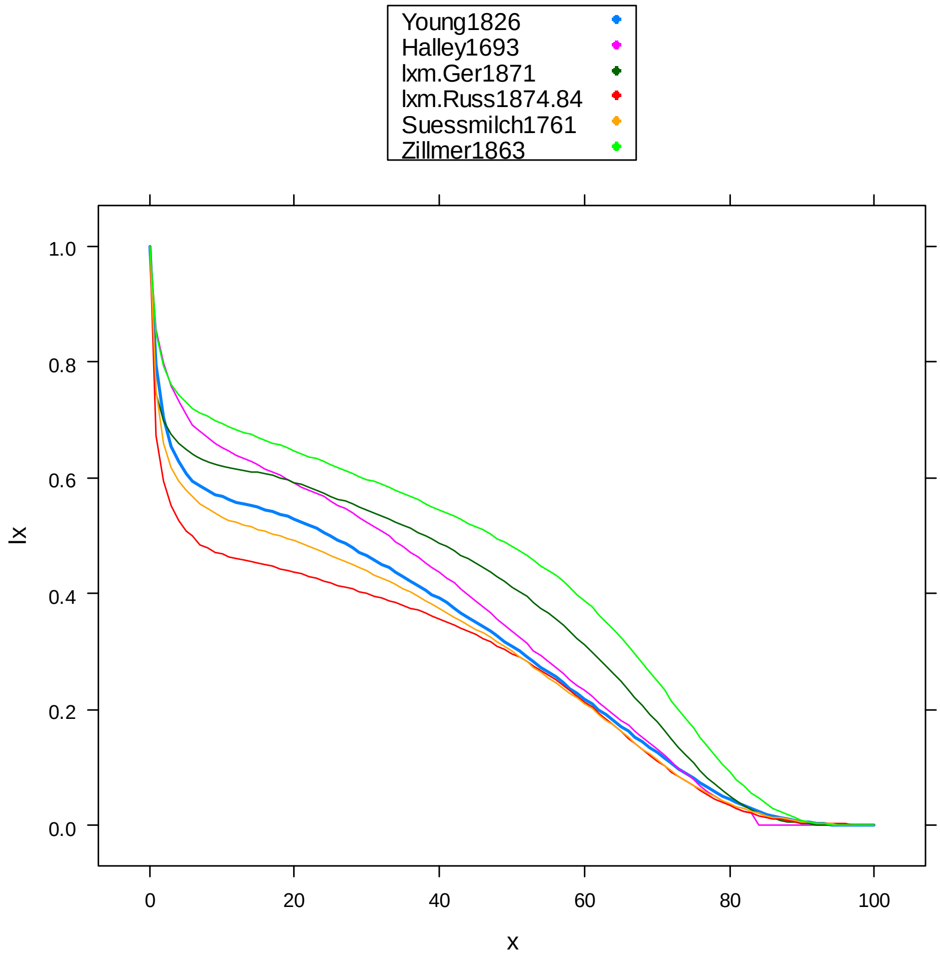

Fig. 6: Young´s Life Table compared to other Life Tables from the 18th and 19th Centuries

4. Young´s Life Table as a Model Life Table

In the broadest sense, model life tables encompass all analytical functions that describe mortality patterns. In a narrower sense, model life tables only refer to standard tables that are based on real data and are limited in time and region; this is usually the interpretation understood as a model life table. An early example of a model life table can be considered the life table by Young (1826). The most well-known example is the model life tables by Coale and Demeny (1966). The purpose of a model life table is to obtain complete life tables with incomplete information (for example, if only life expectancy during a specific period in a region is known).

Coale Demeny Model Life Table

MODEL WEST
LIFE TABLES
LEVEL 5
FEMALES

x	lx	ndx	nqx	ex
0	1.000	0.256	0.256	30.0
1	0.744	0.132	0.178	39.2
5	0.612	0.031	0.050	43.4
10	0.581	0.023	0.039	40.6
15	0.558	0.029	0.051	37.1
..				
70	0.120	0.051	0.424	7.1
75	0.069	0.038	0.557	5.3
80	0.031	0.021	0.696	3.9
85	0.009	0.008	0.837	2.8
90	0.002	0.001	0.938	1.9
95	0.000	0.000	1.000	1.3

The following comparison in Fig. 7 shows that Young's life table corresponds to a specific Coale and Demeny life table, namely the table (MODEL WEST, LEVEL 5, FEMALES) with a life expectancy at birth of 30 years.

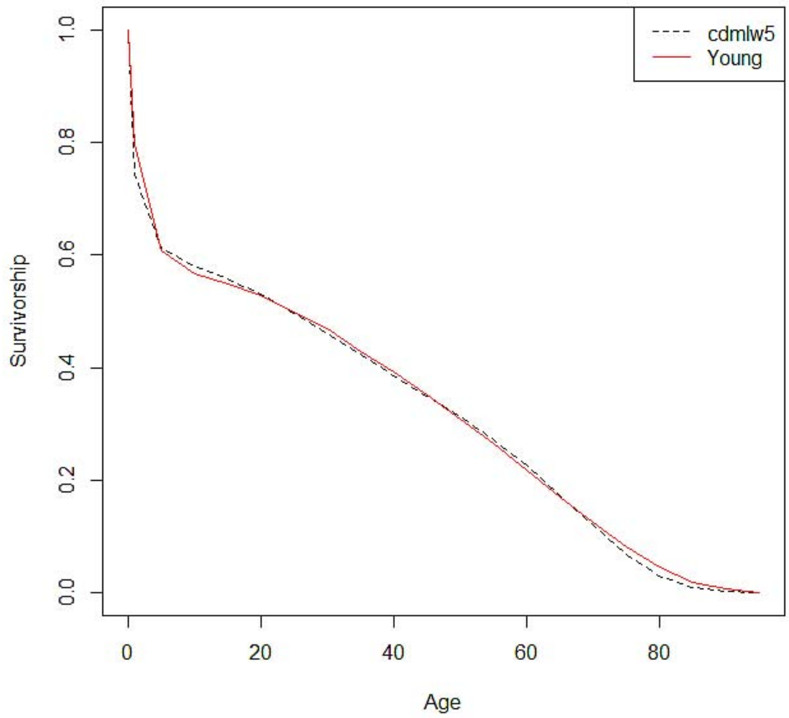

Fig. 7: Young´s Life Table compared to a Model Life Table (MODEL WEST LIFE TABLE, LEVEL 5, FEMALES) (cdmlw5)

5. Young´s Formula of 1816

A predecessor of Young's mortality formula from 1826 was a simpler formula by Young that appeared in the Philosophical Magazine in 1816. This formula, titled „An Algebraical Expression for the Value of Lives”, was later reprinted in Peacock 1855 (pp. 359 ff.).
The complete formula, which represents a death density function, describes the yearly number of deaths dx at age x as follows:

$$dx = \frac{1}{4} \cdot \frac{1}{1+x^2} + 0.000401 \cdot x - 0.0000042 \cdot x^2 \qquad \text{for } 0 \le x \le 95.544 .$$

This formula, often referred to as the „mortality formula of Thomas Young from 1816”, is a simplified mathematical representation of age-specific mortality rates. It does not enjoy wide recognition or extensive citation in modern academic literature or contemporary mortality modeling. Instead, it holds more historical significance.

The formula comprises three terms, each contributing to the overall mortality rate. These terms represent distinct factors influencing mortality across different age groups:

The first term, $\frac{1}{4} \cdot \frac{1}{1+x^2}$, is associated with child mortality and signifies a decreasing mortality rate as age increases.
The second term, $0.000401 \cdot x$, indicates a linear increase in mortality with age.
The third term, $-0.0000042 \cdot x^2$, accounts for a quadratic decrease in mortality with age, capturing patterns of declining mortality at older ages.

In Figure 8, the blue lines represent Young's formula of 1816, while the magenta lines represent Young's formula of 1826. In both cases, the total number of deaths is fixed at 100,003 to facilitate a graphical comparison of the two formulas. The key distinction lies in the formula of 1816, where the minimum number of deaths occurring during youth is higher (1816: x=11.8 years), and the modal age of adults is lower (1816: x=47.2 years). These differences are clearly visible in Figure 8, where both death curves are characterized by high child mortality, with the formula of 1816 exhibiting a slightly higher child mortality rate.

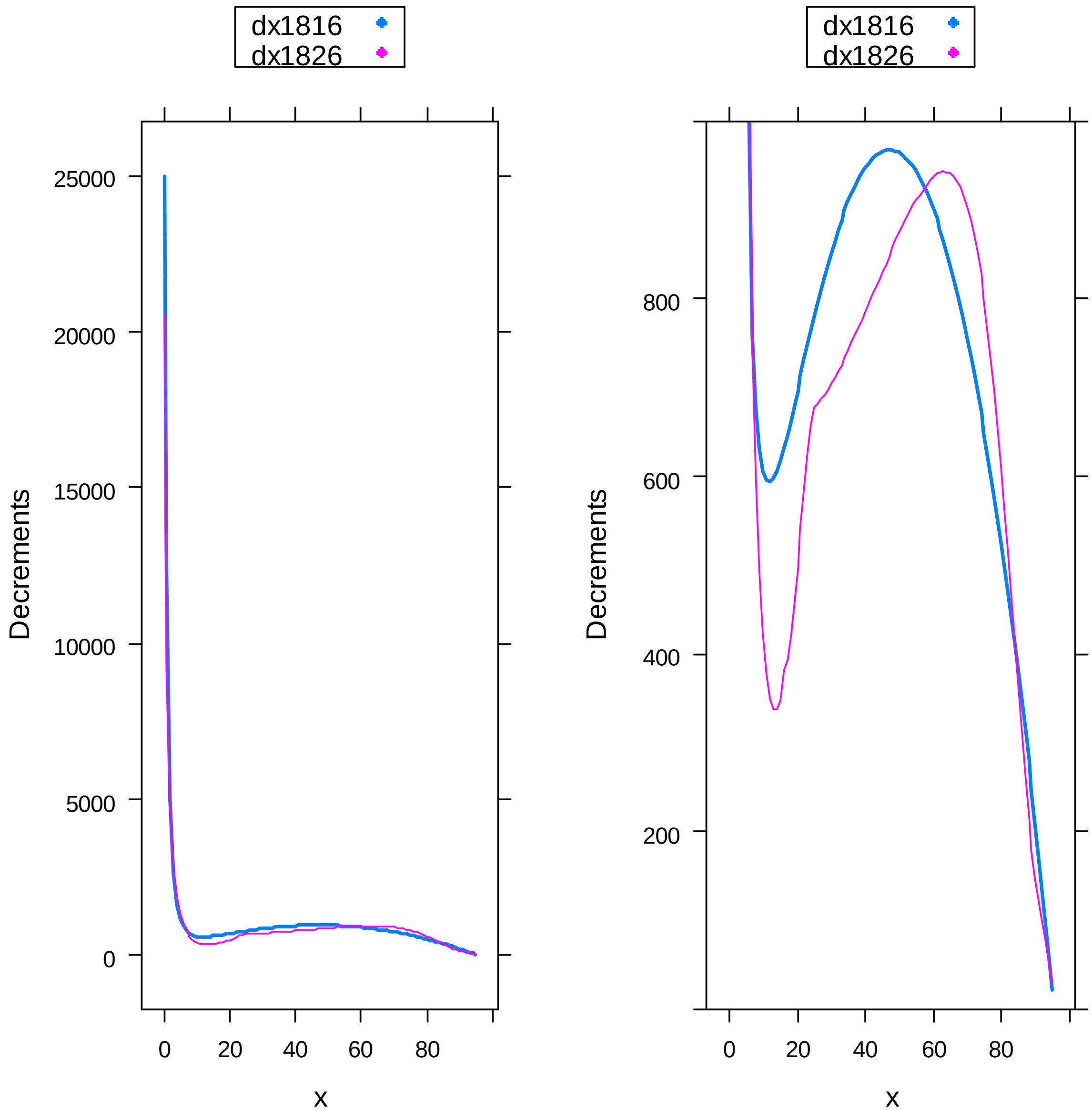

Fig. 8: Comparing the Mortality Functions of 1826 and 1816, each with a Total of 100,003 Deaths

By integrating the death density function, we obtain the distribution function

$$F(x) = \frac{\arctan(x)}{4} - \frac{7 \cdot x^3}{5000000} + \frac{401 \cdot x^2}{2000000},$$

and correspondingly, the survivor function l(x) defined as l(x) = 1 - F(x):

$$l(x) = \frac{14 \cdot x^3 - 2005 \cdot x^2 + 10000000}{10000000} - \frac{\arctan(x)}{4}.$$

This relationship was also examined by Young in 1816 (p. 360 in Peackock, 1855), who compared the formula's values with the recorded deaths in London in 1815, demonstrating a strong fit between the registered deaths and the deaths calculated using the formula.

Similar to our calculations, Thomas Young in 1816 calculated a life expectancy at birth of more than 30 years, with a probable or median value of approximately 27 years. Additionally, he computed life expectancies at various ages, including 1, 5, 10, 20, and so on, and presented a comparative table with these values in contrast to Halley's life table.

In Figure 9, essential life table functions have been graphically depicted based on their corresponding formulas, which are

- life expectancy $e(x) = \frac{\int_{x}^{95.544} l(x)}{l(x)}$
- force of mortality $\mu(x) = -\frac{\frac{dl(x)}{l(x)}}{l(x)}$.

The life expectancy in Young's analysis exhibits the typical form of an 18th or 19th-century life table, where, after high infant mortality, the life expectancy increases up to a maximum and then falls due to the increasing force of mortality. The force of mortality itself shows a distinctive 'bathtub' shape, characterized by a high force of mortality at both low and high ages, and a lower force of mortality at intermediate ages."

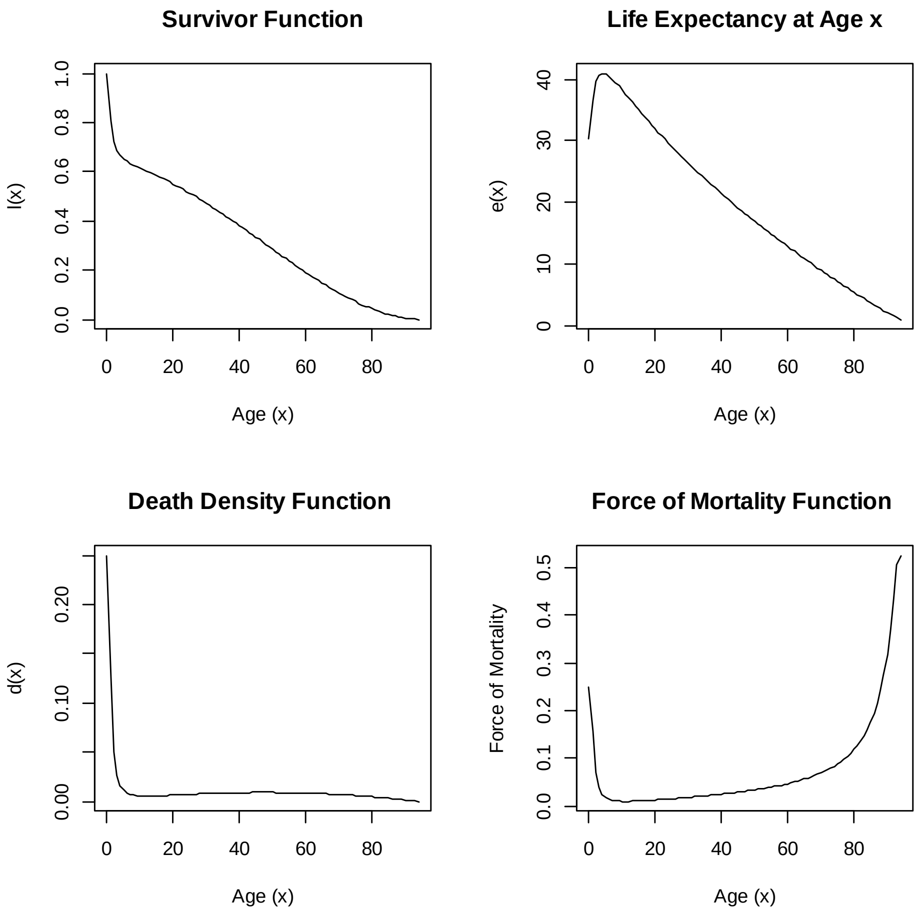

Fig. 9: Important Life Table Functions of Young´s Mortality Formula of 1816

Young's life table of 1816, characterized by its simplicity with just three terms, serves as a foundational stepping stone in mortality modeling. This mathematical life table, beginning with l(0) = 1, provides a clear and intuitive framework that simplifies the understanding of the more complex life table of 1826. By employing techniques of integration and differentiation, this concise model allows us to calculate various vital functions, including life expectancy at age x and the force of mortality, effectively demonstrating the power of mathematical tools in demography and actuarial science (see also Appendix C).

6. Conclusion

Thomas Young (1773-1829) was a brilliant scholar whose contributions spanned across many fields. In addition to his well-known work in medicine, physics, anthropology, and ancient history, Young also developed a formula for the law of mortality. While his formula has been criticized for certain methodological issues, it was an early and noteworthy method for creating a model life table from empirical data. Young's formula also corresponds to a particular life table of Coale and Demeny. As such, Young's contributions to demography should be recognized and appreciated. His life table has practical relevance for historians and demographers who need a suitable and complete life table of England at the beginning of the 19th century, making Young's work an important source for demographic research.

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Appendix A

Table 1: Life table (lx: formula values; lxYoung: Young´s values)

x	lx	lxYoung	x	lx	lxYoung	x	lx	lxYoung
0	100000	100003	33	44407	44391	66	16171	16158
1	79468	79472	34	43681	43665	67	15232	15219
2	70323	70366	35	42947	42931	68	14299	14286
3	65559	65586	36	42204	42189	69	13374	13360
4	62705	62732	37	41454	41438	70	12458	12445
5	60826	60852	38	40695	40679	71	11555	11542
6	59504	59511	39	39928	39911	72	10666	10654
7	58527	58532	40	39152	39135	73	9795	9783
8	57776	57780	41	38367	38350	74	8945	8933
9	57178	57177	42	37573	37555	75	8119	8107
10	56684	56683	43	36770	36751	76	7320	7306
11	56260	56260	44	35958	35938	77	6552	6538
12	55884	55883	45	35137	35117	78	5818	5805
13	55534	55534	46	34307	34286	79	5123	5108
14	55197	55197	47	33469	33447	80	4469	4454
15	54860	54860	48	32621	32599	81	3859	3844
16	54512	54513	49	31764	31742	82	3297	3285
17	54147	54132	50	30899	30876	83	2785	2772
18	53755	53739	51	30025	30002	84	2325	2312
19	53332	53317	52	29143	29120	85	1916	1904
20	52873	52859	53	28253	28230	86	1559	1547
21	52375	52362	54	27355	27332	87	1252	1240
22	51836	51822	55	26450	26426	88	990	982
23	51255	51241	56	25537	25513	89	769	767
24	50634	50620	57	24618	24596	90	582	589
25	49979	49964	58	23693	23673	91	421	425
26	49301	49286	59	22762	22744	92	279	295
27	48619	48604	60	21827	21810	93	151	208
28	47932	47917	61	20888	20872	94	39	148
29	47240	47225	62	19946	19930	95	0	104
30	46542	46527	63	19002	18987	96		73
31	45837	45822	64	18057	18043	…..		
32	45125	45110	65	17113	17100	114		0

Table 2: Formula values of the decrement table and its terms

x	age x-1	dx	dxYoung	deviation	A	B	C	D	E	F
1	0	20532	20531	1	378	255	20408	0.0	0	0
2	1	9145	9106	39	388	293	9050	0.0	0	0
3	2	4765	4780	-15	398	328	4695	0.0	0	0
4	3	2853	2854	-1	408	359	2804	0.0	0	0
5	4	1879	1880	-1	418	386	1847	0.0	0	0
6	5	1322	1341	-19	428	409	1303	0.0	0	0
7	6	976.84	979	-2	438	427	966	0.0	0	0
8	7	751.04	752	-1	448	440	743	0.0	0	0
9	8	598.44	603	-5	458	448	588	0.0	0	0
10	9	493.97	494	0	468	451	477	0.0	0	0
11	10	423.09	423	0	478	448	393	0.0	0	0
12	11	376.88	377	0	488	440	329	0.0	0	0
13	12	349.58	349	1	498	427	279	0.0	0	0
14	13	337.27	337	0	508	409	238	0.0	0	0
15	14	337.19	337	0	518	386	205	0.0	0	0
16	15	347.24	347	0	528	359	178	0.0	0	0
17	16	365.78	381	-15	538	328	155	0.0	0	0
18	17	391.39	393	-2	548	293	136	0.0	0	0
19	18	422.82	422	1	558	255	119	0.0	0	0
20	19	458.84	458	1	568	214	105	0.0	0	0
21	20	498.18	497	1	578	173	93	0.0	0	0
22	21	539.46	540	-1	588	130	82	0.0	0	0
23	22	581.02	581	0	598	89	72	0.0	0	0
24	23	620.74	621	0	608	51	64	0.0	0	0
25	24	655.43	656	-1	618	19	56	0.0	0	0
26	25	677.83	678	0	628	0	50	0.0	0	0
27	26	682.00	682	0	638	0	44	0.0	0	0
28	27	686.84	687	0	648	0	39	0.0	0	0
29	28	692.26	692	0	658	0	34	0.0	0	0
30	29	698.22	698	0	668	0	30	0.0	0	0
31	30	704.64	705	0	678	0	27	0.0	0	0
32	31	711.47	712	-1	688	0	24	0.1	0	0
33	32	718.67	719	0	698	0	21	0.1	0	0
34	33	726.19	726	0	708	0	18	0.1	0	0
35	34	734.00	734	0	718	0	16	0.2	0	0
36	35	742.05	742	0	728	0	14	0.2	0	0
37	36	750.32	751	-1	738	0	13	0.3	0	0
38	37	758.78	759	0	748	0	11	0.4	0	0
39	38	767.39	768	-1	758	0	10	0.5	0	0
40	39	776.12	776	0	768	0	9	0.6	0	0
41	40	784.97	785	0	778	0	8	0.8	0	0
42	41	793.89	795	-1	788	0	7	1.0	0	0
43	42	802.87	804	-1	798	0	6	1.2	0	0
44	43	811.88	813	-1	808	0	5	1.5	0	0
45	44	820.90	821	0	818	0	5	1.9	0	0
46	45	829.91	831	-1	828	0	4	2.4	0	0
47	46	838.87	839	0	838	0	4	3.0	0	0

48	47	847.77	848	0	848	0	3	3.7	0	0
49	48	856.58	857	0	858	0	3	4.5	0	0
50	49	865.26	866	-1	868	0	3	5.5	0	0
51	50	873.77	874	0	878	0	2	6.7	0	0
52	51	882.09	882	0	888	0	2	8.1	0	0
53	52	890.17	890	0	898	0	2	9.8	0	0
54	53	897.96	898	0	908	0	2	11.9	0	0
55	54	905.41	906	-1	918	0	2	14.3	0	0
56	55	912.46	913	-1	928	0	1	17.1	0	0
57	56	919.04	917	2	938	0	1	20.4	0	0
58	57	925.09	923	2	948	0	1	24.3	0	0
59	58	930.51	929	2	958	0	1	28.8	0	0
60	59	935.23	934	1	968	0	1	34.1	0	0
61	60	939.13	938	1	978	0	1	40.2	0	0
62	61	942.11	942	0	988	0	1	47.3	1	0
63	62	944.05	943	1	998	0	1	55.5	1	0
64	63	944.81	944	1	1008	0	1	64.9	1	0
65	64	944.24	943	1	1018	0	1	75.8	1	0
66	65	942.20	942	0	1028	0	1	88.3	2	0
67	66	938.50	939	-1	1038	0	1	102.7	3	0
68	67	932.97	933	0	1048	0	0	119.1	4	0
69	68	925.42	926	-1	1058	0	0	137.8	5	0
70	69	915.64	915	1	1068	0	0	159.1	6	0
71	70	903.44	903	0	1078	0	0	183.3	8	0
72	71	888.59	888	1	1088	0	0	210.9	11	0
73	72	870.90	871	0	1098	0	0	242.0	15	0
74	73	850.17	850	0	1108	0	0	277.3	19	0
75	74	826.21	826	0	1118	0	0	317.2	25	0
76	75	798.86	801	-2	1128	0	0	362.1	33	0
77	76	768.00	768	0	1138	0	0	412.6	43	0
78	77	733.58	733	1	1148	0	0	469.5	55	0
79	78	695.59	697	-1	1158	0	0	533.3	71	0
80	79	654.15	654	0	1168	0	0	604.7	91	1
81	80	609.46	610	-1	1178	0	0	684.7	117	1
82	81	561.90	559	3	1188	0	0	774.1	150	2
83	82	512.01	513	-1	1198	0	0	873.9	191	3
84	83	460.53	460	1	1208	0	0	985.0	243	5
85	84	408.43	408	0	1218	0	0	1108.8	307	8
86	85	356.92	357	0	1228	0	0	1246.4	388	13
87	86	307.43	307	0	1238	0	0	1399.1	489	21
88	87	261.56	258	4	1248	0	0	1568.5	615	33
89	88	220.97	215	6	1258	0	0	1756.2	771	52
90	89	187.13	178	9	1268	0	0	1963.8	964	81
91	90	160.94	148	13	1278	0	0	2193.2	1203	126
92	91	142.09	125	17	1288	0	0	2446.5	1496	196
93	92	128.00	101	27	1298	0	0	2725.8	1857	302
94	93	112.20	80	32	1308	0	0	3033.5	2301	463
95	94	81.91	53	29	1318	0	0	3372.1	2843	707
96	95	14.23	27	-13	1328	0	0	3744.3	3505	1075
97	96	-129.32	0	-129.32	1338	1	0	4153.2	4312	1626

Table 3: MODEL WEST LIFE TABLE, LEVEL 5, FEMALES
(Coale Demeny)

x	lx	ndx	nqx	ex
0	1.000	0.256	0.256	30.0
1	0.744	0.132	0.178	39.2
5	0.612	0.031	0.050	43.4
10	0.581	0.023	0.039	40.6
15	0.558	0.029	0.051	37.1
20	0.529	0.034	0.064	34.0
25	0.496	0.036	0.072	31.1
30	0.460	0.037	0.081	28.3
35	0.423	0.037	0.089	25.6
40	0.385	0.037	0.095	22.8
45	0.349	0.036	0.102	20.0
50	0.313	0.041	0.131	16.9
55	0.272	0.045	0.166	14.1
60	0.227	0.054	0.237	11.4
65	0.173	0.053	0.309	9.1
70	0.120	0.051	0.424	7.1
75	0.069	0.038	0.557	5.3
80	0.031	0.021	0.696	3.9
85	0.009	0.008	0.837	2.8
90	0.002	0.001	0.938	1.9
95	0.000	0.000	1.000	1.3

Appendix B: Tables from Young (1826)

Comparative Decrements from various Tables.

Age.	North-ampton.	Carlisle.	Equitable Office Red.	Mean of Carlisle and Eq. Office.	London Bills.	General Mean.	Living.
0	25751	15390	—	—	17301	19481	99124
1	11734	6820			10493	9682	79643
2	4309	5050			4460	4606	69961
3	2876	2760			3148	2928	65355
4	1691	2010			2242	1981	62327
5	1579	1210	—	—	1469	1419	60346
6	1202	820			945	989	58927
7	944	580			725	750	57938
8	687	430			529	549	57188
9	515	330			441	429	56549
10	446	290	—	—	389	375	56120
11	429	310			346	362	55745
12	429	320			323	357	55383
13	429	330			318	359	55026
14	429	350			315	365	54667

Comparative Decrements from various Tables.

Age.	Northampton.	Carlisle.	Equitable Office Red.	Mean of Carlisle and Equi. Office.	London Bills.	General Mean.	Living.
15	429	390	—	—	317	379	54302
16	455	420			320	398	53923
17	497	430			325	417	53525
18	541	430			335	435	53108
19	575	430			352	452	52673
20	618	430	—	—	372	473	52221
21	644	420			404	489	51748
22	644	420			503	522	51259
23	644	420			608	557	50737
24	644	420			766	610	50180
25	644	430	—	—	882	652	49570
26	644	430			892	655	48918
27	644	450			897	664	48263
28	644	500			902	682	47599
29	644	560			907	704	46917
30	644	570	—	—	913	709	46213
31	644	570			919	711	45504
32	644	560			925	710	44793
33	644	550			931	708	44083
34	644	550			937	710	43375
35	644	550	—	—	943	712	42665
36	644	560			950	718	41953
37	644	570			955	723	41235
38	644	580			961	728	40512
39	644	610			967	740	39784
40	652	660	—	—	974	762	39044
41	661	690			990	780	38282
42	669	710			1010	796	37502
43	669	710			1030	803	36706
44	669	710			1044	808	35903
45	669	700	1346	(765)	1055	830	35095
46	669	690	1346	(821)	1059	850	34265
47	669	670	1346	(873)	1059	867	33415

Decrements of Mortality computed from the Formula.

Age $(x-1)$	$368 + 10x$	$-.11\,(156+20x - xx)^{\frac{3}{2}}$	$+\dfrac{1}{2.85+2.05\,xx+2\left(\frac{x}{10}\right)^{6}}$	Decrement.
0	378	−255	+ 20408	20531
1	388	241	9009	9106
2	398	313	4695	4780
3	408	359	2805	2854
4	418	386	1848	1880
5	428	409	1322	1341
6	438	427	968	979
7	448	440	746	752
8	458	447	592	603
9	468	451	477	494
10	478	447	392	423
11	488	440	329	377
12	498	427	278	349
13	508	409	238	337
14	518	386	205	337

MEAN STANDARD TABLE OF THE DECREMENTS OF LIFE IN GREAT BRITAIN, 1824.

Age.	Decrement.	Living.	Age.	Decrement.	Living.	Age.	Decrement.	Living.	Age.	Decrement.	Living.
0	20531	100003	30	705	46527	60	938	21810	90	164	589
1	9106	79472	31	712	45822	61	942	20872	91	130	425
2	4780	70366	32	719	45110	62	943	19930	92	87	295
3	2854	65586	33	726	44391	63	944	18987	93	60	208
4	1880	62732	34	734	43665	64	943	18043	94	44	148
5	1341	60852	35	742	42931	65	942	17100	95	31	104
6	979	59511	36	751	42189	66	939	16158	96	19	73
7	752	58532	37	759	41438	67	933	15219	97	14	54
8	603	57780	38	768	40679	68	926	14286	98	9	40
9	494	57177	39	776	39911	69	915	13360	99	6	31
10	423	56683	40	785	39135	70	903	12445	100	6	25
11	377	56260	41	795	38350	71	888	11542	101	5	19
12	349	55883	42	804	37555	72	871	10654	102	5	14
13	337	55534	43	813	36751	73	850	9783	103	4	9
14	337	55197	44	821	35938	74	826	8933	104	2	5
15	347	54860	45	831	35117	75	801	8107	105	1	3
16	381	54513	46	839	34286	76	768	7306	106	.25	2
17	393	54132	47	848	33447	77	733	6538	107	.25	1.75
18	422	53739	48	857	32599	78	697	5805	108	.25	1.50
19	458	53317	49	866	31742	79	654	5108	109	.25	1.25
20	497	52859	50	874	30876	80	610	4454	110	.25	1.0
21	540	52362	51	882	30002	81	559	3844	111	.25	.75
22	581	51822	52	890	29120	82	513	3285	112	.25	.50
23	621	51241	53	898	28230	83	460	2772	113	,25	.25
24	656	50620	54	906	27332	84	408	2312	114	0	0
25	678	49964	55	913	26426	85	357	1904			
26	682	49286	56	917	25513	86	307	1547			
27	687	48604	57	923	24596	87	258	1240			
28	692	47917	58	929	23673	88	215	982			
29	698	47225	59	934	22744	89	178	767			

Appendix C: Applying Young's Formula from 1816 to the Life Table of German Males for the Period 1871-1880

We applied Young's formula from 1816 to the life table of German males for the period 1871-1880:

$$l(x) = \frac{a \cdot x^3 - b \cdot x^2 + 10000000}{10000000} - \frac{\arctan(x)}{c}, \qquad 0 \le x \le \omega .$$

For large values of x, the formula can be expressed as,

$$l(x) = \frac{a \cdot x^3 - b \cdot x^2 + 10000000}{10000000} - \frac{\pi}{2 \cdot c},$$

as the limit of arctan(x) approaches π/2 for increasing x. In our case, the approximation is sufficient when x exceeds 30.

The following Table presents the estimation results, while the Figure A3.1 displays both the actual values (in blue) and the estimated values (in red).

```
Parameters:
   Estimate Std. Error t value
a     6.5048     0.6281   10.36
b 1354.0101    60.6585   22.32
c     4.2789     0.0747   57.28
```

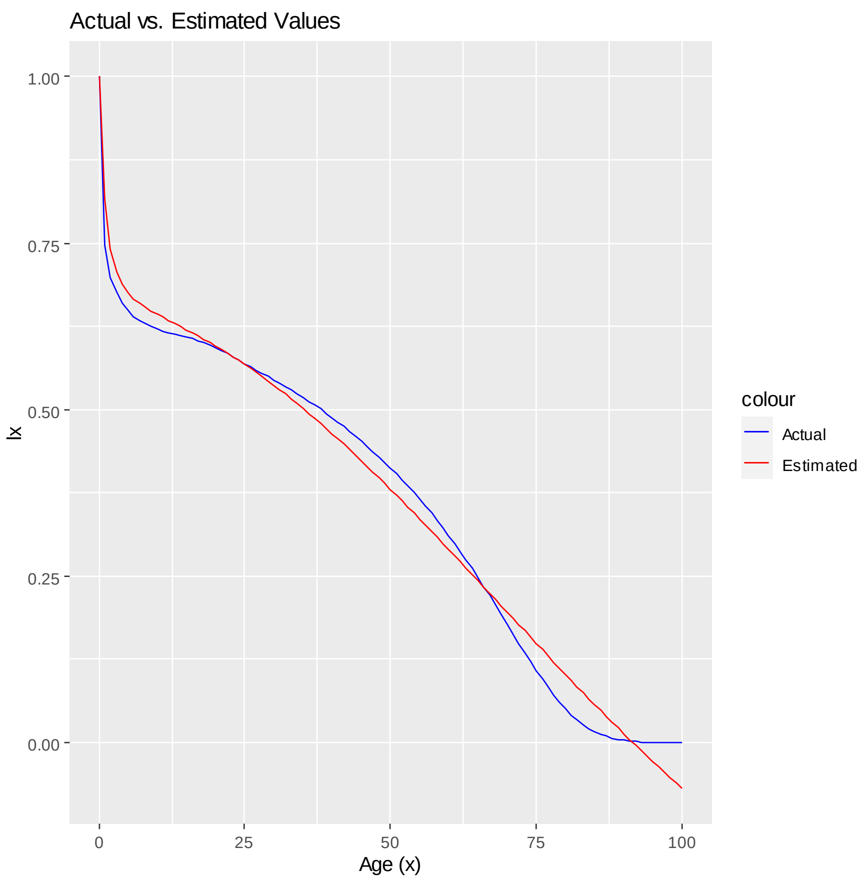

Fig. A3.1: Applying Young's Formula from 1816 to the Life Table of German Males for the Period 1871-1880 ($\omega = 91.5$).

Upon examining the plotted values, several key observations emerge:

1. Lack of Curvature Alignment: The most apparent distinction lies in the failure of the fitted values to precisely replicate the curvature of the actual life table. The genuine life table exhibits a gradual decline in survivorship with advancing age, with a more pronounced decrease among older individuals. In contrast, the fitted values do not faithfully capture this curvature. This suggests that the chosen Young's formula and parameter values may not comprehensively represent the underlying mortality pattern in the dataset.

2. Negative Values at Advanced Ages: Another crucial observation is that the fitted values become negative at older ages. This unmistakably indicates that the selected model and parameterization may not be suitable for modeling mortality at extremely advanced ages. In actual mortality data, survivorship typically diminishes but remains positive even at the highest ages. The appearance of negative values in the fitted life table implies that the model does not behave realistically at extreme ages.

3. Overall Discrepancy: A visual comparison between the actual and fitted values reveals a significant discrepancy between the two. While the fitted values may generally follow a declining trend, they fail to capture the intricacies inherent in the actual data.

These discrepancies, observed in the life tables Young investigated, were likely recognized by him. As a result, he developed a formula with six terms to create a more realistic representation of the life table, a topic that has been extensively discussed in the paper.